13

CHAPTER

Propagation of Light Waves

13.1. INTRODUCTION

Propagation of light is an electromagnetic phenomenon. Light waves are known to behave like any other electromagnetic wave. The physical laws describing propagation, reflection, refraction, attenuation of electromagnetic waves accurately describe the behaviour of light waves. In the electromagnetic theory of light, the mechanical displacement of the medium is replaced by a variation of the electric field at the corresponding point. The light wave consists of varying electric and magnetic fields and can be described by two vectors, the amplitude of electric field strength, **E**, and the amplitude of the magnetic field strength, **H**. These vectors oscillate at right angles to each other and to the direction of propagation. They cannot be separated. Thus, a beam of light is a traveling configuration of electric and magnetic fields. The electromagnetic theory was developed by James Clerk Maxwell in 1873. The first and foremost outcome of Maxwell's equations was the prediction of existence of electromagnetic waves and the brilliant prediction that light waves are electromagnetic waves. Starting from the Maxwell's equations, the propagation of light waves in space and materials can be easily explained and all the fundamental laws of optics such as law of reflection, Snell's law of refraction etc can be derived. Expressions to calculate the reflectivity, transmissivity and absorptivity of material media can be obtained from these equations. Further, the electromagnetic

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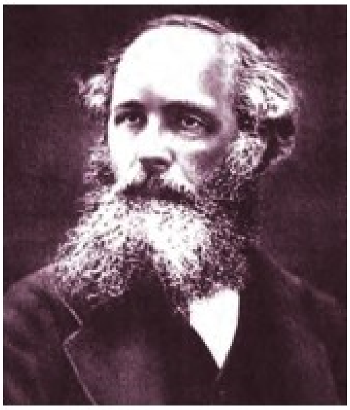
theory explains the origin of refractive index and the dispersion properties of optical media. In this chapter we briefly acquaint ourselves with the propagation of light waves and their interaction with material media.

13.2. MAXWELL'S EQUATIONS



James Clerk Maxwell (1831–1909)

Maxwell unified the theories of electricity and magnetism by way of deducing four very important equations which combine the experimental observations reported by Gauss, Ampere, and Faraday with his concept of displacement current. The equations encapsulate the connection between the electric field and electric charge and between the magnetic field and electric current. The Maxwell's equations also define the bilateral coupling between the electric and magnetic field quantities. They along with some auxiliary equations form the fundamental tenets of electromagnetic theory. When the charge and current sources vary with time, the electric and magnetic fields become interconnected and the coupling between them produces electromagnetic waves capable of traveling through free space and in material media. In all, there are four Maxwell's equations. These equations cannot be derived since they are the fundamental axioms or postulates of electrodynamics, obtained with the help of generalization of experimental results.

The Maxwell's equations are expressed in differential form and integral form in the following way:

Law	Differential Form	Integral Form	
Gauss's law	$\nabla \cdot \mathbf{D} = \rho$	$\oint \mathbf{D} \cdot \mathbf{dS} = \int \rho \, dV$	(13.1)
Faraday's law	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot \mathbf{dl} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{dS}$	(13.2)
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oint \mathbf{B} \cdot \mathbf{dS} = 0$	(13.3)
Ampere's law	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot \mathbf{dl} = \int \left[\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \right] \cdot \mathbf{dS}$	(13.4)

Maxwell's equations contain only the first derivatives of fields **E** and **B** with respect to time and space coordinates and the first powers of densities ρ and J of electric charges and currents. Therefore, these equations are linear and the fields obey superposition principle.

13.2.1. PHYSICAL SIGNIFICANCE OF MAXWELL'S EQUATIONS

The physical significance of Maxwell's equations can be readily interpreted from their mathematical statement in the integral form.

(1) Maxwell's first equation (13.1) shows that the total electric flux density **D** through the surface enclosing a volume is equal to the charge density ρ within the volume. It means that a charge distribution generates a steady electric field.

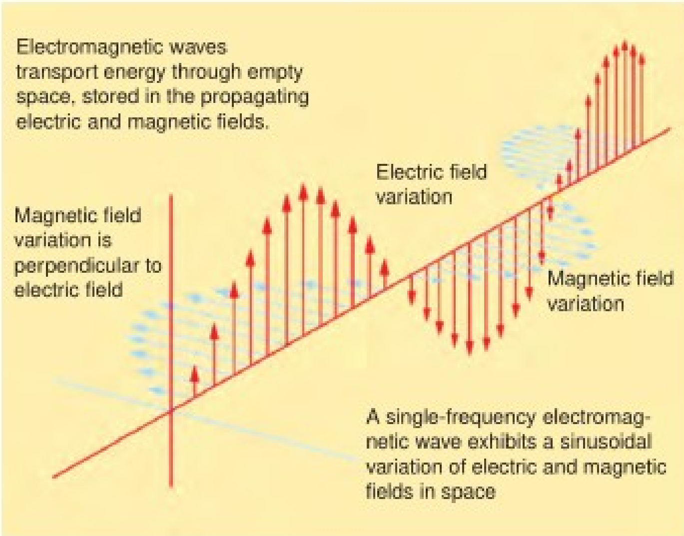
(2) The second equation shows that the e.m.f. around a closed path is equal to the time derivative of the magnetic flux density through the surface bounded by the path. It means that an electric field can also be generated by a time-varying magnetic field.

(3) Maxwell's third equation tells us that the net magnetic flux through a closed surface is zero. It implies that magnetic poles do not exist separately in the way as electric charges do. Thus, in other words, magnetic monopoles do not exist.

(4) Maxwell's fourth equation shows that the magneto motive force around a closed path is equal to the conduction current plus the time derivative of the electric flux density D through any surface bounded by the path. The time derivative of the electric flux density $\partial D / \partial t$ is called displacement current. Thus this equation means that a magnetic field is generated by a time-varying electric field.

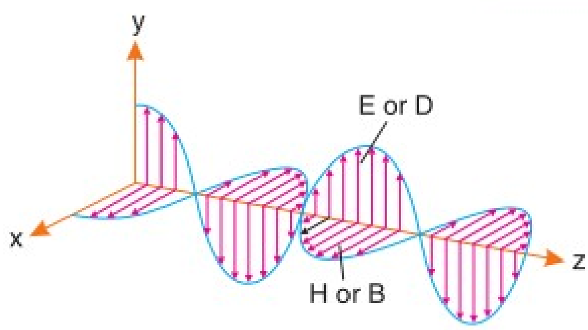
13.2.2. ELECTROMAGNETIC WAVES

Maxwell showed that by combining the four equations a *wave equation* was obtained which described the propagation of waves. The time variation of a magnetic field induces an electric field, while a variation of an electric field, in its turn, induces a magnetic field and electromagnetic fields can exist independently, without electric charges and currents. The continuous inter-conversion of the fields preserves them and an electromagnetic perturbation propagates in space. Such fields are called **electromagnetic waves**. The generation of electromagnetic wave does not require any medium. **The electromagnetic waves propagate through space entirely on their own.** Maxwell's theory placed no restriction on possible wavelengths for the electromagnetic radiation. The vector cross products in Maxwell's second and fourth equations imply that the two fields, **E** and **H** are normal to each other and also normal to the direction of propagation. In a vacuum, these waves always propagate at a velocity equal to the velocity of light c.



The vectors **E** and **B** in an electromagnetic wave always oscillate in phase (see Fig. 13.1). The instantaneous values of **E** and **B** at any point are connected through the relation

$$\sqrt{\varepsilon}\, E = \sqrt{\mu}\, H$$



The directions of E and H in a uniform plane wave.

Fig. 13.1

This means that **E** and **H** (or **B**) simultaneously attain their maximum values, vanish etc.

Note that electromagnetic waves are, in general, neither longitudinal nor transverse. In some types of electromagnetic waves, the electric vector **E** is at right angles to the ray while the magnetic vector **H** is not. Waves of this type are called **transverse electric waves** or TE waves. Since **H** vector is not normal to the ray, it must have a component along the ray direction. In another type of waves the magnetic vector is at right angles to the ray while the electric vector **E** is not. Waves of this type are called **transverse magnetic waves** or TM waves. If both the vectors **E** and **H** are at right angles to the ray, the waves are called **transverse electromagnetic waves** or TEM waves.

13.3. CONSTITUTIVE RELATIONS

The electric and magnetic properties of a medium are described by three quantities: relative permittivity,ε_r, the relative permeability,μ_r, and the conductivity,σ. If Maxwell's equations are to be extended to material media, they should be supplemented with relations which would contain these quantities characterizing individual properties of the medium. Such relations are called **constitutive relations** and also known as *material equations.*

The **permittivity** of a dielectric material is denoted by

$$\varepsilon = \varepsilon_o \varepsilon_r \text{ F/m} \tag{13.5}$$

where ε_r is a dimensionless quantity called the **relative permittivity** or **dielectric constant** of the material. ε_o is the **permittivity of free space** which is given by

$$\varepsilon_o = 8.854 \times 10^{-12} \text{ F/m}$$

In addition to the *electric field intensity* **E**, we often use the related quantity, the *electric flux density* **D**.

$$\mathbf{D} = \varepsilon \, \mathbf{E} \text{ C/m}^2 \tag{13.6}$$

Magnetic **permeability** of a material is denoted by

$$\mu = \mu_o \mu_r \text{ H/m} \tag{13.7}$$

where μ_r is a dimensionless quantity called the **relative permeability** of the material and μ_o is called the magnetic permeability of free space.

$$\mu_o = 4\pi \times 10^{-7} \text{ henry/m}$$

Magnetic field intensity is denoted by **H** and is related to magnetic induction **B** through

$$\mathbf{B} = \mu \, \mathbf{H} \tag{13.8}$$

The point form of Ohm's law describes the flow of current in a material and it is given by

$$\mathbf{J} = \sigma \, \mathbf{E} \tag{13.9}$$

where J is the current density and σ the electrical conductivity of the medium.

We usually assume ε_r,μ_r, and σ are not functions of time and that the medium is linear, homogeneous, isotropic; ε, μ, and σ are therefore constant and uniform throughout the medium. A medium is a **homogeneous medium** when the quantities ε, μ, and σ are constant throughout the medium. The medium is **isotropic** if ε is a scalar constant, so that **D** and **E** have everywhere the same direction. Material equations have the simplest form for sufficiently weak electric fields. Equations (13.6), (13.8), and (13.9) hold good for the case of isotropic non-ferroelectric and non-ferromagnetic media.

When the relations (13.6), (13.8), and (13.9) are inserted in the Maxwell's equations (13.1) to (13.4) we get the following differential equations relating the electric and magnetic field strengths **E** and **H**. If they are then solved as simultaneous equations, they will determine the laws which both **E** and **H** must obey.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon} \tag{13.10a}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{13.10b}$$

$$\nabla . \mathbf{H} = 0 \tag{13.10c}$$

$$\nabla \times \mathbf{H} = J + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \tag{13.10d}$$

13.4. WAVE EQUATION FOR FREE-SPACE

Maxwell's equations are coupled partial differential equations in **E** and **H** which cannot be solved in general. In order to simplify the equations, we should uncouple the set and obtain differential equations in **E** or **B** alone. In order to understand the nature of waves, we consider *free space*, which is a large empty volume of space. Free space is a perfect dielectric and does not contain charges ($\rho = 0$) and there are no conduction currents ($J = 0$) flowing in it. Maxwell's equations for free space (or a dielectric medium) become

$$\nabla \cdot \mathbf{E} = 0 \qquad (13.11a) \qquad\qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (13.11b)$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (13.11c) \qquad\qquad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \qquad (13.11d)$$

In order to describe the propagation of an electromagnetic wave in free space, we need to derive wave equation for **E** and **B** and then solve them to obtain explicit expressions for **E** and **B** as functions of (x,y,z). We start by taking curl of both sides of equ.(13.11b) and obtain

$$\nabla \times (\nabla \times \mathbf{E}) = -\nabla\left(\frac{\partial \mathbf{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{H})$$

Using equ.(13.11d) into the above equation, we get

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu_o \varepsilon_o \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

But $\quad \nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$

But $\quad \nabla \cdot \mathbf{E} = 0$

$\therefore \quad \nabla \times (\nabla \times \mathbf{E}) = -\nabla^2 \mathbf{E}$

$$\therefore \quad \nabla^2 \mathbf{E} = \mu_o \varepsilon_o \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad (13.12a)$$

This is the law that **E** must obey.

A similar procedure for **B** gives us

$$\therefore \quad \nabla^2 \mathbf{B} = \mu_o \varepsilon_o \frac{\partial^2 \mathbf{B}}{\partial t^2} \qquad (13.12b)$$

This is the law that **B** must obey.

Equations (13.12a) and (13.12b) are very much similar to the general wave equation (12.16), and constitute **wave equations**. They are *three-dimensional vector wave equations* and describe the propagation of an electromagnetic wave through a uniform medium. Since the wave equations for **E** and **B** are of the same form, their solutions will also have the same form. The solution to (13.12) leads to waves that can exist in free space. Even though the electric and magnetic fields of the waves start out on charges and currents, they detach themselves from them and move through free space as independent entities.

13.4.1. VELOCITY OF THE ELECTROMAGNETIC WAVE

The propagation characteristics of the electromagnetic wave are contained in the solution of equ.(13.12). To bring out the characteristics, we compare it with the general wave equation (12.16).

$$\nabla^2 \xi = \frac{1}{\upsilon^2} \frac{\partial^2 \xi}{\partial t^2}$$

The comparison gives

$$\upsilon^2 = \frac{1}{\mu_o \; \varepsilon_o}$$

or

$$\upsilon = \frac{1}{\sqrt{\mu_o \; \varepsilon_o}} \qquad (13.13)$$

Substituting the values of μ_o and ε_o , we find that

$$\frac{1}{\sqrt{\mu_o \varepsilon_o}} = \frac{1}{\sqrt{(4\pi \times 10^{-7}\, wb.A.m^2)\left(8.9 \times 10^{-12}\, C^2 / N.m^2\right)}} = 3.0 \times 10^8 \text{ m/s.}$$

$$\therefore \qquad \frac{1}{\sqrt{\mu_o \varepsilon_o}} = c \text{, the velocity of light} \qquad (13.14)$$

Obviously, electromagnetic waves travel with the velocity of light in free space.

13.4.2. RELATION BETWEEN THE REFRACTIVE INDEX AND RELATIVE PERMITTIVITY OF A MEDIUM

In case of a medium other than vacuum, we have to use $\varepsilon_o \varepsilon_r$ (= ε) and μ_o μ_r (= μ), instead of ε_o and μ_o in equ.(13.12). We then get

$$\upsilon = \frac{1}{\sqrt{\mu_o \mu_r \varepsilon_o \varepsilon_r}} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \cdot \frac{1}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

or $$\frac{c}{\upsilon} = \sqrt{\mu_r \varepsilon_r}$$

But $$\frac{c}{\upsilon} = n \qquad \text{(refractive index of the medium)}$$

$$\therefore \qquad n = \sqrt{\mu_r \varepsilon_r}$$

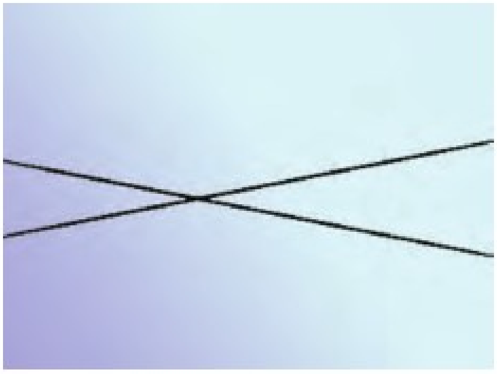
For a non-magnetic medium $\mu_r = 1$. Therefore,

$$n = \sqrt{\varepsilon_r} \qquad (13.15)$$

or $$n^2 = \varepsilon_r$$

13.5. UNIFORM PLANE WAVES

When energy is emitted by a source, it expands outwardly from the source in the form of spherical waves. The spherical wave travels at the same speed in all directions and therefore expands at the same rate. To an observer very far away from the source, the wave front of the spherical wave appears approximately planar. A plane wave is the simplest example of wave motion. In a plane wave, the electric and magnetic intensities are of constant value over any plane perpendicular to the direction of propagation. Such a plane is a surface of equal phase. A **plane wave** is thus a wave for which the phase has the same value at all points on an infinite plane. As the amplitude is also constant over the plane surface, it is called a **uniform plane wave**. Plane waves vary only in the direction of propagation and are uniform in planes normal to the direction of propagation. In this chapter we confine ourselves to the plane wave propagation in unbounded media. Plane wave propagation can be described by Cartesian coordinates, which are easier to work with mathematically than the spherical coordinates needed for describing propagation of a spherical wave.



