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63(FY) SEM–4/MAJ/PHYMAJ2034

2025

PHYSICS

(Major)

Paper : PHYMAJ2034

(Mathematical Physics-II)

Full Marks : 50

Pass Marks : 20

Time : Two hours

The figures in the margin indicate full marks for the questions.

I. Answer *all* the following questions : 1×5=5

(a) Bessel's equation of order n is

(i) $x^2y'' + xy' + (x^2 - n^2)y = 0$

(ii) $y'' + xy' + n^2y = 0$

(iii) $xy'' + y' + ny = 0$

(iv) $y'' + y = 0$

(b) The wave equation in one dimension is given by

(i) $\frac{\partial u}{\partial t} = h\frac{\partial^2 u}{\partial x^2}$

(ii) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

(iii) $\frac{\partial^2 u}{\partial x^2} = 0$

(iv) $\frac{\partial u}{\partial t} = -ku$

(c) A function $f(x)$ is periodic if

(i) $f(x+T) = f(x)$ for all x

(ii) $f(x) = 0$ for all x

(iii) $f(x)$ is differentiable

(iv) $f(x) = f(-x)$

(d) The value of $\Gamma(n)$ for a positive integer n is

(i) n

(ii) $n!$

(iii) $(n-1)!$

(iv) $1/n!$

(e) What is a key characteristic of systematic errors ?

(i) They vary randomly around the true value

(ii) They are always zero

(iii) They are caused by unpredictable environmental factors

(iv) They consistently deviate in one direction from the true value

2. Answer all the following questions : ***(any five)***

2×5=10

(a) Distinguish between even and odd functions with suitable graphical examples.

(b) State the Frobenius method for solving a differential equation.

(c) Express the following function in terms of Legendre polynomials

$$f(x) = 1 + 2x - 3x^2 + 4x^3 .$$

(d) Show that

$$J_{-n}(x) = (-1)^n J_n(x)$$

(e) Find the value of $\dfrac{\Gamma(-3/2)}{\Gamma(3/2)}$

(f) What is the normal law of errors ?

(g) Write the one-dimensional diffusion equation and explain the physical significance.

3. Answer the following questions : ***(any five)***

5×5=25

(a) Define the Fourier coefficients a_n and b_n for a function of period 2π. State the Dirichlet's conditions for a Fourier series. 2+3=5

(b) Prove the recurrence relation

$$xP_n'(x) - P_{n-1}'(x) = nP_n(x)$$

(c) Reduce the following differential equation into Bessel's differential equation.

$$x\frac{d^2y}{dx^2} + a\frac{dy}{dx} + k^2xy = 0$$

(d) Establish the following relation

$$\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$$

(e) Reduce the following integral to gamma function

$$\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta$$

(f) Find the Fourier series of the following periodic function.

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

$$f(x + 2\pi) = f(x)$$

(g) State the method of least-squares fitting. Find expressions for the best-fit slope and intercept for a linear fit.

(h) Form the partial differential equations from

(i) $z = a\left(x^2 - y^2\right)$

(ii) $z = (x + a)(y + b)$

4. Answer the following questions : ***(any one)*** 10×1=10

(a) Deduce the following Rodrigues formula from Legendre's differential equation.

$$P_n(x) = \frac{1}{2^n n!} \frac{d_n\left(x^2 - 1\right)^n}{dx^n}$$

(b) Using the method of separation of variables, obtain the solution of the following wave equation.

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$
