

Total number of printed pages-11

63(FY) SEM-4/MIN/MATMIN2024

2025

MATHEMATICS

(Minor)

Paper : MATMIN2024

(Vector Calculus)

Full Marks : 70

Pass Marks : 28

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Choose the correct answer : $1 \times 10 = 10$
- (a) The value of a scalar triple product, if two of its vectors are equal is
- (i) One
 - (ii) Two
 - (iii) Zero
 - (iv) Three



(b) The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + (\hat{i} \times \hat{k}) \cdot \hat{j}$ is

(i) 0

(ii) 1

(iii) $2\hat{k}$

(iv) $2\hat{j}$

(c) The volume of the parallelepiped whose coterminous edges are $2\hat{i} - 3\hat{j} + \hat{k}$, $\hat{i} - \hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$ is

(i) 1

(ii) 0

(iii) 12

(iv) 14

(d) If $\bar{A} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$ and

$\bar{B} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$ then $\frac{d}{dt}(\bar{A} \cdot \bar{B})$ at

$t=1$ is

(i) 4

(ii) 6

(iii) -4

(iv) -6

(e) The gradient of a scalar function $\varphi(x, y, z)$ is

(i) $\frac{\partial^2 \varphi}{\partial x^2} \hat{i} + \frac{\partial^2 \varphi}{\partial y^2} \hat{j} + \frac{\partial^2 \varphi}{\partial z^2} \hat{k}$

(ii) $\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} + \frac{\partial \varphi}{\partial z}$

(iii) $\frac{\partial \varphi}{\partial x} \hat{i} + \frac{\partial \varphi}{\partial y} \hat{j} + \frac{\partial \varphi}{\partial z} \hat{k}$

(iv) $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}$

(f) Which of the following is equivalent to $\nabla \cdot (\vec{A} \times \vec{B})$?

(i) $\vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$

(ii) $\vec{B} \times (\nabla \cdot \vec{A}) - \vec{A} \times (\nabla \cdot \vec{B})$

(iii) $\vec{B} \cdot (\nabla \cdot \vec{A}) - \vec{A} \cdot (\nabla \cdot \vec{B})$

(iv) $\vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \cdot \vec{B})$

(g) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\nabla \cdot \vec{r} =$

(i) 0

(ii) 3

(iii) -3

(iv) 1

(h) If $\vec{f} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$, then the value of $\nabla \cdot \vec{f}$ is

(i) $2(x + y + z)$

(ii) $x + y + z$

(iii) $x + 2y + z$

(iv) $2x + y + 2z$

(i) The curl of a vector function $\vec{F} = x^2\hat{i} + 2z\hat{j} - y\hat{k}$ is

(i) $-3\hat{k}$

(ii) $-3\hat{i}$

(iii) 0

(iv) $-3\hat{j}$

(j) By Green's theorem $\oint_c (Mdx + Ndy) =$

$$(i) \iint_R \left\{ \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right\} dx dy$$

$$(ii) \iint_R \left\{ \frac{\partial N}{\partial y} - \frac{\partial M}{\partial x} \right\} dx dy$$

$$(iii) \iint_R \left\{ \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right\} dx dy$$

$$(iv) \iint_R \left\{ \frac{\partial M}{\partial x} - \frac{\partial N}{\partial y} \right\} dx dy$$

2. Answer the following questions : **(any five)**
2×5=10

(a) Prove that

$$[\overline{abc}] = [\overline{bca}] = [\overline{cab}]$$

(b) Prove that

$$[\overline{a}(\overline{b} + \overline{c})\overline{d}] = [\overline{abd}] + [\overline{acd}]$$

(c) If \overline{a} and \overline{b} are differentiable vector functions of a scalar t , then show that

$$\frac{d}{dt}(\overline{a} + \overline{b}) = \frac{d\overline{a}}{dt} + \frac{d\overline{b}}{dt}$$

(d) A particle moves along the curve

$$x = 3t^2, \quad y = t^2 - 2t, \quad z = t^3$$

Find the velocity and acceleration at $t = 1$

(e) Prove that

$$\text{div}(\text{curl } \overline{F}) = 0, \text{ where } \overline{F} = x\hat{i} + y\hat{j} = z\hat{k}$$

(f) If $\varphi = x^3 + y^3 + z^3 - 3xyz$ find $\text{div}(\text{grad } \varphi)$.

(g) Evaluate

$$\iint_S x^2 dy dz + y^2 dz dx + 2z(xy - x - y) dx dy$$

where S is the surface of the cube

$$0 \leq x \leq 1, \quad 0 \leq y \leq 1, \quad 0 \leq z \leq 1$$

3. Answer the following questions : (any six)

$$5 \times 6 = 30$$

- (a) Prove that the necessary and sufficient condition for a vector $\vec{f}(t)$ to have constant magnitude is that

$$\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$$

- (b) Find the constant P such that the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + p\hat{j} + 5\hat{k}$ are coplaner.

- (c) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

- (d) Established the identify.

$$\vec{a} = (\hat{i} \cdot \vec{a})\hat{i} + (\hat{j} \cdot \vec{a})\hat{j} + (\hat{k} \cdot \vec{a})\hat{k}$$

- (e) If $\vec{a} = [pqr]$, then show that

$$\frac{d\vec{a}}{dt} = \left[\frac{dp}{dt} \vec{qr} \right] + \left[\vec{p} \frac{dq}{dt} \vec{r} \right] + \left[\vec{pq} \frac{dr}{dt} \right]$$

- (f) Show that

$$\text{curl}(\vec{A} + \vec{B}) = \text{curl} \vec{A} + \text{curl} \vec{B}$$

- (g) Find the unit normal to the surface

$$2x^2y + 3yz = 4 \text{ at the point } (1, -1, -2)$$

- (h) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2\hat{i} + y^3\hat{j}$ and C is the arc parabola $y = x^2$ in the xy -plane from $(0,0)$ to $(1,1)$

- (i) If \vec{F} is a conservative field, prove that $\text{curl} \vec{F} = \nabla \times \vec{F} = 0$.

4. Answer the following questions : **(any two)**
10×2=20

(a) If $\vec{r} = \vec{a} \cos \mu t + \vec{b} \sin \mu t$, where \vec{a} and \vec{b} are two constant vectors and μ is a constant scalar, prove that

$$\frac{d^2 \vec{r}}{dt^2} + \mu^2 \vec{r} = 0$$

Also, show that

$$\vec{r} \times \frac{d\vec{r}}{dt} = \mu(\vec{a} \times \vec{b})$$

(b) If \vec{a} is a constant vector, prove that

$$(i) \quad \nabla(\vec{a} \cdot \vec{r}) = \vec{a}$$

$$(ii) \quad \nabla \cdot (\vec{a} \times \vec{r}) = 0 \quad 5+5=10$$

(c) State and prove the Green's theorem.
2+8=10

(d) Prove that

$$(i) \quad \text{curl}(\phi \vec{A}) = (\text{grad } \phi) \times \vec{A} + \phi \text{curl } \vec{A}$$

$$(ii) \quad \text{div}(\text{grad } \phi) = \nabla^2 \phi \quad 5+5=10$$