Total number of printed pages –8

63 (FY)SEM–2/MIN2/MATMIN1024

2025

MATHEMATICS

(MINOR)

Paper : MATMIN1024

(Integral Calculus and Differential Equations)

Full Marks : 70

Pass Marks : 28

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

(i) $\int e^{-\log x} dx$ is equal to

(a) $-e^{-\log x} + C$

(b) $-xe^{-\log x} + C$

(c) $\log|x| = C$

(d) None of the above

(ii) $\int \frac{dx}{\sqrt{x^2 - a^2}}$ is

(a) $\log \frac{x + \sqrt{x^2 - a^2}}{a} + C$

(b) $\log \frac{x - \sqrt{x^2 - a^2}}{a} + C$

(c) $\log \frac{x + \sqrt{x^2 - a^2}}{x} + C$

(d) $\log \frac{x - \sqrt{x^2 - a^2}}{x} + C$

(iii) $\int_0^{\pi/2} \sin^n x \, dx$ is equal to

(a) $\int_0^{-\pi/2} \sin^n x \, dx$

(b) $\int_0^{\pi/2} \cos^n x \, dx$

(c) $-\int_0^{\pi/2} \cos^n x \, dx$

(d) None of the above

(iv) The integrating factor for the linear differential equation of first order $\frac{dy}{dx} + P(x)y = Q(x)$ is

(a) $e^{\int P dx}$

(b) $e^{-\int P dx}$

(c) $e^{\int P/Q \, dx}$

(d) $e^{\int Q \, dy}$

(v) The general solution of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ is

(a) $y = C_1 e^{2x} + C_2 e^{-2x}$

(b) $y = C_1 e^{-2x} + C_2 e^{-2x}$

(c) $y = C_1 \cos 2x + C_2 \sin 2x$

(d) $y = C_1 \cos 2x + C_2 x \cos 2x$

(vi) The particular integral of the differential equation $y'' - 5y' + 4y = 0$ is

(a) 0

(b) $y = 4e^x + e^{4x}$

(c) $y = C_1 e^x + C_2 e^{4x}$

(d) $y = 2x$

(vii) If $y = Ae^{Bx+C}$ is the solution of homogeneous partial differential equation, then the order of the differential equation is

(a) 1

(b) 2

(c) 3

(d) 4

(viii) Which of the following is the general form of Cauchy-Euler equation for a second order linear differential equation ?

(a) $ax^2y'' + bxy' + cy = 0$

(b) $ax^2y'' + bxy' + c = 0$

(c) $ax^2y'' + by' + c = 0$

(d) $ay'' + bxy + cy' = 0$

(ix) Which of the following will be Wronskian of two functions ?

(a) $W(f,g) = fg' - f'g$

(b) $W(f,g) = ff' - gg'$

(c) $W(f,g) = fg' + f'g$

(d) $W(f,g) = f'g' - fg$

(x) The order of partial differential equation $r^2 + 6S - t = 0$ (symbols have their usual meaning) is

(a) One

(b) Two

(c) Three

(d) Four

2. Answer the following questions : ***(any five)*** 2×5=10

(i) Evaluate $\int_0^{\pi/2} \sin^7 x dx$

(ii) Evaluate $\int_0^{\infty} \frac{dx}{1+x^2}$

(iii) Find the integrating factor of the differential equation $x^2y dx - (x^3 + y^3)dy = 0$

(iv) Solve $(2x^3 + 4y)dx + (4x + y - 1)dy = 0$

Contd.

(v) Solve $(D^2 - 8D + 16)y = 0$

(vi) Solve $p^2 + p - 6 = 0$ where $p = \frac{dy}{dx}$.

3. Answer **any six** from the following questions : 5×6=30

(i) Evaluate $\int \frac{dx}{\sqrt{7 - 6x - x^2}}$

(ii) If $\phi(n) = \int_0^{\pi/4} \tan^n x\, dx$, show that

$$\phi(n) + \phi(n-2) = \frac{1}{n-1}$$

(iii) Show that $x(x-1)^{-1}$ is an integrating factor of the equation

$$x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$$ and hence

solve it.

(iv) Solve

$$(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$$

(v) Find the complete primitive of

$$x^2(y - px) = p^2 y$$

(vi) Solve $(D^2 - 2D + 5)y = e^{2x} \sin x$

(vii) Form the partial differential equation by eliminating the function f and F from

$$y = f(x - at) + F(x + at)$$

(viii) Show that e^{2x} and e^{3x} are linearly independent solution of

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

(ix) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$

4. Answer the following questions : ***(any two)*** 10×2=20

(a) Define Cauchy-Euler equation of homogeneous linear differential equation and also solve 2+8=10

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

(b) (i) Solve by the method of variation of parameter $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ 5

(ii) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$

given that $y(0) = 4, \frac{dy}{dx} = 1$

at $x = 0$. 5

(c) (i) Obtain the reduction formula for

$\int \cos^n x \, dx$ 5

(ii) Evaluate $\int \sec^3 x \, dx$ 5

(d) Prove that the necessary and sufficient condition that a differential equation $Mdx + Ndy = 0$ is exact is that 5+5=10

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

and hence solve

$$(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$$
