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63 (FY)SEM-2/MAJ2/MATMAJ1024

2025

MATHEMATICS

(MAJOR)

Paper : MATMAJ1024

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Answer the following questions : $1 \times 6 = 6$

(i) Let $f(x) = |x|$ be a real valued function.

Then

(A) f is not continuous at $x = 0$

(B) f is continuous at $x = 0$ but not differentiable at $x = 0$

(C) f is both differentiable and continuous at $x = 0$



(D) f is neither continuous nor differentiable at $x=0$.

(ii) Let $f(x) = x^2 - 1$ be defined on $[-1, 1]$. Then which of the following is true for the curve $y = f(x)$?

(A) There exists a tangent at the point $(0, 1)$ parallel to the x -axis

(B) There exists a tangent at the point $(0, 1)$ parallel to the y -axis

(C) There exists a tangent at the point $(0, -1)$ parallel to the x -axis

(D) There exists a tangent at the point $(0, -1)$ parallel to the y -axis

(iii) Let f, g be two functions such that $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$, $f'(a), g'(a)$ exist and $g'(a) \neq 0$. Then

(A) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} \neq \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} =$

(B) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} f'(x)$

(C) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} g'(x)$

(D) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

(iv) $\int_0^{\pi/2} \cos^6 x dx = ?$

(A) $\frac{5\pi}{8}$

(B) $\frac{5\pi}{16}$

(C) $\frac{5\pi}{32}$

(D) $\frac{5\pi}{64}$

(v) The angle of intersection of the curves $r = \sin \theta + \cos \theta$ and $r = 2 \sin \theta$ is

(A) $\pi/4$

(B) $\pi/2$

(C) $\pi/3$

(D) $\pi/6$

(vi) The radius of curvature of the parabola $y^2 = 4x$ at the vertex is

(A) $\frac{1}{4}$

(B) 4

(C) $\frac{1}{2}$

(D) 2

(vii) $\int_0^{\pi/2} \sin^6 \theta \cos^2 \theta d\theta = ?$

(A) $\frac{5\pi}{64}$

(B) $\frac{5\pi}{256}$

(C) $\frac{5\pi}{128}$

(D) $\frac{5\pi}{32}$

2. Answer the following questions : **(any five)**
 $2 \times 5 = 10$

(i) A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

Show that f is continuous at $x = 0$.

(ii) Find $\frac{dy}{dx}$, if $y = \tan^{-1} \frac{\cos x}{1 + \sin x}$.

(iii) Give the geometrical interpretation of Lagrange's Mean Value Theorem.

(iv) Let $f(x)$ be defined on $[a, b]$ such that $f(x)$ is continuous in $[a, b]$ and $f'(x)$ exists in (a, b) . If $f'(x) = 0$ for all $x \in (a, b)$, show that f is constant in $[a, b]$.

(v) If $r = a(1 + \cos \theta)$, then find $\frac{ds}{d\theta}$, where s denotes the arc length with usual meaning.

(vi) Find the centre of curvature of $xy = 12$ at the point (3,4).

(vii) Find the length of the arc of the curve $y = \log \sec x$ between $x = 0$ and $x = \pi/3$

3. Answer the following questions: **(any six)**
 $5 \times 6 = 30$

(i) Define derivability of a function $f(x)$ at a point $x=c$ in an interval. Show that the function

$$f(x) = \begin{cases} 3+2x & \text{for } -3/2 < x \leq 0 \\ 3-2x & \text{for } 0 < x < 3/2 \end{cases}$$

is continuous at $x=0$, but $f'(0)$ does not exist. $1+2+2=5$

(ii) Find y_n , where $y = \frac{1}{x^2 + a^2}$.

(iii) State and prove Cauchy's Mean Value Theorem.

(iv) Evaluate the following **(any one)**:

(a) $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x^2}$

(b) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$

(v) Find the asymptotes of the following curve:

$$x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0$$

(vi) If (α, β) be the coordinates of the centre of curvature of the parabola $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at (x, y) , then show that $\alpha + \beta = 3(x + y)$.

(vii) Find the pedal equation of the curve $y^2 = 4ax$ with regard to its vertex.

(viii) If n is a positive integer, prove that

$$\int_0^{\pi/2} \sin^n x \, dx = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} & \text{if } n \text{ is even} \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot 1 & \text{if } n \text{ is odd} \end{cases}$$

(ix) The part of the parabola $y^2 = 4ax$ bounded by the latus rectum revolves about the tangent at the vertex. Find the area of the curved surface of the reel thus generated.

4. Answer the following question: **(any two)**
 $12 \times 2 = 24$

(i) (a) State and prove Leibnitz's Theorem to find the n th derivative of the product of two functions. $1+5=6$

(b) Let $y = \sin(m \sin^{-1} x)$. Then show that $(1 - x^2)y_2 - 2xy_1 + m^2y = 0$

$$\text{and } (1-x^2)^{n+2} - (2n+1)xy^{n+1} + (m^2-n^2)y_n = 0 \quad 2+4=6$$

(ii) (a) Express $\log(1+x) \forall x \in [-1,1]$ as a power series of x by using Maclaurin's Theorem.

(b) Find $\frac{d}{dx}(\coth x)$.

(c) Verify Rolle's Theorem for the function $f(x) = x(x+3)e^{-x/2}$ in $[-3,0]$. $6+2+4=12$

(iii) (a) If ρ_1 and ρ_2 be the radii of curvature at the ends P and D of conjugate diameters of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ then show that}$$

$$\rho_1^{2/3} + \rho_2^{2/3} = (a^2 + b^2) / (ab)^{2/3}$$

(b) Find the pedal equation of the curve $r^m = a^m \cos m\theta$.

(c) Prove that the curves

$$r^n = a^n \cos n\theta \text{ and } r^n = b^n \sin n\theta \text{ cut orthogonally} \quad 6+3+3=12$$

(iv) (a) Show that for the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature at an extremity of the major axis is equal to half of the latus rectum.

(b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$.

(c) Find the area above the x -axis included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$. $5+4+3=12$