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63/1 (SEM–2) CC3/STSHC2036

2025

STATISTICS

Paper : STSHC2036

(Probability and Probability Distribution)

Full Marks : 60

Pass Marks : 24

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option from the following : ***(any five)*** 1×5=5

   (a) If the events A and B are independent, then

   (i) $P(A+B) = P(A) + P(B)$

   (ii) $P(AB) = P(A)P(B)$

   (iii) $P(AB) = P(A)$

   (iv) $P(AB) = P(B)$

(b) If $P(A \cup B) = 0.7, P(A) = 0.4$ and $P(B) = 0.5$, then the events A and B are

(i) independent

(ii) mutually exclusive

(iii) exhaustive

(iv) both independent and mutually exclusive

(c) The probability of an impossible event is

(i) 0

(ii) 1

(iii) 2

(iv) 3

(d) The parameters of binomial distribution are

(i) n

(ii) p

(iii) n and p

(iv) npr

(e) Which of the following distribution has the mean and variances are equal ?

(i) Binomial

(ii) Poisson

(iii) Normal

(iv) Negative Binomial

(f) If X is a random variable and 'a' is any constant, then

(i) $E(aX)$ = constant

(ii) $E(aX) = 0$

(iii) $E(aX) = XE(a)$

(iv) $E(aX) = aE(X)$

(g) Let X be a random variable and a be any arbitrary value. Then $M_{X-a}(t)$ is equal to

(i) $M_X(at)$

(ii) $M_{aX}(t)$

(iii) $e^{-at}M_X(t)$

(iv) $M_X(et)$

(h) The mean of geometric distribution is

(i) p

(ii) q

(iii) pq

(iv) $\frac{q}{p}$

(i) The mean and variance of standard normal variate z is

(i) $(0,1)$

(ii) $(1,0)$

(iii) (μ,σ^2)

(iv) (σ^2,μ)

(j) The characteristic function of the binomial distribution for the binomial variate $X \sim b(n,p)$ is

(i) $(q+pe^{it})$

(ii) $(p+qe^{it})^n$

(iii) $(p+qe^{t})^n$

(iv) $(q+pe^{it})^n$

2. Answer the following questions : ***(any five)*** $2 \times 5 = 10$

(a) State classical definition of probability.

(b) If A and B are two independent events, then prove that $\bar{A}$ and $\bar{B}$ are also independent events.

(c) If A, B and C are three events, then write down the conditions for their mutual independence.

(d) Define mathematical expectation.

(e) Write *any two* properties of normal distribution.

(f) Find the mean of Poisson distribution.

(g) Find the moment generating function of the random variable X whose probability density function is given by

$$f(x) = \theta . C^{-\theta x}, x \geq 0$$

3. Answer the following questions : ***(any five)*** $5 \times 5 = 25$

(a) State and prove addition law of probability when the events are not mutually exclusive.

(b) State and prove Bayes' theorem.

(c) A coin is tossed until a head occurs. Find the mathematical expectation of the number of tosses required.

(d) Derive Poisson distribution as a limiting case of binomial distribution.

(e) If X and Y are two random variables having joint density function.

$$F(x, y) = \frac{1}{8}(6 - x - y); 0 \le x < 2, 2 \le y < 4$$

$= 0$, otherwise

Find $P(X < 1 \cap Y < 3)$.

(f) Show that the moment generating function of the sum of a number of independent random variables is equal to the product of their respective moment generating functions.

(g) Derive binomial distribution from Bernoulli trials.

(h) Find the mean and variance of the hypergeometric distribution.

(i) Prove that for the normal distribution, the quartile deviation, the mean deviation and standard deviation are approximately 10: 12: 15.

4. Answer the following questions : *(any two)* 10×2=20

(a) What is conditional probability ? An urn contains 5 red and 6 black balls. Two drawings of 2 balls in each draw are made. Find the probability of getting 2 red balls in the first draw and 2 black balls in the second draw if *(i)* balls are not returned to the urn after the first draw *(ii)* balls are returned to the urn after first draw. 2+4+4=10

(b) The joint probability distribution of two random variables X and Y is given by

$$P(X=0, Y=1)=\frac{1}{3}, \quad P(X=1, Y=-1)=\frac{1}{3},$$

and $P(X=1, Y=1)=\frac{1}{3}$

Find *(i)* Marginal distribution of X and Y and *(ii)* The conditional probability distribution of X given $Y=1$.

5+5=10

(c) Define geometric distribution. Find the moment generating function of geometric distribution. Let X_1, X_2 be independent random variables each having geometric distribution $q^k p; k = 0, 1, 2, \ldots$. Show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform.

(d) Define gamma distribution. Find the moment generating function of gamma distribution. Show that the sum of independent gamma variates is also a gamma variate. 2+4+4=10

(c) Define geometric distribution. Find the moment generating function of geometric distribution. Let X_1, X_2 be independent random variables each having geometric distribution $q^k p; k = 0, 1, 2, \ldots$. Show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform.

(d) Define gamma distribution. Find the moment generating function of gamma distribution. Show that the sum of independent gamma variates is also a gamma variate. 2+4+4=10