Two Plane Waves

13.5.1. THE TRANSVERSE NATURE OF PLANE WAVES

Let us assume that the plane waves are traveling along the z-direction and hence **E** is constant over any given plane parallel to the xy-plane. Similarly **H** is constant over the xy-plane. We then have

$$\frac{\partial \mathbf{E}}{\partial x} = \frac{\partial \mathbf{E}}{\partial y} = 0 \text{ and } \frac{\partial \mathbf{H}}{\partial x} = \frac{\partial \mathbf{H}}{\partial y} = 0 \tag{13.16}$$

These relations imply that

$$\frac{\partial E_x}{\partial x} = 0, \frac{\partial E_y}{\partial x} = 0, \frac{\partial E_z}{\partial x} = 0 \text{ and } \frac{\partial E_x}{\partial y} = 0, \frac{\partial E_y}{\partial y} = 0, \frac{\partial E_z}{\partial y} = 0 \tag{13.17a}$$

$$\frac{\partial H_x}{\partial x} = 0, \frac{\partial H_y}{\partial x} = 0, \frac{\partial H_z}{\partial x} = 0 \text{ and } \frac{\partial H_x}{\partial y} = 0, \frac{\partial H_y}{\partial y} = 0, \frac{\partial H_z}{\partial y} = 0 \tag{13.17b}$$

According to Maxwell' equation (13.11a), we have

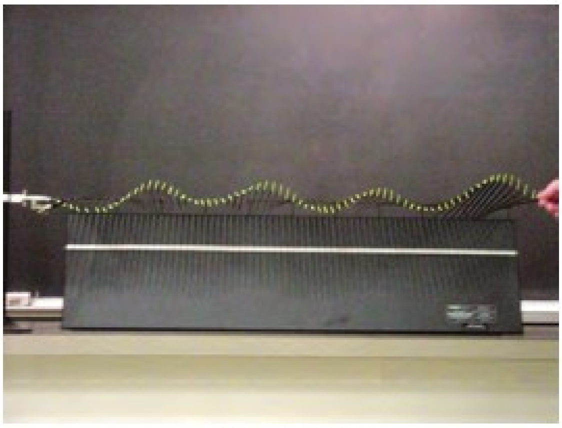
$$\nabla . \mathbf{E} = \mathbf{0}$$

which means that

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

Since $$\frac{\partial E_x}{\partial x} = 0 \text{ and } \frac{\partial E_y}{\partial y} = 0, \text{ then } \frac{\partial E_z}{\partial z} = 0. \tag{13.18a}$$

It means that E_z is independent of z and has the same value all along the z-axis. Wave motion consists of changing values of **E** along the direction of propagation. As E_z is constant along z-direction, it does not contribute to the wave motion and therefore E_z must be zero. $E_z = 0$ implies that **E** lies in a plane perpendicular to the direction of propagation of the wave. Hence, the electric wave is a *transverse* wave.



Transverse wave.

Again, according to Maxwell' equation (13.11c), we have

$$\nabla \cdot \mathbf{B} = \mathbf{0} \text{ and hence } \nabla \cdot \mathbf{H} = \mathbf{0}$$

which means that

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$$

Since $$\frac{\partial H_x}{\partial x} = 0 \text{ and } \frac{\partial H_y}{\partial y} = 0, \text{ then } \frac{\partial H_z}{\partial z} = 0. \tag{13.18b}$$

Thus, H_z is independent of z and has the same value all along the z-axis. As H_z is constant along z-direction, it does not contribute to the wave motion and therefore H_z must be zero. $H_z = 0$ implies that **H** lies in a plane perpendicular to the direction of propagation of the wave. Hence, the magnetic wave also is a *transverse* wave.

13.5.2. RELATION BETWEEN E AND H IN A UNIFORM PLANE WAVE

Now we assume that the variations of **E** are simple harmonic and that **E** is parallel to the y-axis. Then $E_z = 0$ and from equ.(13.12a), we may write the solution for a wave traveling in the positive z-direction,

$$E_y = E_1 \, e^{-i(\omega t - k z)} \tag{13.19a}$$

or in a simpler form as $$E_y = E_1 \cos \left[\omega\left(t - z/c\right)\right] \tag{13.19b}$$

Maxwell's equation (13.11b) can be written into the following three scalar equations.

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t} \qquad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} \qquad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t} \tag{13.20}$$

Using the results of equ.(13.17a) into the first relation of (13.20), we get

$$\frac{\partial E_y}{\partial z} = \frac{\partial B_x}{\partial t} \tag{13.21}$$

The magnetic flux density B_x associated with the electric field E_y can be found by integrating equ.(13.21). Thus,

$$B_x = \int \frac{\partial E_y}{\partial z}\, dt = -\frac{E_1 \omega}{c} \int \sin\left[\omega(t - z/c)\right] dt$$

$$= \frac{1}{c} E_1 \cos\left[\omega(t - z/c)\right]$$

The constant of integration is omitted since we are not interested in a constant component of the field.

$$\therefore \qquad B_x = \frac{1}{c} E_y \qquad (13.22)$$

or

$$H_x = \frac{1}{\mu_o c} E_y = \frac{\sqrt{\mu_o \varepsilon_o}}{\mu_o} E_y$$

$$\therefore \qquad H_x = \sqrt{\frac{\varepsilon_o}{\mu_o}}\, E_y \qquad (13.23)$$

Since E_y and H_x differ only by a scalar and have the same time dependence, **E** and **H** are in phase at all points in space and are mutually perpendicular. Their cross product **E × H** points in the direction of propagation denoted by the vector **k**. The three vectors are oriented as in a left-handed co-ordinate system, as depicted in Fig. 13.1, where *k* points along *z*-direction.

13.5.3. CHARACTERISTIC IMPEDANCE

Equ.(13.23) may be rewritten as

$$\frac{E_y}{H_x} = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

The above equation states that the ratio of the amplitudes of the vectors **E** and **H** always equal to the square root of the ratio of μ_o and ε_o. The ratio has the dimensions of impedance and hence it is called the **characteristic impedance** or **intrinsic impedance** of free space. It is denoted by η_o. We generalize equ.(13.23) by writing it as

$$\frac{\mathbf{E}}{\mathbf{H}} = \eta_o = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad (13.24)$$

Using the values of μ_o and ε_o into the above equation, we get

$$\eta_o = \sqrt{\frac{4\pi \times 10^{-7}\, H/m}{10^{-19}/36\pi\, F/m}} = 120\,\pi = 377\,\text{ohms} \qquad (13.25)$$

In a dielectric material, $\eta = 377/\varepsilon_r^{1/2} = 377/n$, where n is the index of refraction of the material.

13.6. WAVE POLARIZATION

The polarization of an electromagnetic wave is taken as the direction of the **E** vector. It describes *the shape and locus of the tip of the* **E** *vector* (in the plane orthogonal to the direction of propagation) *at a given point in space as a function of time*. The uniform plane wave described by equ.(13.19) has only y-component of electric field. The direction of the field vector at some point in space and time lies along a line in a plane perpendicular to the direction of propagation.

14 CHAPTER

Interference

14.1. INTRODUCTION

In 1680 Christian Huygens suggested that light propagates in the form of waves. Huygens did not know anything about the nature of the light wave; whether it is a transverse wave or a longitudinal wave; he had no knowledge about the speed of light or its wavelength. The wave theory of light received the first experimental evidence in 1801 from the interference experiments conducted by Thomas Young. Using the principle of superposition and Huygens' wave concept, Young explained the interference effects observed in various instances such as double slit experiment, the striking colours observed on oil slicks exposed to sunlight and Newton's rings.

14.2. LIGHT WAVES

A light wave is a harmonic electromagnetic wave consisting of periodically varying electric and magnetic fields oscillating at right angles to each other and also to the direction of propagation of the wave (see Fig.14.1a). The

Light wave.

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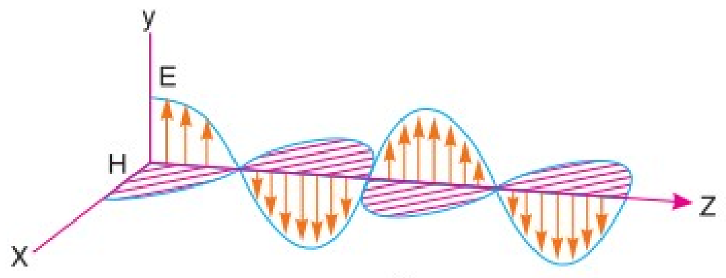
electric field in the wave is defined by the electric field strength vector **E** and the magnetic field by the vector of magnetic induction **B**. Vectors **E** and **B** are of equal importance to the wave. However, a light wave is often represented by the **E** wave (see Fig.14.1b), since many of the effects of light such as photoelectric effect, photochemical and physiological actions are found to be mostly due to the electric vector **E**. The vector **E** is often referred to as the **light vector** or **optical vector.** The electric field is known as **optical field**, radiation field, wave field or light field. The magnetic field is implied to be oscillating in a plane normal to the plane of the electric field oscillations and is not shown specifically in the diagrams.

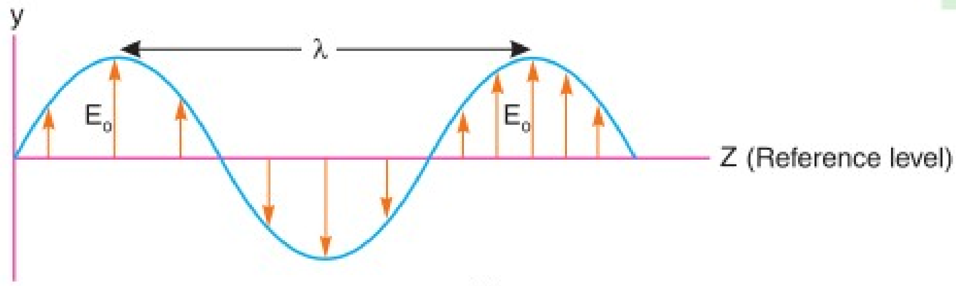


(a)

Fig. 14.1

The light wave depicted in Fig. 14.1(b) is mathematically represented by the expression

$$E = E_0 \sin(kz - \omega t) \tag{14.1}$$



(b)

Fig. 14.1

In fact, the wave shown in Fig. 14.1 (b) and represented by equ. (14.1) is an **ideal** electromagnetic wave. **No real light source emits such perfect waves.**

1. We note the following features about such a mathematical wave.

(*i*) The wave has a *single* definite frequency ν $(= \omega / 2\pi)$. In optics, waves having a single frequency and wavelength are called **monochromatic** waves.

(*ii*) It is a *harmonic* wave and is of **infinite extension** consisting of a continuous **train of waves**. At any instant the wave extends from $z = -\infty$ to $z = +\infty$, and at any point whose position corresponds to a particular value of *z* the wave continues from $t = -\infty$ to $t = +\infty$.

(*iii*) The amplitude of the wave E_0 stays constant as the wave propagates through air. Hence it is a plane wave and its wavefront is normal to the *z*-axis.

(*iv*) The electric vector **E** of the wave oscillates always parallel to a fixed direction in space, i.e. *y*-direction. In other words, the E-vibrations are confined to *yz* – plane. Therefore the wave is **plane polarized** (or **linearly polarized**).

Helium neon laser emitting red light.

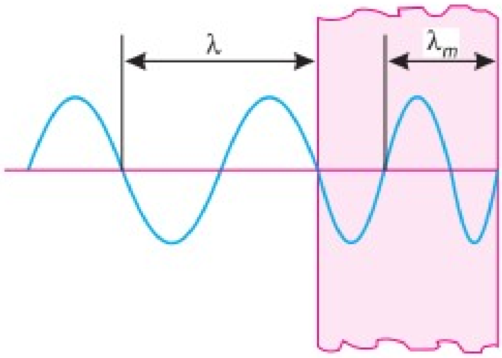
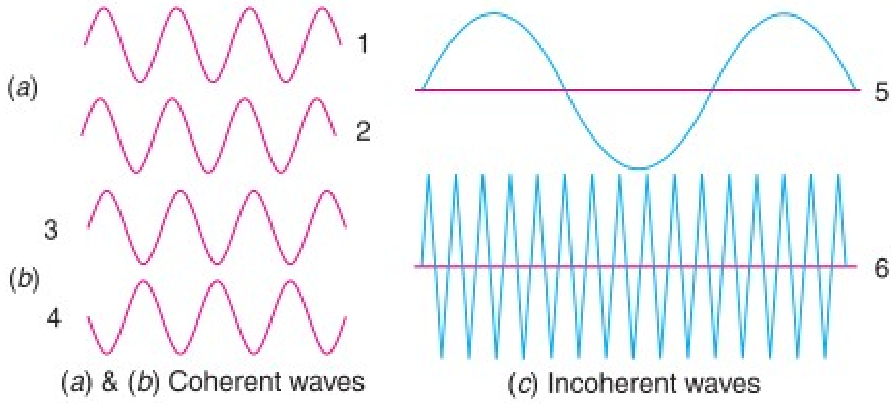


Fig. 14.2

The real light waves emitted by common light sources are far from ideal. They are wave trains of limited length, have a spread of frequencies, and are not plane polarized. Interference effects are easily observed when sinusoidal waves with a single frequency, ν, and wavelength, λ, overlap. While it is fairly easy to produce sound waves and r.f. waves of single frequency, common sources of light *do not emit* monochromatic light. The common light sources emit a continuous distribution of wavelengths. The sodium lamp used in optics laboratories is a fairly monochromatic source of light, which emits light in the yellow region at the wavelength 5893 Å. The most nearly monochromatic source is the helium-neon laser that emits red light at 6328 Å.

2. A light wave travels *slower* in an optical medium than in air or a vacuum. It travels with a velocity υ, which is less than 'c'. The wavelength of light wave *decreases in the medium*, as shown in Fig. 14.2 while its frequency remains constant.

3. Phase difference and coherence: Suppose two waves are passing through a point in space. If the frequencies of the two waves are different, the phase difference between the vibrations changes with time. The waves will drift out of phase because the crests of the higher frequency wave will arrive ahead of the crests of the lower frequency wave (see Fig. 14.3c). Also, if one (or both) of the waves undergoes changes in frequency irregularly, the phase difference changes irregularly. Under these conditions the two waves are said to be **incoherent**. The light emitted by most of the light sources is incoherent as the frequency of light changes abruptly and irregularly, though we can



(*a*) Waves 1 and 2 are in-phase, (*b*) Waves 3 and 4 are in opposite phase and (*c*) Waves 5 and 6 are incoherent.

