2023

PHYSICS

Paper : PHYHE6036/PHYHE6056

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

OPTION—I

Paper : PHYHE6036

(Classical Dynamics)

1. Choose the correct answer (any six) : 1×6=6

(a) Angular momentum of a particle is given by

(i) $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

(ii) $\mathbf{L} = \mathbf{r} \cdot \mathbf{p}$

(iii) $\mathbf{L} = \mathbf{r} + \mathbf{p}$

(iv) $\mathbf{L} = \mathbf{r} - \mathbf{p}$

(b) The total linear momentum of an isolated system is always a _____ quantity.

(i) non-conserved *(ii)* conserved

(iii) scalar *(iv)* variable

(c) d'Alembert's principle is given by

(i) $\sum_{i=1}^{3N} (F_i^a - \dot{p}_i) \cdot \delta r_i = 0$

(ii) $\sum_{i=0}^{N} (F_i^a - p_i) \delta r_i = 0$

(iii) $\sum_{i=1}^{3N} (F_i^a + \dot{p}_i) \cdot \delta r_i = 0$

(iv) $\sum_{i=0}^{N} (F_i^a + \dot{p}_i) \delta r_i = 0$

(d) Euler-Lagrange equation of motion is given by

(i) $\frac{\partial L}{\partial q_i} + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0$

(ii) $\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) = 0$

(iii) $\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0$

(iv) $\frac{\partial L}{\partial \dot{q}_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) = 0$

(e) The Lagrangian of a one-dimensional system with generalized coordinate x and generalized velocity $\dot{x}$ of mass m is given by

(i) $L = \frac{1}{2}mx^2 - V(x)$

(ii) $L = \frac{1}{2}m\dot{x}^2 + V(x)$

(iii) $L = \frac{1}{2}m^2\dot{x} - V(x)$

(iv) $L = \frac{1}{2}m\dot{x}^2 - V(x)$

(f) The Hamilton's equations of motion are

(i) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$

(ii) $q_i = \frac{\partial H}{\partial \dot{p}_i}, p_i = \frac{\partial H}{\partial \dot{q}_i}$

(iii) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$

(iv) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$

(g) For linear oscillators, the oscillation frequencies are independent of the _____ of oscillation.

(i) amplitude (ii) time period

(iii) wavelength (iv) velocity

(h) The speed of electromagnetic wave is given by

(i) $c = \sqrt{\mu_0 \varepsilon_0}$

(ii) $c = \frac{1}{\mu_0 \varepsilon_0}$

(iii) $c = \frac{\mu_0}{\varepsilon_0}$

(iv) $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

(i) The expression for relativistic time dilation is given by

(i) $\Delta t = \frac{\Delta \tau}{\sqrt{1 - \frac{v^2}{c^2}}}$

(ii) $\Delta t = \frac{\Delta \tau}{\sqrt{1 + \frac{v^2}{c^2}}}$

(iii) $\Delta t = \sqrt{\frac{\Delta \tau}{1 - \frac{v^2}{c^2}}}$

(iv) $\Delta t = \frac{\Delta \tau}{\sqrt{1 - \frac{v^2}{c^2}}} + m_0^2 c^4$

(Continued)

(j) The equation of continuity in differential form is given by

(i) $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$

(ii) $\frac{\partial \rho}{\partial t} + \nabla \times (\rho \mathbf{v}) = 0$

(iii) $\frac{\partial \rho}{\partial t} - \nabla \cdot (\rho \mathbf{v}) = 0$

(iv) $\frac{\partial \rho}{\partial t} - \nabla \times (\rho \mathbf{v}) = 0$

2. Answer any *five* of the following questions : 2×5=10

(a) Define generalized coordinates. Write the advantage of using generalized coordinates.

(b) What is phase space trajectory? What is the other name of it?

(c) What are rheonomic and scleronomic constraints?

(d) Find an expression of virtual work.

(e) State Hamilton's action principle.

(f) What is the normal mode of a system?

(g) Write the postulates of special theory of relativity.

3. Answer any *six* of the following questions : 5×6=30

(a) Obtain the expressions of the trajectory of a charged particle moving in a magnetic field.

(b) Establish the equation of motion of a simple pendulum using Lagrangian.

(c) Determine the equations for planetary motion using Hamilton's equations.

(d) A particle of mass m moves on the x-axis under the force field whose potential energy is

$$V = \frac{x(x^2 - 9x + 24)}{3}$$

Show that there is a single position of stable equilibrium and find the approximate period of small oscillations about this point.

(e) Two beads of masses $2m$ and m are connected to a fixed wall with two springs of spring constants $2k$ and k. If the masses can move without friction along a horizontal wire, then find the eigenfrequencies of small amplitude oscillations.

(f) Explain the meaning of stable and unstable equilibrium in terms of kinetic and potential energies of a system.

(g) What is light cone? Draw the diagram of light cone showing past, future and the elsewhere.

(h) A block of mass m negligible in size slides on a frictionless inclined plane of mass M at an angle θ with horizontal. The plane itself rests on a smooth horizontal table. Determine the acceleration of the block on the inclined plane.

(i) If two cosmic-ray protons approach the earth from opposite directions at speeds $V_1 = 0 \cdot 6c$ and $V_2 = -0 \cdot 8c$, then find the velocity of each particle relative to the earth.

(j) Derive the differential equation for the conservation of mass.

4. Answer any *two* of the following questions : $10 \times 2 = 20$

(a) Construct the Hamiltonian for a spherical pendulum and find the equations of motion.

(b) Derive the equation of motion for N particles in terms of normal coordinates q_i and normal frequencies ω_i.

(c) Explain the terms *(i)* time dilation and *(ii)* length contraction.

(d) State and derive Poiseuille's law for an incompressible fluid with steady, laminar flow in circular tube.

5. Answer any *one* of the following questions : 14

(a) Derive Euler-Lagrange equations of motion. Obtain the Lagrangian equation of motion for a free particle in cylindrical and spherical polar coordinates. $6+4+4=14$

(b) Write short notes on any *two* of the following : $7 \times 2 = 14$

(i) Minkowski space-time diagram

(ii) Twin paradox

(iii) Relativistic Doppler effect

(c) What are streamlined and turbulent flows of fluid? Derive Bernoulli's equation for a fluid flowing in streamlined motion. $4+10=14$