

Total number of printed pages-7

63 (FY)SEM-3/MAJ/MATMAJ2024

2024

**MATHEMATICS**

Paper : MATMAJ2024

**[ Differential Equations (ODE) ]**

Full Marks : 50

Pass Marks : 20

Time : Two hours

**The figures in the margin indicate full marks for the questions.**

1. Choose the correct answer :  $1 \times 5 = 5$

a. The order of the differential equation

$$\frac{d^4 x}{dt^4} + 5 \frac{d^2 x}{dt^2} + 3x = \sin t \text{ is}$$

(i) 1

(ii) 2

(iii) 3

(iv) 4

b. Which one of the following ordinary differential equation is linear ?

(i)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 6y^2 = 0$

(ii)  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$

(iii)  $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 6y = 0$

(iv)  $\frac{d^2y}{dx^2} + 5y\frac{dy}{dx} + 6y = 0$

c. If the Wronskian of the functions  $y_1(x)$  and  $y_2(x)$  is zero, i.e.  $W(y_1, y_2) = 0$ , then the functions  $y_1(x)$  and  $y_2(x)$  are called

(i) linearly dependent

(ii) non-linearly dependent

(iii) linearly independent

(iv) non-linearly independent

d. The particular integral of

$$(D^2 - 3D + 2)y = e^{5x} \text{ is}$$

(i)  $xe^{5x}$

(ii)  $\frac{e^{5x}}{5}$

(iii)  $\frac{e^{5x}}{12}$

(iv)  $\frac{e^{5x}}{25}$

e. The differential equation

$$\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + x^3y = 0$$

is a second-order

(i) linear homogeneous

(ii) non-linear homogeneous

(iii) linear non-homogeneous

(iv) non-linear non-homogeneous

2. Answer the following questions : (any five)  
2×5=10

- a. Show that the function  $f$  defined for all real  $x$  by  $f(x) = 2\sin x + 3\cos x$  is an explicit solution of the differential equation

$$\frac{d^2y}{dx^2} + y = 0.$$

- b. Find the integrating factor of

$$(1 + y^2)dx = (\tan^{-1} y - x)dy$$

- c. Examine the following differential equation is exact or not :

$$(3x + 2y)dx + (2x + y)dy = 0$$

- d. Draw input-output compartmental diagram for  $CO_2$ .

- e. Find the general solution of the differential equation

$$y'' - 4y = 0.$$

- f. Find the Wronskian of the functions  $x, x^2, x^3$ . Are they linearly independent?

- g. Write the formulation of the equation of limited growth with harvesting.
3. Answer the following questions : *(any five)*  
5×5=25
- a. Show the relation  $x^2 + y^2 - 25 = 0$  is an implicit solution of the differential equation  $x + y \frac{dy}{dx} = 0$  on the interval  $I$  defined by  $-5 < x < 5$ .
- b. Solve :  $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$
- c. Solve :  $(x - 4)y^4 dx - x^3(y^2 - 3)dy = 0$
- d. Solve the initial value problem (IVP) in equation  $\frac{dN}{dt} = -kN$ ,  
with initial condition  $N(t_0) = n_0$ .
- e. Find a differential equation for the amount of salt in the tank at any time  $t$ . (Note that concentration can be defined as the mass of salt per unit volume of mixture).

- f. Using undetermined co-efficients method to solve

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = x^2 - 2x + 2.$$

- g. Solve by the method of variation of parameter the equation

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$

- h. Solve :

$$(D^2 - 2D + 5)y = e^{2x} \sin x.$$

4. Answer the following questions : **(any one)**

$$10 \times 1 = 10$$

- (a) (i) Solve the following differential equation by finding an integrating factor 5

$$(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0$$

- (ii) Write down Bernoulli's equation and solve it. 1+4=5

(b) Write the differential equation of drug assimilation model of a single cold pill and solve it.

$$2+8=10$$

---