Fig. 14.3

think of an average frequency associated with the light wave. On the other hand, if we consider two waves of same frequency, they may differ in the amplitudes but they maintain a predictable phase relationship (see Fig. 14.3a). The difference in their phases may have any value from zero radians to a maximum of 2π radians; but the phase difference remains constant. Thus, two or more waves of the *same* frequency can maintain the same phase or constant phase difference over a distance and time. Such waves are said to be **coherent waves**.

When coherent waves rise or fall together, reaching the crest (or trough) at the same time, they are said to be **in phase,** as shown in Fig. 14.3 (a). The phase difference will be 0 or 2π radians, which remains constant as the waves propagate in space. The path difference between the waves will then be zero or an integral multiple of a wavelength, λ. Such waves move through space with a *crest-to-crest correspondence.* When one wave reaches its crest while the other falls to its trough, then the phase difference between the waves is π and the waves are said to be in **opposite phase** and the phase difference remains 180^o (see Fig. 14.3b). They are inverted with respect to each other everywhere. Then the path difference between the waves will be $\lambda / 2$ or an odd integral multiple of $\lambda / 2$.

The waves may have also any *constant phase difference* other than zero or π radians.

If two (or more) waves maintain a constant phase difference over a long distance and time, then they are said to be coherent.

Two waves of different frequency (see Fig. 14.3c) can never maintain a constant phase difference, because their phase difference goes on fluctuating and changes arbitrarily. They are said to be **incoherent**, as the phase difference between the waves fluctuates with time.

Thus, **coherence** means the coordinated motion of several waves in a medium maintaining a fixed and predictable phase relationship over a length of time.

The waves shown in Fig. 14.3 (a) and (b) are coherent waves. Sources, which produce coherent waves, are called **coherent sources**.

In the study of interference, we focus our attention mainly on the two specific types of disposition of the waves, namely in phase and opposite phase dispositions, at the point of observation.

4. Optical path and Phase change: Optical path length indicates the number of light waves that fit into that path. Thus, $\Delta = N\lambda$, where Δ is the optical path length and N is an integer or a mixed fraction. Optical path is related to the geometric path and the relation may be found as follows. The distance traversed by light in a medium of refractive index μ in time t is given by

$$L = \upsilon\, t \tag{14.2}$$

where υ is the velocity of light in the medium. The distance travelled by light in a vacuum in the same time t, is $\Delta = c\, t = c \cdot \dfrac{L}{\upsilon} = \mu\, L$

The distance L is called the *geometric path length* (GPL). Δ is the equivalent distance in a vacuum and is called *optical path length* (OPL). Thus,

$$O.P.L. = \mu \times G.P.L.$$

or
$$\Delta = \mu\, L \tag{14.3}$$

The above result means that *more number of waveforms is accommodated along the optical path than in the corresponding geometrical path.* Therefore, ***in the study of interference we always must calculate the optical paths travelled by light rays.***

(a) Effect of Optical path: Optical path determines the **phase** of a light wave arriving at a point. We know that if a wave covers in air a distance of one wavelength, i.e. 1λ, its phase changes by 2π radians. Therefore, we compute that if a wave travels a distance L in air, its phase change is given by

$$\delta = \frac{2\pi L}{\lambda} \tag{14.4}$$

When the wave travels the distance L in a medium, then

$$\delta = \frac{2\pi\Delta}{\lambda} = \frac{2\pi\mu L}{\lambda} \tag{14.5}$$

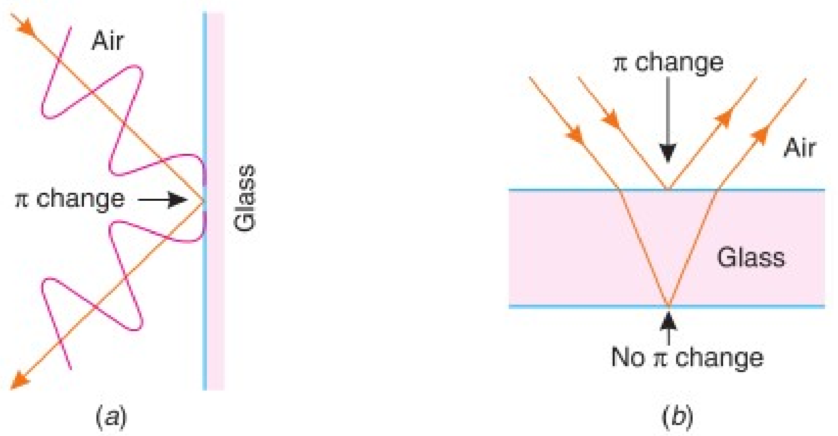
where Δ is the optical path or optical path difference.

Comparing equns. (14.4) and (14.5), we find that a light path of geometric length L in a medium of refractive index μ produces the same phase change as a light path of length μL in a vacuum.

(b) Effect of reflection: The process of reflection also affects the phase of a light wave. When light is incident on a surface, part of the light gets reflected while a major portion may be transmitted or absorbed. The quantity characterizing the reflectivity of a surface is called the **reflection coefficient,** ρ. ρ depends on the nature of surface, the angle of incidence of light and many other factors. Augustin Jean Fresnel, about 150 years ago, derived a set of expressions which allow us to calculate the amount of light reflected and transmitted at an interface. For normal incidence it was shown that

$$\rho = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \tag{14.6}$$

It may be seen that ρ is positive when $\mu_2 < \mu_1$. It implies that the oscillations in the incident and reflected waves occur in the same phase. On the other hand, when $\mu_2 > \mu_1$, ρ is *negative* signifying that the *oscillations in the incident and reflected waves are in opposite phase*. Hence, we draw the following conclusions:



Phase change due to reflection.

Fig. 14.4

(*i*) A light wave travelling from a rarer medium (μ_1) to a denser medium (μ_2) undergoes a phase change of π radians when it gets reflected at the boundary of denser medium, as shown in Fig. 14.4 (a). The wave loses a half-wave on reflection at the boundary of rarer-to-denser medium.

(*ii*) A light wave travelling from a denser medium (μ_2) to a rarer medium (μ_1) does not undergo a change in phase on reflection at the boundary of denser-to-rarer medium (Fig. 14.4b). Therefore, the change in path is zero.

14.3. SUPERPOSITION OF WAVES

Frequently it is necessary to find the resultant disturbance at a point when a number of disturbances arrive simultaneously. According to the **principle of superposition—**

when two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

It means that the resultant is simply the sum of the disturbances. The principle of superposition applies to electromagnetic waves also and is the most important principle in wave optics. In case of electromagnetic waves, the term *displacement* refers to the amplitude of the electric field vector.

Interference is an important consequence of superposition of *coherent* waves.

14.4. INTERFERENCE

If two or more light waves of the same frequency overlap at a point, the resultant effect depends on the *phases* of the waves as well as their *amplitudes*. The resultant wave at *any point* at any instant of time is governed by the **principle of superposition.** The combined effect at each point of the region of superposition is obtained by adding algebraically the amplitudes of the individual waves. Let us assume here that the component waves are of the *same amplitude*.

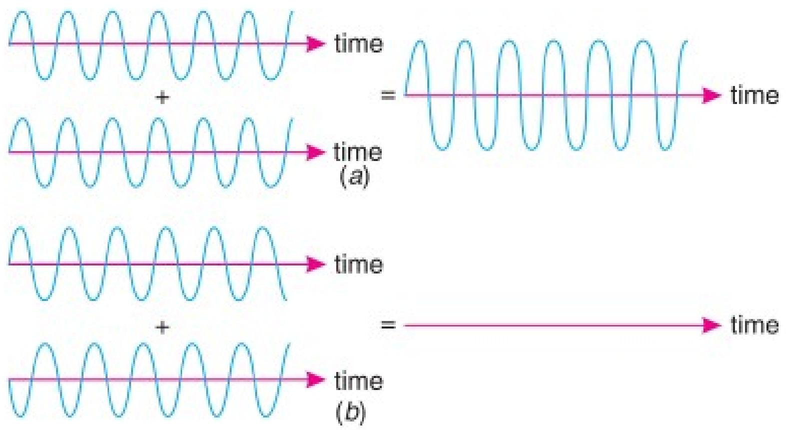


Fig. 14.5

At certain points, the two waves may be *in phase*. The amplitude of the resultant wave will then be equal to the sum of the amplitudes of the two waves, as shown in Fig.14.5(a). Thus, the amplitude of the resultant wave

$$A_R = A + A = 2A. \qquad (14.7)$$

Hence, the intensity of the resultant wave is

$$I_R \propto A_R^2 = 2^2 A^2 = 2^2 I. \qquad (14.8)$$

It is obvious that the resultant intensity is greater than the sum of the intensities due to individual waves.

$$I_R > I + I = 2I \qquad (14.9)$$

Therefore, the interference produced at these points is known as **constructive interference**. A **stationary bright band** of light is observed at points of constructive interference.

At certain other points, the two waves may be *in opposite phase*. The amplitude of the resultant wave will then be equal to the sum of the amplitudes of the two waves, as shown in Fig.14.4 (b). Thus, the amplitude of the resultant wave

$$A_R = A - A = 0. \tag{14.10}$$

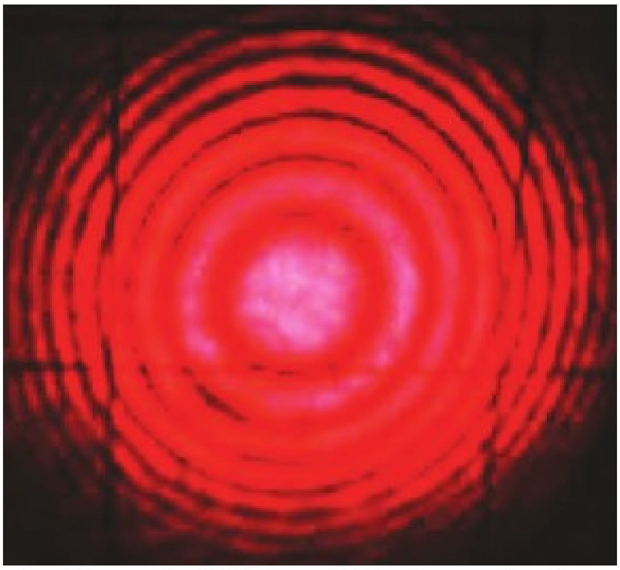
Hence, the intensity of the resultant wave is

$$I_R \propto 0^2 = 0. \tag{14.11}$$

It is obvious that the resultant intensity is less than the sum of the intensities due to individual waves.

$$I_R < 2I \tag{14.12}$$

Therefore, the interference produced at these points is known as **destructive interference. A stationary dark band** of light is observed at points of destructive interference. Thus, we see that a redistribution of energy took place in the region.



In the central bright spot, there is constructive interference and then a destructive interference ring and then constructive, and so on.

Thus, when two or more coherent waves of light are superposed, the resultant effect is **brightness** in certain regions and **darkness** at other regions. The regions of brightness and darkness alternate and may take the form of straight bands, or circular rings or any other complex shape. The alternate bright and dark bands are called **interference fringes. The phenomenon of redistribution of light energy due to the superposition of light waves from two or more coherent sources is known as interference.**

Whether the condition (14.7) occurs or (14.10) occurs at a point is *solely* determined by the difference in the optical paths traversed by the waves that are superposing at that point.

Let us consider two sources of light S_1 and S_2, as shown in Fig. 14.6. Let us assume that the sources are identical and produce harmonic waves of same wavelength and that the waves are in the same phase at S_1 and S_2. Light from these sources travel along different paths, S_1P and S_2P, and meet at a point P. We now wish to know whether we get brightness or darkness at P due to the superposition of waves.

Referring to Fig. 14.6, we find that the waves move along the geometric paths $S_1P = r_1$ and $S_2P = r_2$, which are different in length. Also, the media through which the two waves travelled, may be different. As a result, the optical path lengths are different. If μ_1 is the refractive index of the medium in which the ray S_1P travelled, the corresponding optical path length is $\mu_1 r_1$. Similarly, if μ_2 is the refractive index of the medium in which the ray S_2P travelled, the corresponding optical path length is $\mu_2 r_2$. These optical paths accommodate different number of waveforms along their lengths. The optical path difference between the waves at the point P is $(\mu_2 r_2 - \mu_1 r_1)$. It may come to a few full waves or a mixed fraction of waves. It means that though the waves started with the same phase, they may arrive at P with different phases because they travelled along different optical path lengths.

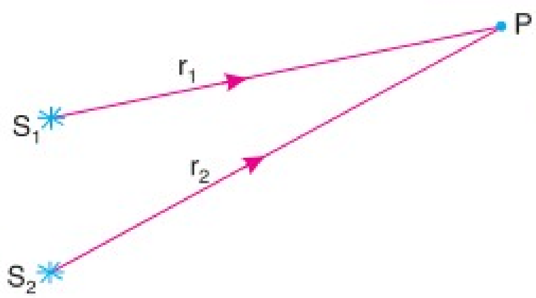


Fig. 14.6

If the optical path difference $\Delta = (\mu_2 r_2 - \mu_1 r_1)$ is equal to zero or an integral multiple of wavelength λ, then the waves arrive in phase at P and superpose with crest-to-crest correspondence. That is, if

$$\Delta = m\lambda \tag{14.13}$$

where m is an integer and takes values, m = 0,1,2,3,4,5,……,then the waves are in phase (see Fig.14.5a) and their overlapping at P produces constructive interference or brightness.

On the other hand, if the optical path difference $\Delta = (\mu_2 r_2 - \mu_1 r_1)$ is equal to an odd integral multiple of half-wavelength, $\lambda/2$, then the waves arrive out of phase at P and superpose with crest-to-trough correspondence. That is, if

$$\Delta = (2m+1)\frac{\lambda}{2} \tag{14.14}$$

where m is an integer and takes values, m = 0,1,2,3,4,5,…….,then the waves are inverted with respect to each other (see Fig. 14.5b) and their overlapping at P produces destructive interference or darkness.

The regions of brightness and darkness are also known as regions of **maxima** and **minima**.

