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63/1 (SEM-2) CC4/MATHC2046

2025

MATHEMATICS

Paper : MATHC2046

(Differential Equations)

Full Marks : 60

Pass Marks : 24

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : **(any five)**

1×5=5

(a) The degree and order of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 \text{ are}$$

~~(i)~~ 2, 3

(ii) 3, 2

~~(iii)~~ 2, 2

(iv) 3, 3

(b) If $Mdx + Ndy = 0$ is homogeneous differential equation and $Mx + Ny \neq 0$ then the integrating factor is

(i) $\frac{1}{Mx - Ny}$

(ii) $\frac{1}{Mx + Ny}$

(iii) $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$

(iv) $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M}$

(c) The necessary and sufficient condition for the ordinary differential equation $Mdx + Ndy = 0$ to be exact is that

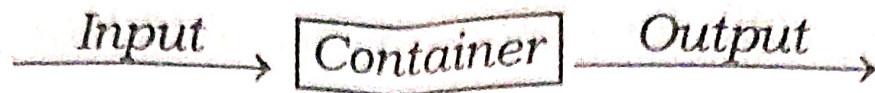
(i) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(ii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iii) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

(iv) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

(d) Input output compartmental diagram



Could not describe

(i) the decay process of radioactive element.

(ii) births and deaths in a population.

(iii) pollution into and out of a lake

(iv) None of the above.

(e) Which one of the following ordinary differential equation is linear.

(i) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 6y^2 = 0$

(ii) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$

(iii) $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 6y = 0$

(iv) $\frac{d^2y}{dx^2} + 5y\frac{dy}{dx} + 6y = 0$

(f) If the Wronskian of the functions $y_1(x)$ and $y_2(x)$ is zero, i.e. $W(y_1, y_2) = 0$, then the functions $y_1(x)$ and $y_2(x)$ are called

(i) linearly dependent

(ii) non-linearly dependent

1. [Illegible text]

2. [Illegible text]

3. [Illegible text]

4. [Illegible text]

5. [Illegible text]

6. [Illegible text]

7. [Illegible text]

8. [Illegible text]

9. [Illegible text]

10. [Illegible text]

11. [Illegible text]

(i) $y = x$ is a part of complementary function of $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$, if

(i) $P + Qx = 0$

(ii) $P - Qx = 0$

(iii) $1 - P + Q = 0$

(iv) $1 + P + Q = 0$

(j) If $f(D) = 0$ and the values of D are imaginary say $\alpha \pm i\beta$ then complementary function corresponding to these values of D will be

(i) $c_1 e^{\alpha x} + c_2 e^{\beta x}$

(ii) $e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x)$

(iii) $e^{\beta x} (c_1 \cos \alpha x + c_2 \sin \alpha x)$

(iv) $e^{\alpha x} (c_1 \sin \alpha x + c_2 \cos \beta x)$

2. Answer the following questions : **(any five)**
 $2 \times 5 = 10$

(a) Show that the function f defined for all real x by $f(x) = 2 \sin x + 3 \cos x$ is an explicit solution of the differential

equation $\frac{d^2y}{dx^2} + y = 0$.

(b) Find the integrating factor of

$$\frac{dy}{dx} = y \tan x - 2 \sin x.$$

(c) Examine the following differential equation is exact or not

$$(3x + 2y)dx + (2x + y)dy = 0$$

(d) Find the Wronskian of the functions $1, x, x^2$. Are they linearly independent?

(e) Solve : $(D^2 + 4)y = e^x$

(f) Draw input-output compartmental diagram for CO_2 .

(g) Write the formulation of the equation of limited growth with harvesting.

3. Answer the following questions : **(any five)**
5×5=25

(a) Show the relation $x^2 + y^2 - 25 = 0$ is an implicit solution of the differential equation $x + y \frac{dy}{dx} = 0$ on the interval 1 defined by $-5 < x < 5$.

(b) Solve : $(1 + y^2)dx = (\tan^{-1} y - x)dy$.

(c) Solve : $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$.

(d) Solve :

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$

(e) Solve the initial value problem (IVP)

in equation $\frac{dN}{dt} = -KN$ with initial

condition $N(t_0) = n_0$.

(f) How long will it take for the lake's pollution level to reach 5% of its initial level, if only fresh water flow into the lake.

(g) Solve by the method of variation of parameter the equation

$$\frac{d^2y}{dx^2} + a^2y = \sec ax.$$

(h) Show that the differential equation

$$\frac{dy}{dx} + \frac{ax + hy + g}{hx + by + f} = 0$$
 is exact and

hence solve it.

(i) Solve : $(D^2 - 2D + 5)y = e^{2x} \sin x$.

4. Answer the following questions : *(any two)*
10×2=20

(a) (i) Solve the following differential equation by finding an integrating factor

$$(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0$$

5

(ii) Write down Bernoulli's equation and solve it.
1+4=5

(b) Define Cauchy-Euler equation of homogeneous linear differential equation also solve

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$$

2+8=10

(c) State and prove the necessary and sufficient condition for first-order first degree differential equation to be an exact differential equation. 2+8=10

(d) Write the differential equation of drug assimilation model of a single cold pill and solve it. 2+8=10