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63/1 (SEM-2) CC4/STSHC2046

2025

STATISTICS

Paper : STSHC2046

(Algebra)

Full Marks : 80

Pass Marks : 32

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : ***(any six)*** 1×6=6

(a) If n is even, the number of real roots of $x^n - 1 = 0$ is

(i) 1

(ii) 0

(iii) 2

(iv) n

(b) If α, β, γ are the roots of the equation $ax^3 + bx + c = 0$, then $\Sigma\alpha$ is

(i) 0

(ii) 1

(iii) 2

(iv) 3

(c) A subset of linearly dependent set of vectors is

(i) linearly dependent

(ii) linearly independent

(iii) both linearly independent and dependent

(iv) neither linearly independent nor dependent

(d) The rank of a matrix A is $r(\neq 0)$. The rank of its transpose A' is

(i) $\frac{1}{r}$

(ii) r

(iii) $r+1$

(iv) $r-1$

(e) If A is a non-singular matrix and $|A| = 3$, then $|A^{-1}|$ is

(i) 0

(ii) 3

(iii) $1/3$

(iv) 1

(f) If $f(x) = \begin{vmatrix} 1 & 1 & x \\ 1 & 2 & x^2 \\ 1 & 3 & x^3 \end{vmatrix}$, then $f(1)$ is

(i) 1

(ii) 2

(iii) 0

(iv) 3

(g) If w_1 and w_2 are sub-spaces of a finite dimensional vector space $V(F)$, then

(i) $dim(W_1 + W_2) = dim\, W_1\, dim\, W_2 - dim(W_1 \cap W_2)$

(ii) $dim(W_1 + W_2) = dim W_1 + dim W_2 + dim(W_1 \cap W_2)$

(iii) $dim(W_1 + W_2) = dim W_1 + dim W_2 - dim(W_1 \cup W_2)$

(iv) $dim(W_1 + W_2) = dim W_1 + dim W_2 + dim(W_1 \cup W_2)$

(h) If the number of variables in a non-homogeneous system $AX = B$ is n, then the system possesses an unique solution if—

(i) $\rho(A) < \rho[AB]$

(ii) $\rho(A) > \rho[AB]$

(iii) $\rho(A) = \rho[AB] = n$

(iv) $\rho(A) = \rho[AB] \neq n$

(i) The product of all the characteristic roots of a square matrix A is

(i) 0

(ii) 1

(iii) $|A|$

(iv) $1/|A|$

(j) A quadratic form is a homogeneous expression of

(i) First degree

(ii) Second degree

(iii) Third degree

(iv) n-th degree

2. Answer the following questions : ***(any five)*** 2×5=10

(a) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\Sigma\alpha^2$.

(b) Show that the vectors (1,2,3), (2,−2,0) form a linearly independent set.

(c) If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$, and $A^{-1} = \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix}$, then find x.

(d) If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 8 \\ -1 & 0 \end{bmatrix}$, find AB.

(e) Express the following system of equations in matrix form :

$$x + 2y + z = 9$$

$$2x - y + z = 3z.$$

(f) Find the characteristic roots of the orthogonal matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(g) Define trace of a matrix. Find the trace of the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$$

3. Answer the following questions : *(any six)* 5×6=30

(a) Solve the equation $x^3 + x^2 - 16x + 20 = 0$, the difference between two roots being 7.

(b) Find the condition that the sum of two roots of the equation

$x^4 + px^3 + qx^2 + rx + s = 0$ is zero.

(c) Prove that every superset of a linearly dependent set is linearly dependent.

(d) If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find K, such that

$A^2 = KA - 2I_3$

(e) Show that the set of vectors (1,0,0), (0,2,1), (1,1,0) form a basis of the real vector space R^3.

(f) The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7, By adding second and the third number to the three times of the first number, we get 12, use determinant to find the numbers.

(g) Define inverse of a matrix. If A^{-1} and B^{-1} are the inverses of A and B respectively, then show that

$$(AB)^{-1} = B^{-1}A^{-1}$$

(h) Find an orthogonal matrix P that will diagonalise the real symmetric matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

(i) Prove that corresponding to an eigen-vector X of a square matrix A, there exists one and only one eigenvalues. Is the converse true ?

(j) Define Hermitian and skew-Hermitian matrices. If A be any square matrix, prove that $A + A^{\theta}$ is Hermitian and $A - A^{\theta}$ is skew-Hermitian.

4. Answer the following questions : *(any two)* 10×2=20

(a) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $Y = \begin{bmatrix} -1 \\ 1 \\ -5 \end{bmatrix}$

Find the inverse of the matrix A and hence solve the equation $AX = Y$, Verify $AA^{-1} = I_3$. 5+3+2=10

(b) Define :

(i) Characteristic polynomial

(ii) Characteristic equation

(iii) Characteristic roots of a square metrix A.

Find the icharacteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{bmatrix}$$

and hence find A^3. 3+3+4=10

(c) (i) Define Involutory and Idempotent matrix.

Show that $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is nilpotent. What is the order of it ?

2+3+1=6

(ii) Using properties of determinant, show that

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$$ 4

(d) (i) The equation $2x^3 - 9x^2 + 12x + \lambda = 0$ has two equal roots. Find λ and solve the equation completely. 5

(ii) Solve the equation $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$, the roots being in *A.P.* 5

5. Answer the following : ***(any one)***

14×1=14

(a) *(i)* Define subspace of a vector space. Prove that the set of all linear combinations of a given set of r vectors of $V_n(F)$ is a subspace of $V_n(F)$. 2+4=6

(ii) Write down the quadratic form corresponding to the symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$ 4

(iii) Show that the rank of the sum of two matrices cannot exceed the sum of their ranks. 4

(b) *(i)* Define linear space *L(S)* of a subset S of a vector space *V(F)* and show that *L(S)* is a subspace of *V(F)* containing the set *S*. 4

(ii) If W_1 and W_2 are any two subspaces of a vector space V over the field F, then prove that $W_1 \cap W_2$ is also a subspcace of V. 5

(iii) If A is an n-square matrix, then show that 5

$$|adj\ A| = |A|^{n-1}$$

(c) (i) State Cayley-Hamilton theorem on characteristic polynomial and verify it for

$$A = \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -1 \\ 4 & 0 & -2 \end{bmatrix}$$ 2+4=6

(ii) Reduce the matrix

$$A = \begin{bmatrix} 2 & 4 & -2 & 2 \\ 1 & 2 & -3 & 0 \\ 3 & 6 & -4 & 3 \end{bmatrix}$$

to Echelon form and find the rank of A.

4

(iii) Show that $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. Mention the corresponding eigenvalue. 4