14.4.1. THEORY OF INTERFERENCE

(a) Analytical Method: Let us assume that the electric field components of the two waves arriving at point P vary with time as

$$E_A = E_1 \sin \omega t \tag{14.15}$$

and

$$E_B = E_2 \sin (\omega t + \delta) \tag{14.16}$$

where δ is the phase difference between them. According to Young's principle of superposition, the resultant electric field at the point P due to the simultaneous action of the two waves is given by

$$E_R = E_A + E_B \tag{14.17}$$

$$= E_1 \sin \omega t + E_2 \sin (\omega t + \delta)$$

$$= E_1 \sin \omega t + E_2 (\sin \omega t \cos\delta + \cos \omega t \sin \delta)$$

$$= (E_1 + E_2 \cos \delta) \sin \omega t + E_2 \sin \delta \cos \omega t \tag{14.18}$$

Equ. (14.18) shows that ***the superposition of two sinusoidal waves having the same frequency but with a phase difference produces a sinusoidal wave with the same frequency but with a different amplitude E.***

Let

$$E_1 + E_2 \cos \delta = E \cos \phi \tag{14.19}$$

and

$$E_2 \sin \delta = E \sin \phi \tag{14.20}$$

where E is the amplitude of the resultant wave and ϕ is the new initial phase angle. In order to solve for E and ϕ, we square the equ.(14.19) and (14.20) and add them.

$$(E_1 + E_2 \cos \delta)^2 + E_2^2 \sin^2 \delta = E^2 (\cos^2 \phi + \sin^2 \phi)$$

or

$$E^2 = E_1^2 + E_2^2 \cos^2 \delta + 2E_1 E_2 \cos \delta + E_2^2 \sin^2 \delta$$

or

$$E^2 = E_1^2 + E_2^2 + 2E_1 E_2 \cos \delta \tag{14.21}$$

Thus, it is seen that the square of the amplitude of the resultant wave is not a simple sum of the squares of the amplitudes of the superposing waves, there is an additional term which is known as the *interference term.*

(b) Phasor diagram *and* phasor addition: A wave may be viewed either sideways or end-on. In sideways view, as the wave travels through a distance λ, the phase angle changes from 0 to 2π radians. In the end-on view we find a point on the profile of the wave oscillating linearly. The two perspectives may be combined as follows. A circle having a radius OA equal to the amplitude of the wave motions is drawn (see Fig. 14.7). Now consider a point P on the circumference of the circle; Q is the projection of P on the vertical axis. As P moves around the circumference with constant angular velocity ω, Q oscillates vertically. This is the end-on view. The Fig. 14.7 is called the **phasor diagram**. OP is called a **rotating vector** or a **phasor**. It means that the *length* of a phasor is proportional to the *amplitude* of the sinusoidal wave and the *projection* of a phasor on the vertical axis is proportional to the *instantaneous value* of the alternating quantity.

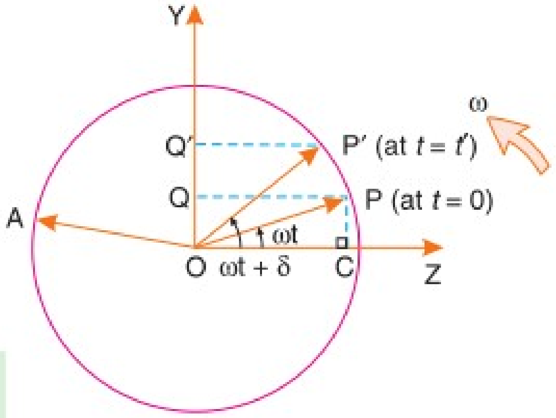


Fig. 14.7

Phasor representation may be used to *add sinusoidal functions* with a phase difference. The wave E_A is represented with a vector of amplitude E_1 rotating about the origin in a counter clockwise direction with an angular frequency ω (See Fig. 14.8 a). As the phasor E_1 rotates, the projection E_A oscillates along the vertical axis. The second wave, E_B, has amplitude E_2 and angular frequency ω but its phase is δ with respect to wave E_A. It is also shown in Fig. 14.8 (a). The resultant E_R is the sum of E_A and E_B obtained by drawing the phasors end to end, by placing the foot of one arrow at the head of the other (as in Fig. 14.8b), maintaining the proper phase difference. The whole assembly rotates counterclockwise about the origin. The sum of the projections on the vertical axis at any time gives the instantaneous value of the total field at a point. The amplitude E_R of the resultant sinusoidal wave at P is the *vector sum* of the other two phasors, as shown in Fig. 14.8 (b).

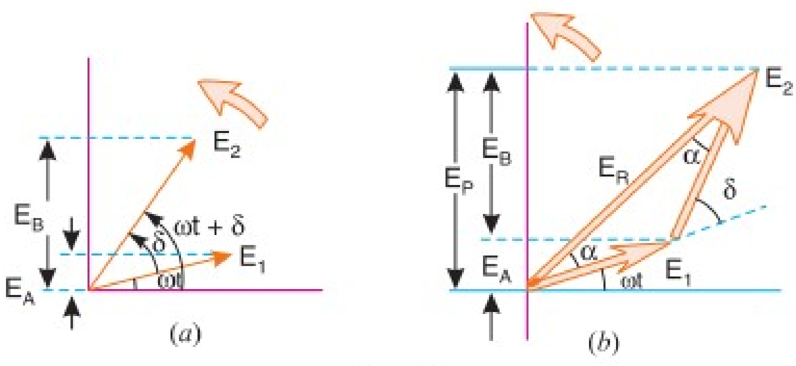


Fig. 14.8

To find E_R, we use the law of cosines.

$$E_R^2 = E_1^2 + E_2^2 - 2E_1E_2 cos(\pi\text{-}\delta)$$

or

$$E_R^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos\,\delta$$

This is the same as the equ.(14.21). This method is particularly convenient when several wave amplitudes have to be added.

14.4.2. INTENSITY DISTRIBUTION

The intensity of a light wave is given by the square of its amplitude.

$$I = \tfrac{1}{2}\varepsilon_o cE^2 \propto E^2$$

Using this relation into (14.21), we get

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\delta \tag{14.22}$$

We see that the resultant intensity at P on the screen is not just the sum of the intensities due to the separate waves. The term $2\sqrt{I_1 I_2}\ \cos\delta$ is known as the **interference term**. Whenever the phase difference between the waves is zero, i.e. $\delta = 0$, we have maximum amount of light. Thus,

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \tag{14.23}$$

When $I_1 = I_2 = I_o$ $\qquad I_{\max} = 4I_o$ (14.23a)

It means that the resultant intensity I will be *more than the sum* of the intensities due to the two sources.

When the phase difference is $\delta = 180^o$, $\cos 180^o = -1$ and we have a minimum amount of light.

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \tag{14.24}$$

which, when $I_1 = I_2$, becomes

$$I_{\min} = 0 \tag{14.24a}$$

It means that the resultant intensity I will be *less than the sum* of the intensities due to the two sources.

At points that lie between the maxima and minima, when $I_1 = I_2 = I_0$, we get

$$I = I_o + I_o + 2I_o\ \cos\delta$$
$$= 2I_o\left(1 + \cos\ \delta\right)$$

Then using the identity $1 + \cos\ \delta = 2\ \cos^2\left(\frac{1}{2}\delta\right)$, we get

$$I = 4I_o \cos^2\left(\tfrac{1}{2}\delta\right) \tag{14.25}$$

Equ. (14.25) shows that the intensity varies along the screen in accordance with the ***law of cosine square***. Fig. 14.9 shows the variation of intensity as a function of phase angle δ.

It is seen from the plot that the intensity varies from zero at the fringe minima to $4I_o$ at the fringe maxima.

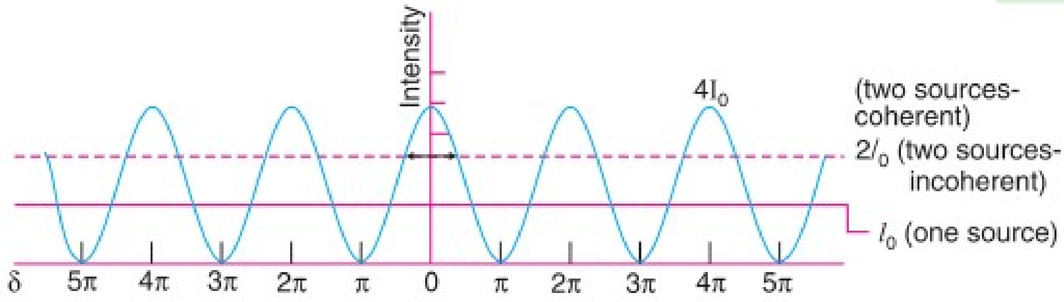


Fig. 14.9

14.4.3. SUPERPOSITION OF INCOHERENT WAVES

Incoherent waves are the waves that do not maintain a constant phase difference. Then the phase of the waves fluctuate irregularly with time and independently of each other. In case of light waves the phase fluctuates randomly at a rate of about 10^8 per second. Light detectors such as human eye, photographic film etc cannot respond to such rapid changes. The detected intensity is always the average intensity, averaged over a time interval which is very much larger than the time of fluctuation. Thus,

$$I_{ave} = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \langle \cos\delta \rangle$$

The average value of the cosine over a large time interval will be zero and hence the interference term becomes zero. Therefore, the average intensity of the resultant wave is

$$I_{ave} = I_1 + I_2$$

If $I_1 = I_2$, then $$I_{ave} = 2\,I \qquad (14.26)$$

It implies that the superposition of incoherent waves does not produce interference but gives a uniform illumination. The average intensity at any point is simply equal to the sum of the intensities of the component waves.

14.4.4. SUPERPOSITION OF MANY COHERENT WAVES

The result (14.23) may be written as

$$I_{max} = 2^2 I_o$$

which gives the resultant intensity when two coherent waves superpose. The resultant maximum intensity due to N coherent waves will be therefore

$$I_{max} = N^2 I_o \qquad (14.27a)$$

and the minimum intensity $$I_{min} = 0 \qquad (14.27b)$$

where N represents the number of coherent waves superposing at a point.

14.5. YOUNG'S DOUBLE SLIT EXPERIMENT – WAVEFRONT DIVISION

As early as in 1665 Grimaldi attempted to produce interference between two beams of light. He directed sunlight into a dark room through two pinholes in a screen, with an expectation that bright and dark bands would be observed in the area where the beams overlap on each other. He observed uniform illumination instead. In 1801, about one hundred thirty six years later, Thomas Young gave the first demonstration of the interference of light waves. Young admitted the sunlight through a single pinhole and then directed the emerging light onto two pinholes. Finally the light was received on a screen. The spherical waves emerging from the pinholes interfered with each other and a few coloured fringes were observed on the screen. The amount of light that emerged from the pinhole was very small and the fringes were faint and difficult to observe. The pinholes were later replaced with narrow slits that let through much more light. The sunlight was replaced by monochromatic light. Young's experiment is known as **double-slit experiment**.

Fig. 14.10 shows a plan view of the basic arrangement of the double slit experiment. The primary light source is a monochromatic source; it is generally a sodium lamp, which emits yellow light of wavelength at around 5893Å. This light is not suitable for causing interference because emissions from different parts of any ordinary source are ***not*** coherent. Therefore, the monochromatic light is allowed to pass through a narrow slit at S. The light coming out of the slit originated from only a small region of the light source and hence behaves more nearly like an ideal light source. Cylindrical wavefronts are produced from the slit S, the primary light source, which fall on the two narrow closely spaced slits, S_1 and S_2 as shown in Fig. 14.10. The slits at S_1 and S_2 are very narrow. The cylindrical waves emerging from the slits overlap. If the slits are *equidistant* from S, the *phase* of the wave at S_1 will be the same as the *phase* of the wave at S_2. Further, waves leaving S_1 and S_2 are therefore always *in phase*. Hence, sources S_1 and S_2 act as *secondary coherent sources*. The waves leaving from S_1 and S_2 interfere and produce alternate bright and dark bands on the screen at T.

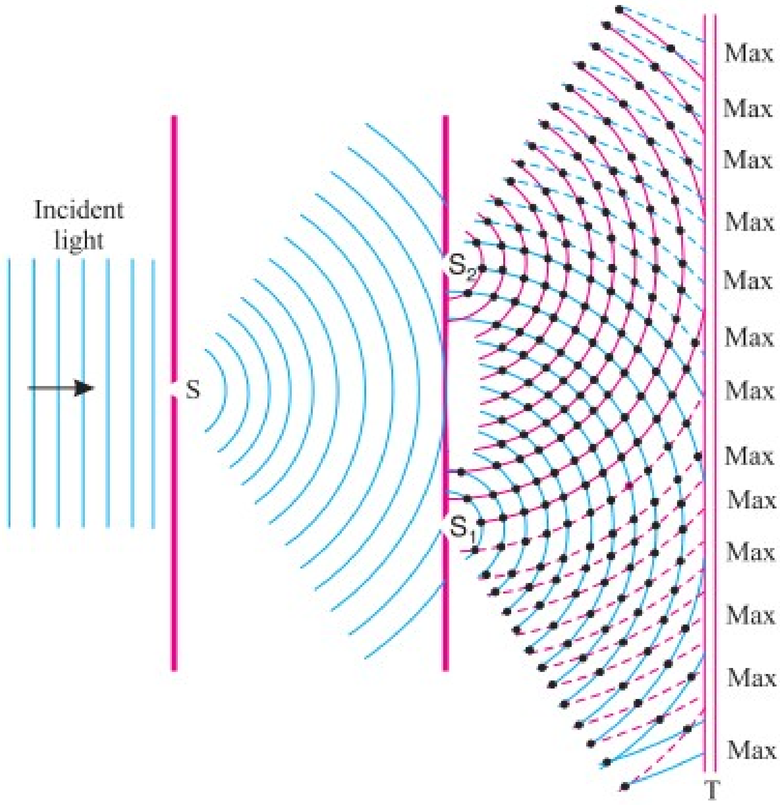


Fig. 14.10

14.5.1. OPTICAL PATH DIFFERENCE BETWEEN THE WAVES AT P:

Let P be an arbitrary point on screen T, which is at a distance D from the double slits. Let θ be the angle between MP and the horizontal line MO. Let S_1N be a normal on to the line S_2P. The distances

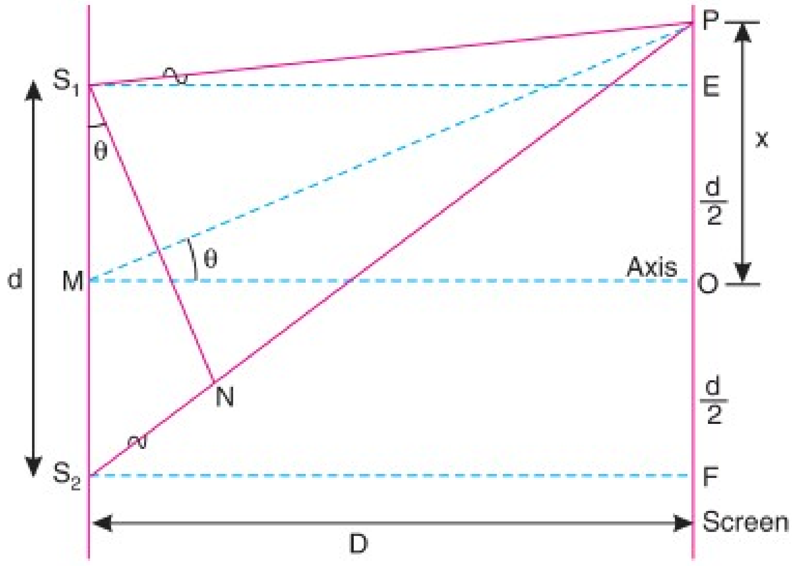


Fig. 14.11

PS_1 and PN are equal. The waves emitted at the slits, S_1 and S_2 are initially in phase with each other. The difference in the path lengths of these two waves is S_2N. We assume that the experiment is carried out in air. Therefore, the optical paths are identical with geometrical paths. The nature of the interference of the two waves at P depends simply on how many waves are contained in the length of the path difference S_2N. If S_2N contains an integral number of wavelengths, the two waves interfere constructively, producing a maximum in the intensity of light on the screen at P. If it contains an odd number of half-wavelengths, then the waves interfere destructively and produce a minimum intensity at P.

Let the point P be at a distance x from O (Fig.14.11). Then

PE = $x - d/2$ and PF = $x + d/2$.

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$(S_2P)^2 - (S_1P)^2 = 2xd$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

We can approximate that $S_2P \cong S_1P \cong D$.

$$\therefore \text{Path difference} = S_2P - S_1P = \frac{xd}{D} \qquad (14.28)$$

We now find out the conditions for observing bright and dark fringes on the screen.

