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63(FY) SEM–4/MAJ/MATMAJ2054

2025

MATHEMATICS

(Major)

Paper : MATMAJ2054

(Analytical Geometry (2D) and Vector Calculus)

Full Marks : 70

Pass Marks : 28

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : 1×6=6

(a) If an equation $hxy + gx + fy + c = 0$ represents a pair of lines, then

(i) $gh = cf$

(ii) $fh = cg$

(iii) $fg = ch$

(iv) $hf = -cg$

(b) When the origin is shifted to $(-1, 2)$ by the translation of axes, the transformed equation of $x^2 + y^2 + 2x - 4y + 1 = 0$ is

(i) $x'^2 + y'^2 = 4$

(ii) $x'^2 + y'^2 = 16$

(iii) $x'^2 + 2x' + y'^2 = 4$

(iv) $x'^2 - 2x' + y'^2 = 16$

(c) Vector triple product $\vec{a} \times (\vec{b} \times \vec{c})$ of the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is given by

(i) $(\vec{a}.\vec{b})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

(ii) $(\vec{b}.\vec{c})\vec{a} - (\vec{a}.\vec{c})\vec{b}$

(iii) $(\vec{b}.\vec{c})\vec{a} - (\vec{a}.\vec{b})\vec{c}$

(iv) $(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

(d) If the pair of lines given by $(x \cos\alpha + y \sin\alpha)^2 = (x^2 + y^2)\sin^2\alpha$ are perpendicular to each other, then α is

(i) 0

(ii) $\frac{\pi}{2}$

(iii) $\frac{\pi}{4}$

(iv) $\frac{\pi}{6}$

(e) If vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + p\hat{j} + 5\hat{k}$ are coplanar, then the value of p is

(i) 2

(ii) -2

(iii) -1

(iv) -4

(f) If $\vec{r} = 3xz\hat{i} + 2xy\hat{j} - yz^2\hat{k}$, then $div\,\vec{r}$ is

(i) $3x + 2y + 2z$

(ii) $2x + 3y + 2z$

(iii) $x + 2y + z$

(iv) $3z + 2x - 2yz$

2. Answer the following questions : ***(any five)*** $2 \times 5 = 10$

(a) What will be the form of $x \cos\alpha + 7 \sin\alpha = P$, when the axes are rotated through an angle α ?

(b) Find the polar form of the equation $(x^2 + y^2)^2 = a^2(x^2 - y^2)$

(c) Prove that $curl\ (grad\ \phi) = 0$

(d) The co-ordinates of new origin are (2, 1) and the axes are rotated through an angle 60°. If the co-ordinates of a point in the new system are $\left(\frac{3-4\sqrt{3}}{2}, -\frac{4+3\sqrt{3}}{2}\right)$. Find the co-ordinate of it in the old system.

(e) Evaluate : $\left(2\hat{i}-3\hat{j}\right).\left[\left(\hat{i}+\hat{j}-\hat{k}\right)\times\left(3\hat{i}-\hat{k}\right)\right]$

(f) Find the condition that the line $lx + my = n$ is a tangent to the parabola $y^2 = 4ax$.

(g) If r be the distance of $P(x, y, z)$ from the origin and $\vec{r}$ be the position vector of P relative to the origin, show that $curl\,\vec{r} = 0$.

3. Answer the following questions : **(any six)** 5×6=30

(a) Transform the equation $x^2 + 2xy\tan 2\alpha - y^2 = a^2 \sec 2\alpha$ to the rectangular axes inclined at an angle α to the old rectangular axes.

(b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors then, show that

$$\left[\vec{b}\times\vec{c} \;\; \vec{c}\times\vec{a} \;\; \vec{a}\times\vec{b}\right] = \left[\vec{a}\;\vec{b}\;\vec{c}\right]^2$$

(c) Prove that the locus of the point whose polar with respect to the ellipse $x^2/a^2 + y^2/b^2 = 1$ subtends a right angle at the origin, is the ellipse $x^2/a^4 + y^2/b^4 = 1/a^2 + 1/b^2$.

(d) Prove that

$$curl\left(\vec{u}\times\vec{v}\right) = \vec{u}\,div\,\vec{v} - \vec{v}\,div\,\vec{u} + \left(\vec{v}.\nabla\right)\vec{u} - \left(\vec{u}.\nabla\right)\vec{v}$$

(e) Prove that the locus of poles of tangents to the hyperbola $x^2 - y^2 = a^2$ with respect to the parabola $y^2 = 4ax$ is the ellipse $x^2/a^2 + y^2/(4a^2) = 1$.

(f) Find the equation of the normal at $(a\cos\theta, b\sin\theta)$ to the ellipse $x^2/a^2 + y^2/b^2 = 1$.

(g) Find $\vec{\nabla}\phi$ at (1, – 2, – 1) in the direction of $2\hat{i}-\hat{j}-2\hat{k}$, where $\phi = x^2yz + 4xz^2$.

(h) Find the equation of the bisectors of the angles between the lines $ax^2 + 2hxy + by^2 = 0$.

(i) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplaner vectors, then show that any vector $\vec{r}$ can be expressed as

$$\vec{r} = \frac{\left[\vec{b}\ \vec{c}\ \vec{r}\right]\vec{a} + \left[\vec{c}\ \vec{a}\ \vec{r}\right]\vec{b} + \left[\vec{a}\ \vec{b}\ \vec{r}\right]\vec{c}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}$$

4. Answer the following questions : ***(any two)*** 12×2=24

(a) (i) Reduce the following equation to the standard form :

$3x^2 - 6xy - 5y^2 - 6x + 22y - 17 = 0$

(ii) The polar of a point with respect to the parabola $y^2 = 4ax$ touches the parabola $x^2 + 4by = 0$; Prove that the locus of the point is a rectangular hyperbola. 7+5=12

(b) (i) Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is

$$\frac{\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}$$

(ii) Find the equation of the pair of straight lines through the origin perpendicular to the pair $ax^2 + 2hxy + by^2 = 0$.

(iii) Choose the new origin (h, k) without changing the directions of the axes, such that the equation $5x^2 - 2y^2 - 30x + 8y = 0$ may reduce to the form $Ax'^2 + By'^2 = 1$.

6+4+2=12

(c) (i) Define Green's theorem. Evaluate by Green's theorem. 6

$$\oint_C \{(\cos x \sin y - xy)dx + \sin x \cos y\, dy\}$$

where C is the circle $x^2 + y^2 = 1$ in the xy – plane described in the positive sense.

(ii) Define Stokes' theorem. 6

Evaluate : $\oint_C \left(e^x dx + 2y\, dy - dz\right)$

by using Stokes' theorem, where c is the curve $x^2 + y^2 = 4$, $z = 2$.

(d) (i) Find the equation of the tangent to the conic represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ at the point (x', y').

(ii) Prove that the locus of the poles of normal chords of the ellipse

$$x^2/a^2 + y^2/b^2 = 1$$ is given by

$$a^6/x^2 + b^6/y^2 = (a^2 - b^2)^2.$$

7+5=12