14.5.2. BRIGHT FRINGES

Bright fringes occur wherever the waves from S_1 and S_2 interfere constructively. The first time this occurs is at O, the axial point. There, the waves from S_1 and S_2 travel the same optical path length to O and arrive in phase. The next bright fringe occurs when the wave from S_2 travels one complete wavelength further the wave from S_1. In general constructive interference occurs if S_1P and S_2P differ by a whole number of wavelengths.

The condition for finding a bright fringe at P is that

$$S_2P - S_1P = m\lambda$$

Using the equation (14.28), it means that

$$\frac{xd}{D} = m\lambda \qquad (14.29)$$

where m is called the **order of the fringe**.

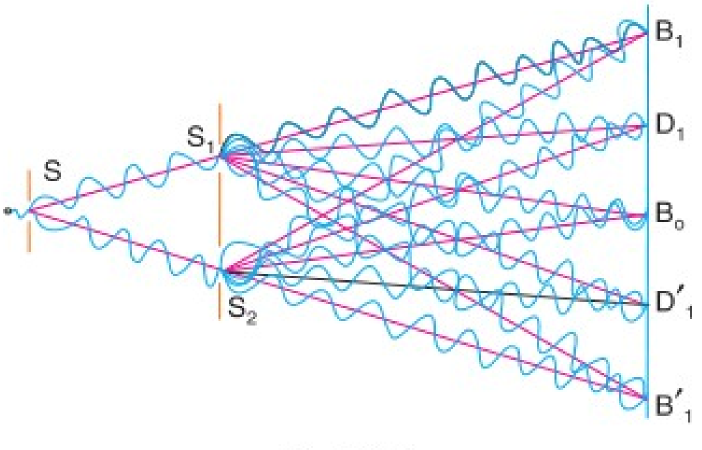
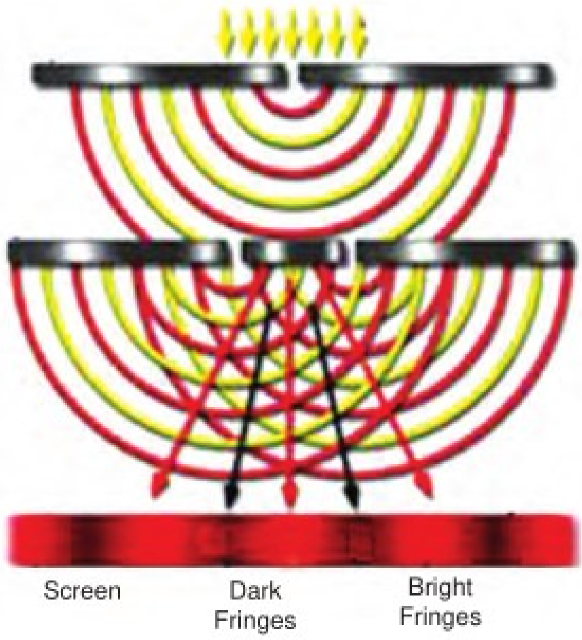


Fig. 14.12

The bright fringe B_o (at O), corresponding to m = 0, is called the *zero-order* fringe. It means the path difference between the two waves reaching at O is zero. Fringe at B_1 is the *first-order bright fringe* from the axis corresponding to m = 1; the path difference between the two waves reaching at B_1 is one λ. The *second order bright fringe* (m = 2) will be located where the path difference is 2λ and so on.



14.5.3. DARK FRINGES

The first dark fringe occurs when $(S_2P - S_1P)$ is equal to λ / 2. The waves are now in opposite phase at P. The second dark fringe occurs when $(S_2P - S_1P)$ equals 3λ / 2. The mth dark fringe occurs when

$$(S_2P - S_1P) = (2m + 1)\,\lambda / 2$$

The condition for finding a dark fringe is $$\frac{xd}{D} = (2m+1)\frac{\lambda}{2} \qquad (14.30)$$

The *first-order dark fringe* D_1 (Fig. 14.12) from the axis corresponds to m = 0, where the path difference between the two waves is λ/2. The second order dark fringe (m = 1) will be produced where the path difference is 3λ/2 and so on.

14.5.4. SEPARATION BETWEEN NEIGHBOURING BRIGHT FRINGES

The mth order fringe occurs when $x_m = \dfrac{m\lambda\, D}{d}$

and the (m+1)th order fringe occurs when $x_{m+1} = \dfrac{(m+1)\lambda\, D}{d}$

The fringe separation, β is given by $$\beta = x_{m+1} - x_m = \frac{\lambda D}{d} \qquad (14.31)$$

The same result will be obtained for dark fringes. Thus, the distance between any two consecutive bright or dark fringes is known as the ***fringe width*** and is the *same* everywhere on the screen. Further, the width of the bright fringe is equal to the width of the dark fringe. Therefore, the alternate bright and dark fringes are *parallel*. From the equ.(14.31), we find the following:

(*i*) The fringe width β is independent of the *order* of the fringe.It is directly proportional to the wavelength of light, i.e.$\beta \propto \lambda$. The fringes produced by red light are less closer compared to those produced by blue light.

(*ii*) The width of the fringe is *directly proportional* to the distance of the screen from the two slits, $\beta \propto D$. The farther the screen, the wider is the fringe separation.

(*iii*) The width of the fringe is *inversely proportional* to the distance between the two slits. The closer are the slits, the wider will be the fringes.

14.6. COHERENCE

Interference fringes did not appear on the screen in the experiment of Grimaldi as he did not keep the slit S before the double slit arrangement. He obtained only a uniform illumination. It was so because the beams arriving at the screen were not coherent and the phase difference between them varied with time in a haphazard way. The reason for the lack of coherence lies in the very process of light emission. In ordinary sources of visible light, individual atoms are responsible for the emission of light. An atom leaving an excited state gives up the excess energy in the form of a burst of light (photon) and jumps to the lower normal state. The process of transition of the atom from an upper state to a lower state lasts for a brief time of about 10^{-8}sec. Therefore, the light emitted by an atom is not a continuous harmonic wave of infinite extension but is a *wave train* of finite length having a certain limited number of oscillations. It is impossible to say exactly when an atom may emit light because the emission is completely a random process.

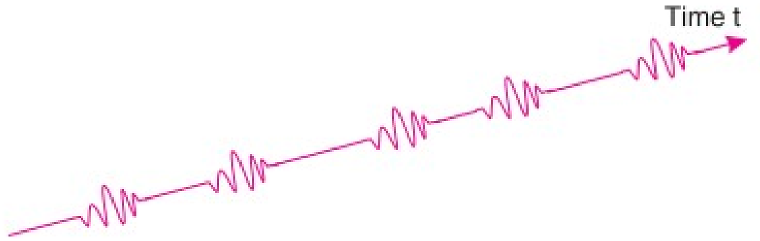


Fig. 14.13

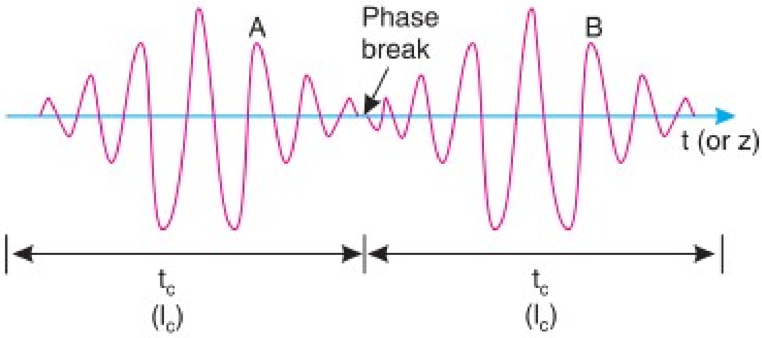


Fig. 14.14

Fig. 14.13 pictorially describes the emission of light by a single atom in terms of wave trains. Other atoms in the source behave similarly but with different emission times. Adding together the wave trains generated by all atoms in the light source produces a succession of wave trains which have their phases distributed randomly. In passing from one wave train to the next there is an abrupt change in phase. Therefore, it is not possible to relate the phase at a point in wave train B to a point in wave train A. The phase of the wave train from an atom would remain constant with respect to the phase of the wave train from another atom only for about 10^{-8} sec. It implies that the two wave trains can be coherent for a maximum time of about 10^{-8} sec. Therefore, light from conventional sources is characterized by two important parameters, namely coherence time and coherence length.

Coherence time: It is *the average time during which the wave remains sinusoidal and phase of the wave packet can be predicted reliably.*

Coherence length: It is *the length of the wave packet over which it may be assumed to be sinusoidal and has predictable phase.*

Light from a sodium discharge lamp has a coherence length of about 2 to 3 cm, while the coherence length of white light is a fraction of a cm. In the double slit experiment, the presence of slit S ensures that the same group of wave trains are incident on slits S_1 and S_2. When the phase of the wave changes at S this change is communicated simultaneously to S_1 and S_2.Therefore, the waves emerging from S_1 and S_2 will be coherent with respect to each other and a *stationary* interference pattern is produced on the screen.

14.7. CONDITIONS FOR INTERFERENCE

We may now summarize the conditions that are to be fulfilled in order to observe a distinct well-defined interference pattern.

(A) Conditions for sustained interference:

(*i*) The waves from the two sources must be of the same frequency.

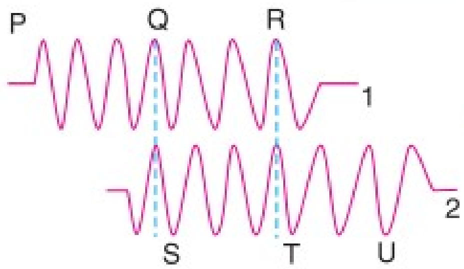
If the light waves differ in frequency, the phase difference fluctuates irregularly with time. Consequently, the intensity at any point fluctuates with time and we will not observe steady interference.

(*ii*) The two light waves must be coherent.

If the light waves are coherent, then they maintain a fixed phase difference over a time and space. Hence, a stationary interference pattern will be observed.

(*iii*) The path difference between the overlapping waves must be less than the coherence length of the waves.

We have already learnt that light is emitted in the form of wave trains and a finite coherence length characterizes them. If we consider two interfering wave trains, having constant phase difference, as in Fig. 14.15, the interference effects occur due to parts QR of wave 1 and ST of wave 2.



Partial overlap of the wavetrains

Fig. 14.15

For the parts PQ and TU interference will not occur. Therefore, the interference pattern does not appear distinctly. When the entire wave train PR overlaps on the wave train SU, interference pattern will be distinct. On the other hand, when the path difference between the waves 1 and 2 becomes very large, the wave trains arrive at different times and do not overlap on each other. Therefore, in such cases interference does not take place. The interference pattern completely vanishes if the path difference is equal to the coherence length. It is hence required that

$$\Delta < l_{coh} \tag{14.32}$$

(*iv*) If the two sets of waves are plane polarized, their planes of polarization must be the same. Waves polarized in perpendicular planes cannot produce interference effects.

(B) Condition for formation of distinct fringe pattern:

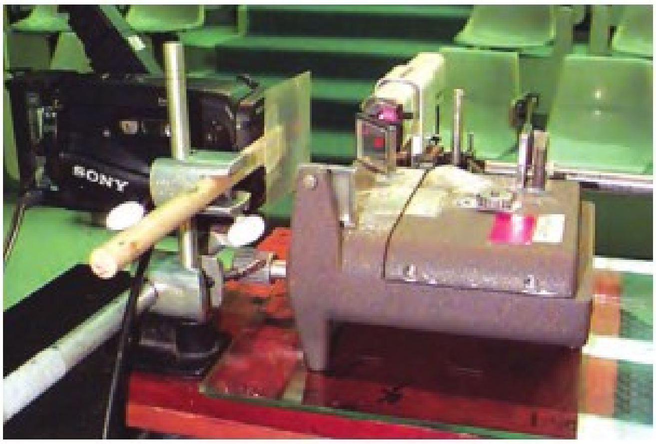
(v) The two coherent sources must lie close to each other in order to discern the fringe pattern. If the sources are far apart, the fringe width will be very small and fringes are not seen separately.

(*vi*) The distance of the screen from the two sources must be large.

(*vii*) The vector sum of the overlapping electric field vectors should be zero in the dark regions for obtaining distinct bright and dark fringes. The sum will be zero only if the vectors are anti-parallel and have the same magnitude.

14.8. TECHNIQUES OF OBTAINING INTERFERENCE

The phase relation between the waves emitted by two independent light sources rapidly changes with time and therefore they can *never* be coherent, though the sources are identical in all respects. However, if two sources are derived from a single source by some device, then any phase change occurring in one source is simultaneously accompanied by the same phase change in the other source. Therefore, the phase difference between the waves emerging from the two sources remains constant and the sources are *coherent*. The techniques used for creating coherent sources of light can be divided into the following two broad classes.



Michelson's Interferometer.

(a) Wavefront splitting: One of the methods consists in dividing a light wavefront, emerging from a narrow slit, by passing it through two slits closely spaced side by side. The two parts of the same wavefront travel through different paths and reunite on a screen to produce fringe pattern. This is known as **interference due to division of wavefront.** This method is useful only with *narrow* sources. Young's double slit, Fresnel's double mirror, Fresnel's biprism, Lloyd's mirror, etc employ this technique.

(b) Amplitude splitting: Alternately, the amplitude (intensity) of a light wave is divided into two parts, namely reflected and transmitted components, by partial reflection at a surface. The two parts travel through different paths and reunite to produce interference fringes. This is known as **interference due to division of amplitude.** Optical elements such as beam splitters, mirrors are used for achieving amplitude division. Interference in thin films (wedge, Newton's rings etc), Michelson's interferometer etc interferometers utilize this method. This method requires *extended* source.

14.9. FRESNEL BIPRISM

Fresnel used a biprism to show interference phenomenon. The biprism consists of two prisms of very small refracting angles joined base to base. In practice, a thin glass plate is taken and one of its faces is ground and polished till a prism (Fig. 14.16 a) is formed with an obtuse angle of about 179° and two side angles of the order of 30'.

When a light ray is incident on an ordinary prism, the ray is bent through an angle called the *angle of deviation*. As a result, the ray emerging out of the prism appears to have emanated from a source S′ located at a small distance above the real source, as shown in Fig.14.16(b). We say that the prism produced a *virtual image* of the source. A biprism, in the same way, creates two virtual sources S_1 and S_2, as seen in Fig.14.16(c). These two virtual sources are images of the same source S produced by refraction and are hence *coherent*.

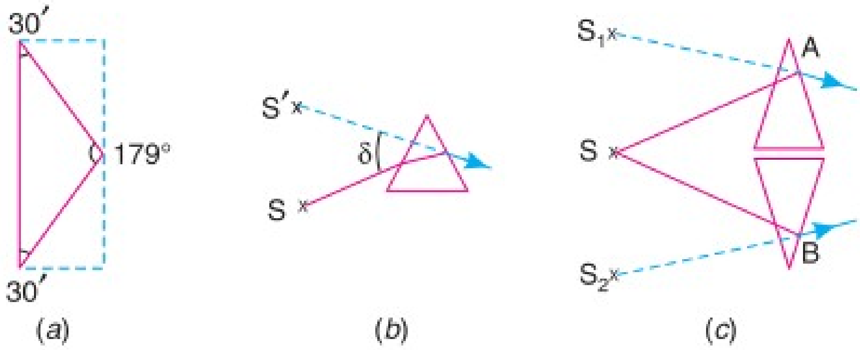


Fig. 14.16

14.9.1. EXPERIMENTAL ARRANGEMENT

The biprism is mounted suitably on an optical bench. An optical bench consists of two horizontal long rods, which are kept strictly parallel to each other and at the same level. The rods carry uprights on which the optical components are positioned. A monochromatic light source such as sodium vapour lamp illuminates a vertical slit S. Therefore, the slit S acts as a narrow linear monochromatic light source. The biprism is placed in such a way that its refracting edge is parallel to the length of the slit S. A single cylindrical wavefront impinges on both prisms. The top portion of wavefront is refracted downward and appears to have emanated from the virtual image S_1. The lower segment, falling on the lower part of the biprism, is refracted upward and appears to have emanated from the virtual source S_2. The virtual sources S_1 and S_2 are coherent (see Fig.14.17), and hence the light waves are in a position to interfere in the region beyond the biprism. If a screen is held there, interference fringes are seen. In order to observe fringes, a micrometer eyepiece is used.

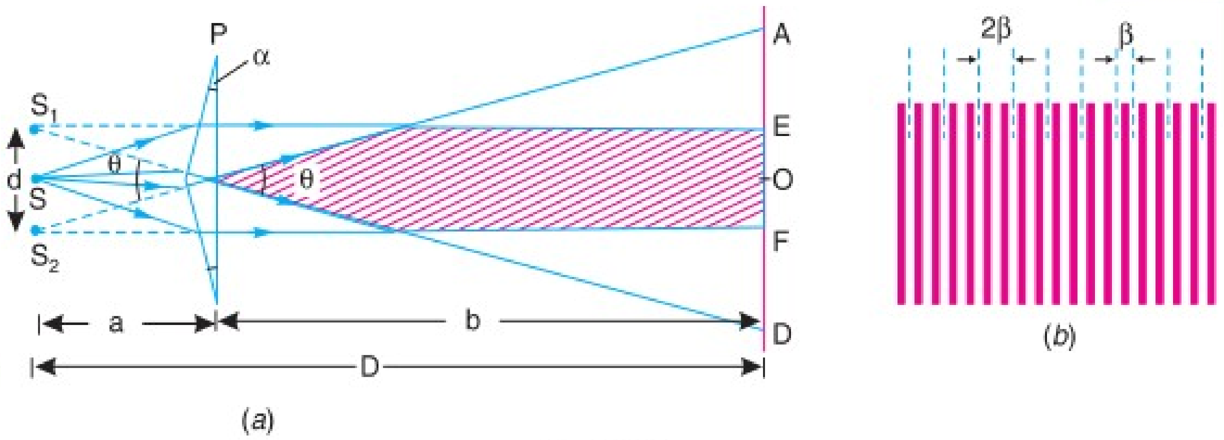


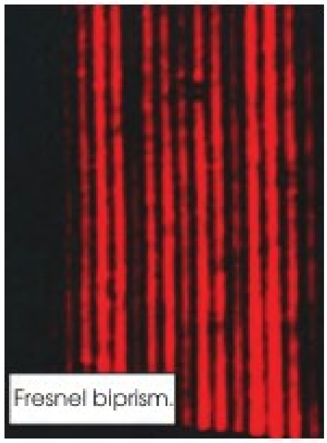
Fig. 14.17

Theory:

The theory of the interference and fringe formation in case of Fresnel biprism is the same as described in § 14.6 for the double-slit. As the point O is equidistant from S_1 and S_2, the central bright fringe of maximum intensity occurs there. On both sides of O, alternate bright and dark fringes, as shown in Fig.14.17(b), are produced. The width of the dark or bright fringe is given by equ.(14.31).

$$\beta = \frac{\lambda D}{d}$$

where D(= a + b) is the distance of the sources from the eye-piece.



Dark or bright fringes.

14.9.2. DETERMINATION OF WAVELENGTH OF LIGHT

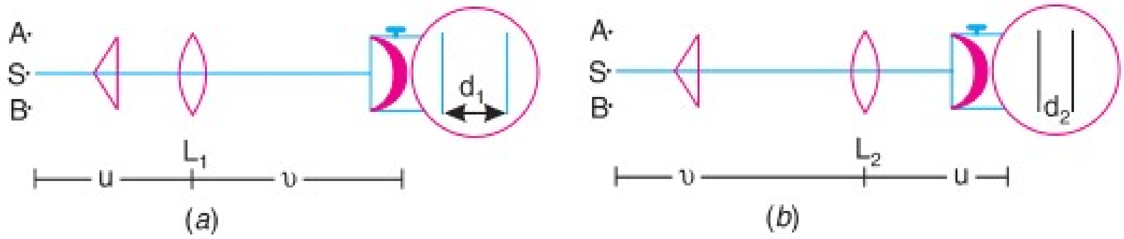
The wavelength of the light can be determined using the equ.(14.31). For using the relation, the values of β, D and d are to be measured. These measurements are done as follows.

Adjustments:

A narrow adjustable slit S, the biprism, and a micrometer eyepiece are mounted on the uprights and are adjusted to be at the same height and in a straight line. The slit is made vertical and parallel to the refracting edge of the biprism by rotating it in its own plane. It is illuminated with the light from the monochromatic source. The biprism is moved along the optical bench till, on looking through it along the axis of the optical bench, two equally bright vertical slit images are seen. Then the eyepiece is moved till the fringes appear in the focal plane of the eyepiece.

(*i*) Determination of fringe width β: When the fringes are observed in the field view of the eyepiece, the vertical cross-wire is made to coincide with the centre of one of the bright fringes. The position of the eyepiece is read on the scale, say x_o. The micrometer screw of the eyepiece is moved slowly and the number of the bright fringes N that pass across the cross-wire is counted. The position of the cross-wire is again read, say x_N. The fringe width is then given by $\beta = \frac{x_N - x_o}{N}$

(*ii*) Determination of 'd': (*a*) A convex lens of short focal length is placed between the slit and the eyepiece without disturbing their positions. The lens is moved back and forth near the biprism till a sharp pair of images of the slit is obtained in the field view of the eyepiece. The distance between the images is measured. Let it be denoted by d_1.



Measurement of the distance etween the two virtual sources.

Fig. 14.18

If u is the distance of the slit and υ that of the eyepiece from the lens (Fig. 14.18a.), then the magnification is

$$\frac{\upsilon}{u} = \frac{d_1}{d} \tag{14.33}$$

The lens is then moved to a position nearer to the eyepiece, where again a pair of images of the slit is seen. The distance between the two sharp images is again measured. Let it be d_2. Again magnification is given by

$$\frac{u}{\upsilon} = \frac{d_2}{d} \tag{14.34}$$

Note that the magnification in one position is the reciprocal of the magnification in the other position.

Multiplying the equations (14.33) and (14.34), we obtain

$$\frac{d_1 d_2}{d^2} = 1$$

$$d = \sqrt{d_1 d_2} \tag{14.35}$$

Using the values of β, d and D in the equation (14.31), the wavelength λ can be computed.

(b) Alternatively, the value of d can be determined as follows. The deviation δ produced in the path of a ray by a thin prism is given by

$$\delta = (\mu - 1)\alpha$$

where α is the refracting angle of the prism. From the Fig. 14.17, it is seen that $\delta = \theta/2$. Since d is very small, we can also write $d = a\theta$.

$$\therefore \quad \frac{\theta}{2} = \frac{d}{2a} = (\mu - 1)\alpha$$

$$\therefore \quad d = 2a(\mu - 1)\alpha \tag{14.36}$$

14.9.3. INTERFERENCE FRINGES WITH WHITE LIGHT

In the biprism experiment if the slit is illuminated by white light, the interference pattern consists of a central **white fringe** flanked on its both sides by a few coloured fringes and general illumination beyond the fringes. The central white fringe is the *zero-order* fringe.

With monochromatic light all the bright fringes are of the same colour and it is not possible to locate the zero-order fringe. Therefore, in order to locate the zero order fringe the biprism is to be illuminated by white light.

14.9.4. LATERAL DISPLACEMENT OF FRINGES

The biprism experiment can be used to determine the thickness of a given thin sheet of transparent material such as glass or mica. If a thin transparent sheet is introduced in the path of one of the two interfering beams, the fringe system gets displaced towards the beam in whose path the sheet is introduced. By measuring the amount of displacement, the thickness of the sheet can be determined.

Suppose S_1 and S_2 are the virtual coherent monochromatic sources. The point O is equidistant from S_1 and S_2, where we obtain the *central bright fringe*. Therefore, the optical path $S_1O = S_2O$. Let a transparent plate G of thickness t and refractive index μ be introduced in the path of one of the beams (see Fig. 14.19). The optical path lengths S_1O and S_2O are now not equal and the *central bright fringe* shifts to P from O. The light waves from S_1 to P travel partly in air and partly in the sheet G; the distance travelled in air is $(S_1P - t)$ and that in the sheet is t.

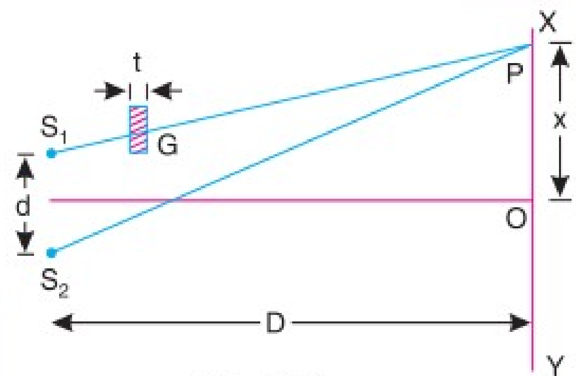


Fig. 14.19

The optical path $\Delta_{S_1P} = (S_1P - t) + \mu\, t = S_1P + (\mu - 1)\, t$

The optical path $\Delta_{S_2P} = S_2P$

The optical path difference at P is $\Delta_{S_1P} - \Delta_{S_2P} = 0$, since in the presence of the thin sheet, the optical path lengths S_1P and S_2P are *equal* and central zero fringe is obtained at P.

$$\therefore \quad \Delta_{S_1P} = \Delta_{S_2P}$$

$$\left[S_1P + (\mu - 1)\, t\right] = S_2P$$

$$\therefore \quad S_2P - S_1P = (\mu - 1)\, t$$

But according to the relation (14.28), $S_2P - S_1P = \dfrac{x\, d}{D}$

where x is the **lateral shift** of the central fringe due to the introduction of the thin sheet.

$$\therefore \quad (\mu - 1)\, t = \frac{xd}{D}$$

Hence, the thickness of the sheet is $t = \dfrac{xd}{D(\mu - 1)}$ (14.37)

14.10. LLOYD'S SINGLE MIRROR

In 1834, Lloyd devised an interesting method of producing interference, using a single mirror and using almost grazing incidence. The Lloyd's mirror consists of a plane mirror about 30 cm in length and 6 to 8 cm in breadth (see Fig. 14.20). It is polished on the front surface and blackened at the back to avoid multiple reflections. A cylindrical wavefront coming from a narrow slit S_1 falls on the mirror which reflects a portion of the incident wavefront, giving rise to a virtual image of the slit

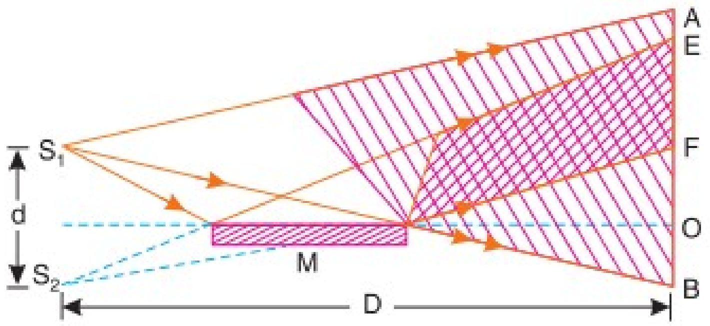
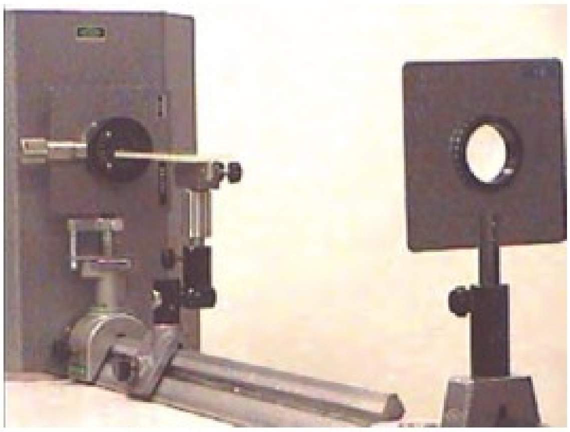


Fig. 14.20

S_2. Another portion of the wavefront proceeds directly from the slit S_1 to the screen. The slits S_1 and S_2 act as two coherent sources. Interference between direct and reflected waves occurs within the region of overlapping of the two beams and fringes are produced on the screen placed at a distance D from S_1 in the shaded portion EF.

The point O is equidistant from S_1 and S_2. Therefore, central (zero-order) fringe is expected to lie at O (the perpendicular bisector of S_1S_2) and it is also expected to be bright. However it is not usually seen since the point O lies outside the region of interference (only the direct light and not the reflected light reaches O).



Lloyd's mirror - white light.

By moving the screen nearer to the mirror such that it comes into contact with the mirror, the point O can be just brought into the region of interference. With white light the central fringe at O is expected to be white but in practice it is *dark*. The occurrence of dark fringe can be understood taking into the consideration of the phase change of π that light suffers when reflected from the mirror. The phase change leads to a path difference of $\lambda/2$ and hence destructive interference occurs there.

14.10.1. DETERMINATION OF WAVELENGTH

The fringe width is given by equ. (14.31). Thus,

$$\beta = \frac{\lambda D}{d}$$

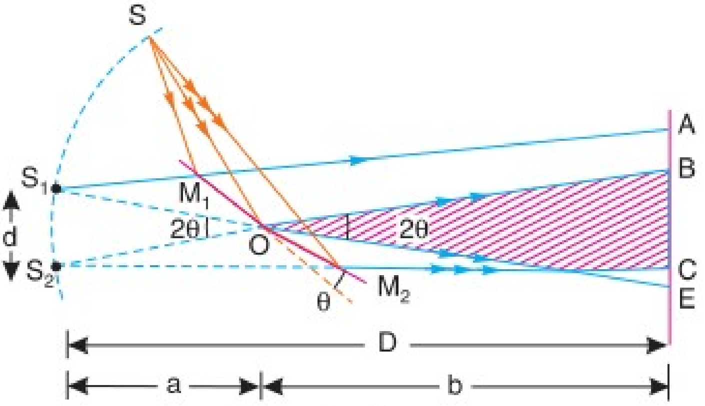
Measuring β,D and d, the wavelength λ can be determined.

Comparison between the fringes produced by biprism and Lloyd's mirror:

1. In biprism the complete set of fringes is obtained. In Lloyd's mirror a few fringes on one side of the central fringe are observed, the central fringe being itself invisible.
2. In biprism the central fringe is bright whereas in case of Lloyd's mirror, it is dark.
3. The central fringe is less sharp in biprism than that in Lloyd's mirror.

14.11. FRESNEL'S DOUBLE MIRROR

Fresnel's double mirror is an arrangement for obtaining two coherent sources by using the phenomenon of reflection. It consists of two plane mirrors inclined to each other at a very small angle, as shown in Fig. 14.21.The mirrors are silvered on their front surfaces and are arranged at nearly 180º such that their surfaces are nearly coplanar.



Fresnel's mirrors.

Fig. 14.21

A narrow slit S is placed parallel to the line of intersection of the mirror surfaces and is illuminated with monochromatic light. One portion of the cylindrical wavefront coming from slit S is reflected from the first mirror and another portion of the wavefront is reflected from the second mirror. After reflection, the light appears to diverge from S_1 and S_2, which are the virtual images of S. As the images S_1 and S_2 of the slit are derived from the same source S, they behave as two coherent sources, placed at a distance *d* apart. The waves diverging from S_1 and S_2 overlap and interference fringes are produced in the overlapping region EF on the screen. The fringes are of equal width.

Fringe width:

It is seen from the geometry of the figure (Fig. 14.21) that $OS_1 = OS_2 = OS$. That is, S_1, S_2 and S lie on a circle with O as a centre. Let 'a' be the distance of the sources and 'b' be the distance of the screen from O. Then the fringe width is given by

$$\beta = \frac{\lambda D}{d} = \frac{(a+b)\lambda}{d} \tag{14.38}$$

OE and OB are the reflected rays from OM_1 and OM_2 respectively, corresponding to the incident ray SO. Therefore, the angle between OE and OB is twice the angle between the mirrors. Hence, $\angle S_1OS_2 = \angle BOE = 2\theta$. Now,

$$\text{Arc } S_1S_2 = a \times 2\theta$$

$$\therefore \quad d = a \times 2\theta$$

Using the above result into equ.(14.38), we get

$$\beta = \frac{(a+b)}{2a\theta}\lambda \tag{14.39}$$

Comparison between the fringes produced by biprism and double mirror :

The fringes in both cases are similar in appearance. However, the double mirror fringes are narrower than the biprism fringes.

14.12. ACHROMATIC FRINGES

A system of white and dark fringes, without any colours, obtained by white light is called *achromatic fringes*.

When the slit is illuminated by white light in any interference experiment, we obtain a central white fringe flanked by a few coloured fringes. Coloured fringes are obtained because the fringe width is dependent on the wavelength $(\beta = \lambda D / d)$. For example, the width of the red fringe is more than the blue fringe. Rayleigh designed an experiment where white and dark fringes were obtained. It can be done if the fringe width is independent of the wavelength of light and is the same for all wavelengths. The fringe width β can be kept constant for all wavelengths, if λ / d is the same in all cases. Then the maxima of each order for all wavelengths coincide, resulting in achromatic fringes.

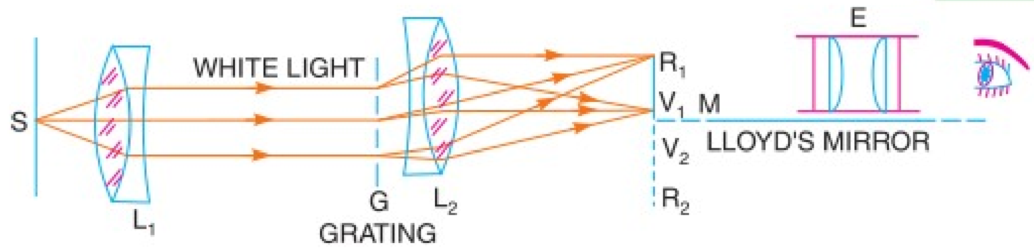


Fig. 14.22

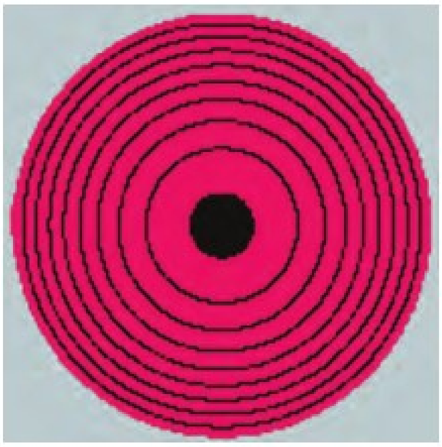
In practice, achromatic fringes may be obtained as follows. S is a narrow source of white light at the focal plane of the converging lens L_1. A grating G having 800 to 1200 lines per cm is placed normal to light emerging from L_1. Another achromatic lens L_2 is used to form the second order spectrum on an opaque screen with a narrow opening in it. The narrow opening is adjusted so that only the first order spectrum is allowed to pass through it. The violet end is nearer to the highly polished Lloyd's mirror M than the red end. The position of M is adjusted such that V_2 and R_2 are the images of V_1 and R_1. Interference occurs between the beams from V_1R_1 and those from V_2R_2. The violet fringes are produced by V_1 and V_2 while red fringes are produced by R_1 and R_2.

Suppose $V_1V_2 = d_1$ and $R_1R_2 = d_2$.

If $\frac{\lambda_V}{d_1} = \frac{\lambda_R}{d_2}$, the fringe width β will be the same and interference fringes due to different colours will overlap and white achromatic fringes are produced in the field of view. The white and dark fringes are seen through the eyepiece or can be projected on a screen.

14.13. NON-LOCALIZED FRINGES

Point sources produce fringes, which can be seen at different distances from the source. As the screen is moved far from the source, the fringe spacing increases and conversely, when the screen is moved nearer to the source, the fringes come closer. Therefore, we say the fringes are **non-localized**. Narrow sources produce non-localized fringes.



Non-localized fringes.

14.14. VISIBILITY OF FRINGES

The contrast of the interference fringes can be quantitatively described by the parameter called *visibility*. Visibility is defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{14.40}$$

the value of visibility varies between 0 and 1. When the fringes are of maximum intensity in the bright areas and totally dark in the dark areas, the visibility is equal to 1. As the phase difference increases, the coherence between the light waves decreases and the visibility is reduced. Finally, when the coherence between the two light waves disappears, I_{max} and I_{min} become equal and the visibility goes to zero. Then fringes are not observed and instead we observe uniform illumination.

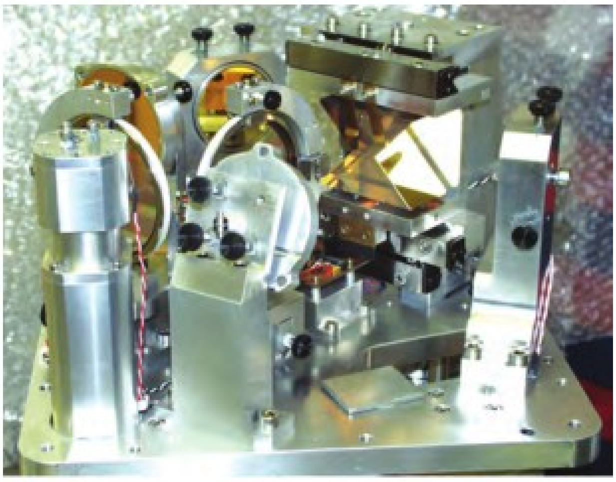
From equ.(14.23) and (14.24), the visibility of the fringes produced by two beams can be expressed as

$$V = \frac{(E_1+E_2)^2-(E_1-E_2)^2}{(E_1+E_2)^2+(E_1-E_2)^2} = \frac{2E_1E_2}{E_1^2+E_2^2} = \frac{2\sqrt{I_1I_2}}{I_1+I_2} = \frac{2\sqrt{I_1/I_2}}{1+I_1/I_2} \quad (14.41)$$

It is seen from the above equation that the closer the intensities of the two waves, the higher is visibility of the fringes. When $I_1 = I_2$, V = 1. It may be noted that V is always equal to 1 when monochromatic light is used.

14.15. FRINGE PATTERN WITH WHITE LIGHT

When the slit is illuminated with white light in any interference experiment, we obtain a central white fringe flanked by a few coloured fringes. At the centre of the screen O, there is zero path difference and the bright fringes produced by all the colours there, add to each other. As a result a white fringe is produced at O. At other places away from O, the bright fringes in each order separate, because the fringe width is dependent on the wavelength ($\beta = \lambda D / d$). For example, the width of the red fringe is more than the blue fringe. Hence coloured fringes are produced on either side of the central white fringe.



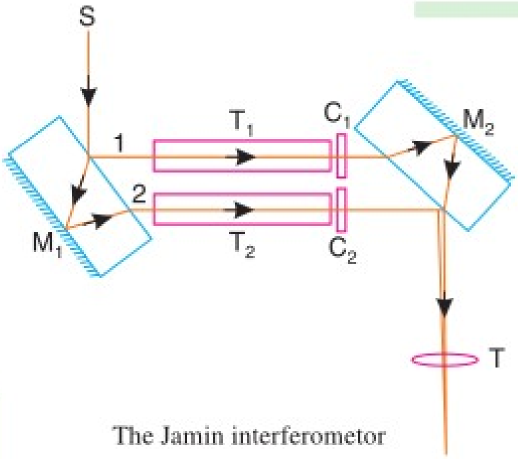
Interferometer is based on the principle of interference of light

14.16. INTERFEROMETRY

Instruments based on the principle of interference of light are known as **interferometers**. The instruments designed by Jamin and Rayleigh are used to determine the refractive index of gases and are known as *refractometers*.

14.16.1. JAMIN'S REFRACTOMETER

Jamin's refractometer consists of two exactly identical and optically plane glass blocks M_1 and M_2 (cut from the same block) and silvered on their back surfaces and arranged with their faces parallel to each other, as shown in Fig. 14.23. Light from an extended monochromatic source S incident on M_1 at an angle 45° is broken into two parallel rays (1) and (2) by reflection at the upper and lower surfaces of M_1. These two rays combine after suffering reflections at the two surfaces of M_2 as shown and form interference fringes as observed in a telescope. If the plates M_1 and M_2 are parallel, the light paths will be identical.



The Jamin interferometer

Fig. 14.23

Measurement of refractive index :

To measure the refractive index of a gas at a given temperature and pressure, two similar evacuated tubes T_1 and T_2 are placed in the paths of the two parallel beams. The gas whose refractive index is to be measured is then slowly allowed to enter into one of the tubes. As the gas enters, the optical path of the beam passing through the tube increases. The fringes therefore move past the cross-wire of the telescope. The fringes are counted until the gas entering the tube attains the given pressure and temperature.

If l is the length of the tube and μ the refractive index of the gas, then the change in the optical path difference due to the presence of the gas is $(\mu - 1)\, l$. If N be the number of fringes passed across the field of view, then

$$(\mu - 1)\, l = N\lambda \tag{14.42}$$

The refractive index, μ can be calculated using the above expression.

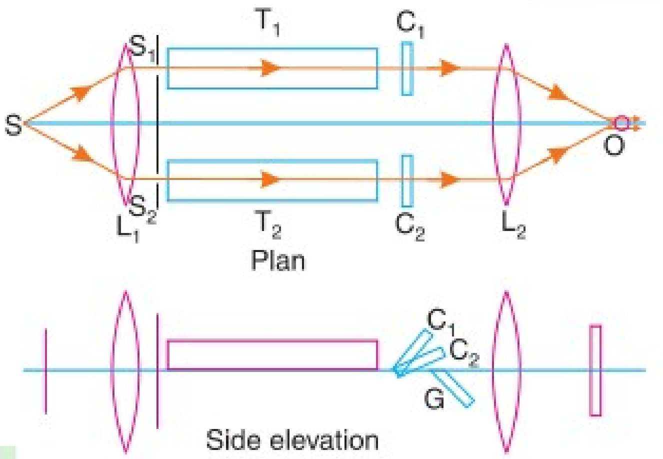
Compensator:

To avoid counting of fringes, a device called the "Jamin's compensator" is used. The compensator consists of two equally thick glass plates C_1 and C_2 cut from the same piece of glass and inclined at a small angle. The plates can be rotated together about a horizontal axis and the rotation is read on a divided circle D. One plate of the compensator is placed in the path of each beam. When the plates are equally inclined to the incident beam, the optical paths through the plates are the same. When the plates are rotated, the angles of incidence of the two beams change and a relative phase difference is introduced which varies as the compensator is rotated. By using monochromatic light and observing the passage of fringes across the field of view as the compensator is rotated, the scale can be calibrated to read the optical path difference in terms of wavelength.

Now, in the actual experiment, white light is used and the central achromatic fringe is adjusted on the cross wire by adjusting the compensator. The gas is then introduced in one of the tubes at the given temperature and pressure which results in shifting of the fringes. The compensator is now rotated so as to bring the central fringe back on the cross wire again. The change in the optical path difference due to this rotation is determined as the compensator is already calibrated. This must be equal to $(\mu - 1)\, l$, from which μ can be calculated.

14.16.2. RAYLEIGH REFRACTOMETER

The Rayleigh refractometer is mainly used to determine the refractive indices of inert gases and slight variation in the refractive index of solutions and gases. Light from a monochromatic source passes through a slit S and is then incident on the lens L_1. The parallel beam then passes through the slits S_1 and S_2. The upper parts of the parallel beams emerging from S_1 and S_2 are allowed through the separate chambers T_1 and T_2. After passing through the chambers T_1 and T_2 and the compensating plates C_1 and C_2, the beams are recombined by the lens L_2 in its focal plane. Interference fringes are obtained in the focal plane of lens and are observed with the help of the eyepiece or a telescope. Because of the gas chambers S_1 and S_2 are widely separated and fringes are closely spaced. The lens L_2 forms a *second*



The Rayleigh interferometer

Fig. 14.24

system of interference pattern, which is due to the superposition of the lower parts of the two beams emerging from S_1 and S_2 and passing beneath the chambers T_1 and T_2. The upper edge of the lower fringe system is made to coincide with the lower edge of the upper system with the help of an inclined thick plate P (not shown in the figure) held across the lower parts of the slits. The centres of the two sets of interference patterns coincide and the fringe spacing in the two sets is the same. The first fringe system shifts with the change of gas pressure etc whereas the second fringe system remains *stationary*.

It is used to determine the refractive index of inert gases.

The circular disc D attached to the compensating plates C_1 and C_2 is previously calibrated in terms of wavelength and refractive index. Initially, both the tubes T_1 and T_2 are evacuated. Using white light, the central white fringe is observed in the field of view of the eyepiece. The gas at a known pressure and temperature is introduced into the tube T_1. The central white fringe shifts from the field of view.

By rotating the circular disc D, which in turn displaces the compensating plates C_1 and C_2, the central white fringe is brought back to the centre of the field of view. The number of wavelengths graduated on the circular disc is noted. The change in the path difference is

$$(\mu - 1)\, l = N\lambda$$

where N is the number of fringes that crossed the field of view. Knowing l, N and λ, the refractive index μ can be calculated.

WORKED OUT PROBLEMS

Example 14.1: Green light of wavelength 5100 Å from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, find the slit separation.

Solution: The fringe width $\beta = \frac{\lambda D}{d}$

It is given that D = 200 cm, $\lambda = 5100 \times 10^{-8}$cm and $10\beta = 2$ cm.

$\therefore \quad \beta = 0.2$ cm.

The slit separation $d = \frac{\lambda D}{\beta} = \frac{5100 \times 10^{-8}\, cm \times 200 cm}{0.2 cm} = \mathbf{0.05\ cm.}$

Example 14.2: Two coherent sources are 0.18 mm apart and the fringes are observed on a screen 80 cm away. It is found that with a certain monochromatic source of light, the fourth bright fringe is situated at a distance of 10.8 mm from the central fringe. Calculate the wavelength of light.

Solution: The distance of the n^{th} fringe from the central fringe is $x = \frac{n\lambda D}{d}$.

It is given that D = 80 cm, d = 0.18 mm = 0.018 cm, x = 10.8 mm = 1.08 cm and n = 4.

$\therefore \quad \lambda = \frac{xd}{n\,D} = \frac{1.08\,\text{cm} \times 0.018\,\text{cm}}{4 \times 80\,\text{cm}} = 6075 \times 10^{-8}\text{cm} = \mathbf{6075\ Å.}$

Example 14.3: A light source emits light of two wavelengths 4300Å and 5100Å. The source is used in a double slit experiment. The distance between the sources and the screen is 1.5 m and the distance between the slits is 0.025mm. Calculate the separation between the third order bright fringes due to these two wavelengths.

Solution: It is given that $\lambda_1 = 4300\text{Å} = 4300 \times 10^{-8}\text{cm}$, $\lambda_2 = 5100\text{Å} = 5100 \times 10^{-8}\text{cm}$, $n = 3$, $D = 1.5\text{ m} = 150\text{ cm}$ and $d = 0.025\text{mm} = 0.0025\text{cm}$.

Now $$x_1 = \frac{n\lambda_1 D}{d} \text{ and } x_2 = \frac{n\lambda_2 D}{d}$$

$$\therefore \quad x_2 - x_1 = \frac{nD}{d}(\lambda_2 - \lambda_1) = \frac{3\times 150\,\text{cm}}{0.0025\,\text{cm}}(5100-4300)\times 10^{-8}\text{ cm} = \mathbf{1.44\ cm}$$

Example 14.4: Interference fringes are observed with a biprism of refracting angle 1° and refractive index 1.5 on a screen 80 cm away from it. If the distance between the source and the biprism is 20 cm, calculate the fringe width when the wavelength of light used is 6900Å.

Solution: The fringe width $\beta = \frac{\lambda D}{d}$ and $d = 2(\mu - 1)\alpha\, a$

It is given that $\mu = 1.5$, $\alpha = 1° = \frac{\pi}{180°}$, $a = 20$ cm and $b = 80$ cm, $\lambda = 6900 \times 10^{-8}$cm

$$\therefore \quad D = (20 + 80)\text{ cm} = 100\text{ cm}.$$

$$\beta = \frac{\lambda D}{2(\mu-1)\alpha\, a} = \frac{(6900\times 10^{-8}\text{cm})\times 100\,\text{cm}}{2(1.5-1)\times(\pi/180)\times 20\,\text{cm}} = \mathbf{0.02\ cm}$$

Example 14.5: In a biprism experiment the eyepiece is placed at a distance of 1.2 m from the source. The distance between the virtual sources was found to be 7.5×10^{-4}m. Find the wavelength of light, if the eyepiece is to be moved transversely through a distance of 1.888cm for 20 fringes.

Solution: The fringe width $\beta = \frac{\lambda D}{d}$; But $\beta = \frac{l}{n}$ $\therefore\ \lambda = \frac{ld}{nD}$

It is given that $l = 1.888\text{cm} = 0.01888\text{m}$, $d = 7.5 \times 10^{-4}$m, $n = 20$ and D = 1.2m.

$$\therefore \quad \lambda = \frac{0.01888\text{m}\times 7.5\times 10^{-4}\text{m}}{20\times 1.2\text{m}} = 5900\times 10^{-10}\text{m} = \mathbf{5900\text{Å}}$$

Example 14.6: A thin sheet of a transparent material (μ =1.60) is placed in the path of one of the interfering beams in a biprism experiment using sodium light, λ = 5890 Å. The central fringe shifts to a position originally occupied by the 12th bright fringe. Calculate the thickness of the sheet.

Solution: The thickness of the sheet, $t = \frac{n\lambda}{\mu - 1}$

It is given that $\mu = 1.60$, $n = 12$, $\lambda = 5890$ Å.

$$\therefore \quad t = \frac{12\times 5890\times 10^{-10}\text{m}}{1.60-1} = 1.18\times 10^{-5}\text{m} = \mathbf{0.12\ \mu m}$$

Example 14.7: When a thin sheet of transparent material of thickness 6.3×10^{-4}cm is introduced in the path of one of the interfering beams, the central fringe shifts to a position occupied by the sixth fringe. If λ = 5460 Å, find the refractive index of the sheet.

Solution: $(\mu - 1)\, t = n\lambda$ $\therefore$ The refractive index of the sheet $\mu = \frac{n\lambda}{t} + 1.$

It is given that $t = 6.3\times 10^{-4}$cm, $n = 6$ and $\lambda = 5460$ Å $= 5460 \times 10^{-8}$cm.

$$\therefore \quad \mu = \frac{6\times 5460\times 10^{-8}\text{cm}}{6.3\times 10^{-4}\text{cm}} + 1 = 1.52$$

Example 14.8: In Lloyd's single mirror interference experiment, the slit source is at a distance of 2 mm from the plane of the mirror. The screen is kept at a distance of 1.5 m from the source. Calculate the fringe width. Wavelength of light is 5890 Å.

Solution: The fringe width $\beta = \frac{\lambda D}{d}$

It is given that $\lambda = 5890$ Å $= 5890 \times 10^{-10}$m, $D = 1.5$ m and $d/2 = 2$mm. $\therefore d = 4 \times 10^{-3}$m.

$$\beta = \frac{5890 \times 10^{-10}\,\text{m} \times 1.5\text{m}}{4 \times 10^{-3}\,\text{m}} = 22\text{mm}$$

QUESTIONS

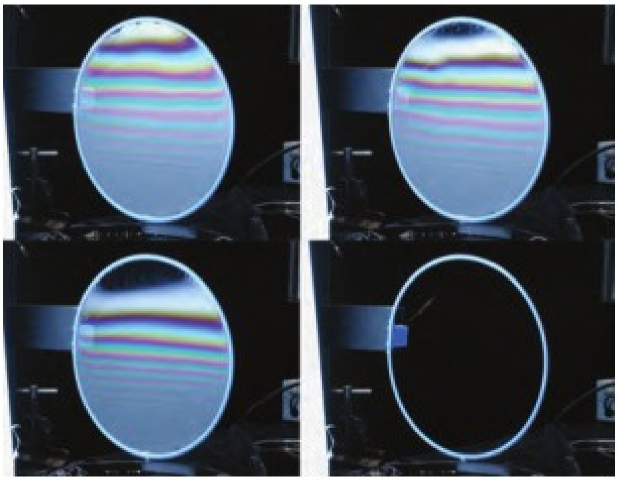
1. What are the conditions necessary for observing interference fringes?
2. Why is the condition of coherence necessary to observe interference fringes?
3. Is it possible to observe interference fringes with light emanating from two independent sources? If not, why?
4. How can coherent sources be obtained in practice?
5. Is it necessary that the interfering waves should have the same frequency? If so, why?
6. Is it necessary that the interfering waves should have equal amplitudes? Explain.
7. What are coherent sources? How are they realized in practice? Describe a method for determining the refractive index of a gas using the interference phenomenon. **(Madhurai Kamaraj, 2003)**
8. How would you determine the wavelength of light with the Lloyd's mirror experiment? In what respect do the fringes in this case differ from those obtained with Fresnel's biprism? How would you obtain achromatic fringes with this arrangement?
9. Describe Fresnel's biprism. Explain how the wavelength of light can be determined with its help.
10. What are coherent sources? Explain the importance of such sources in interference phenomenon. Two coherent sources form interference fringes. Obtain an expression for the distance between two consecutive bright fringes. **(Nagpur, 2004)**
11. What are coherent sources? Discuss why two independent sources of light of the same wavelength cannot produce interference fringes? Give a diagram showing clearly how coherent sources are produced in a biprism. Derive the formula for the fringe width in the biprism experiment.
12. What is meant by interference of light? State the fundamental conditions for the production of interference fringes.
13. State the basic conditions for the phenomenon of interference of light. Briefly discuss the effect of introducing a thin plate in the path of one of the interference beams in a biprism experiment. Deduce an expression for the displacement of the fringes. Show how this method is used for finding the thickness of a mica sheet.
14. Discuss the conditions for interference. Describe Young's experiment and derive an expression for (i) intensity at a point on the screen and (ii) fringe width. **(Punjab, 2005; Nagpur 2004)**
15. Derive an expression for the resultant intensity when two coherent beams of light are superposed.
    What is the visibility of fringes:-
    (*a*) for two slits of equal intensities?
    (*b*) if intensity of one slit is 4 times the other?
    What will be the intensity when the two sources are incoherent?
16. Show that the distance between the two virtual coherent sources in Fresnel's biprism arrangement is $2d\,(n-1)\,\theta$ where d is the distance between the source and the biprism, θ is the angle of the biprism and n is the refractive index of the material of the prism.

17. State the principle of superposition of light waves.
18. Why two independent sources can not produce observable interference pattern?
19. How are the fringes of equal inclination obtained? **(Nagpur, 2005)**
20. Derive the equation for optical path difference between two reflected rays from a thin film. **(Nagpur, 2005)**
21. Explain optical path of light in denser medium. **(Nagpur, 2005)**
22. Draw a well-labelled diagram of Rayleigh interferometer. **(Nagpur, 2004)**
23. What are the conditions for sustained interference pattern? **(Kovempu, 2005)**
24. How are coherent sources formed in the Biprism? On what factors does the separation between coherent sources depend?
25. State and explain conditions for the interference of light. **(Garhwal, 2005)**
26. Calculate the displacement of fringes when a thin transparent film is introduced in the path of one of the interfering beams in biprism.
27. State the principle of superposition of light waves.
28. Explain the phase change when light is reflected from a denser surface. **(RTMNU, 2010)**
29. Explain the conditions for interference of light. **(A.P.University, 2010)**
30. (*i*) Explain lateral displacement of fringes in double-slit experiment.
    (*ii*) How is it useful to determine the thickness of a sample? **(RTMNU, 2010)**

PROBLEMS FOR PRACTICE

1. Two coherent sources, whose intensity ratio is 9:4, produce interference fringes. Deduce the ratio of maximum to minimum intensity of the fringe system. **(Purvanchal, 2002)** [**Ans:** 25:1]
2. Interference fringes are formed by a biprism whose acute angle is 20' and refractive index is 1.5. The slit is at 10 cm from the biprism and is illuminated by light of wavelength 6000 Å. Find the fringe width on a screen placed at a distance of one metre from the biprism. **(Lucknow, 2000)** [**Ans:** 1.1 mm]
3. A biprism is placed 5 cm from a slit illuminated by sodium light ($\lambda = 5890$ Å). The width of the fringes obtained on a screen 75 cm from the biprism is 9.424×10^{-2} cm. What is the distance between the two coherent sources? **(Kanpur, 2002)** [**Ans:** 0.5 mm]
4. A biprism forms interference fringes with monochromatic light of wavelength 5450 Å. On introducing a thin glass plate ($\mu = 1.5$) in the path of one of the interfering beams, the central fringe shifts to the position previously occupied by the third bright fringe. Does fringe width change? Find the thickness of the plate. **(Awadh, 2001)** [**Ans:** 0.03 μm]
5. A parallel beam of light of wavelength 5890 Å is incident on a thin glass plate of refraction in glass is 60°, calculate the smallest thickness of the plate, which will appear dark by reflected light. **(Nagpur, 2005)**
6. A Biprism of obtuse angle 176° is made of glass of Refractive index 1.5. A slit illuminated with monochromatic light is placed 20 cm. behind and the width of interference fringes formed on a screen 80 cm in front of Biprism is found to be 9.25×10^{-3} cm. Calculate the wavelength of light. **(Lucknow, 2004)**
7. A thin mica sheet ($\mu = 1.6$) of 7×10^{-4} cm thickness introduced in the path of one of the interfering beams in a biprism arrangement shifts the central fringe to a position occupied by the 7th bright fringe from the center. Find the wavelength of the light used. **(Garhwal, 2005)**
8. In a double slit experiment, when a thin plate of transparent material is introduced in the path of one of the interfering beams, the central fringe is displaced by 3.6 fringes. Calculate the thickness of the plate. Given $\mu = 1.4$, $\lambda = 5500$Å. **(RTMNU, 2010)**

15 CHAPTER



Interference In Thin Films

15.1. THIN FILM

An optical medium is called a **thin film** when its thickness is about the order of 1 wavelength of light in visible region. Thus, a film of thickness in the range 0.5 μm to 10 μm may be considered as a thin film. A thin film may be a thin sheet of transparent material such as glass, mica, an air film enclosed between two transparent plates or a soap bubble. When light is incident on such a film, a small part of it gets reflected from the top surface and a major part is transmitted into the film. Again, a small part of the transmitted component is reflected back into the film by the bottom surface and the rest of it emerges

Soap bubble.

At a Glance

- Thin Film
- Plane Parallel Film
- Interference Due to Transmitted Light
- Haidinger Fringes
- Variable Thickness (Wedge-Shaped) Film
- Newton's Rings
- Michelson's Interferometer
- Applications of Michelson Interferometer
- Twyman and Green Interferometer
- Mach-Zehnder Interferometer
- Multiple Beam Interference
- Fabry-Perot Interferometer and Etalon
- Lummer and Gehrcke Plate
- Applications of Thin Film Interference
- Antireflection Coatings
- Dielectric Mirrors
- Interference Filters

out of the film. A small portion of the light thus gets reflected partially several times in succession within the film (see Fig. 15.1).

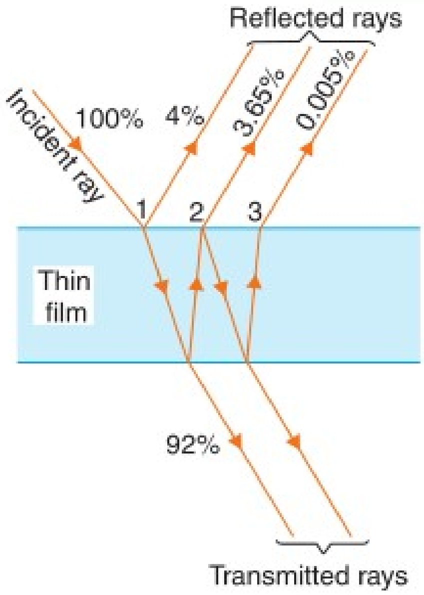


Fig. 15.1

In transparent thin films, the two bounding surfaces strongly transmit light and only weakly reflect the incident light. Therefore, only the first reflection at the top surface and the first reflection at the bottom surface will be of appreciable strength. For example, if we consider a glass plate, having a refractive index 1.52, the reflectivity of the top surface is given by

$$r = \left[\frac{1.52-1}{1.52+1}\right]^2 = 0.042$$

It means that about 4% of the incident light is reflected by the top surface of the glass plate, while 96% of it is transmitted into the plate. Out of the light reaching the bottom surface, again 3.8% is reflected and 92% is transmitted out of the plate. Then, again out of the 3.8% of the light 0.15% is reflected at the inner boundary of the top surface and about 3.65% is transmitted out into the air. After two reflections, the intensity will become insignificantly small. At each reflection, the intensity and hence the *amplitude of light wave is divided* into a reflected component and a refracted component. The reflected and refracted components travel along different paths and subsequently overlap to produce interference. Therefore, the interference in thin films is called interference by division of amplitude. Newton and Robert Hooke first observed the thin film interference. However, Thomas Young gave the correct explanation of the phenomena. A thin film may be uniform or non-uniform in its structure. However, as long as its thickness lies within the specified limits, interference of light occurs.

15.2. PLANE PARALLEL FILM

A transparent thin film of uniform thickness bounded by two parallel surfaces is known as a *plane parallel thin film.*

When light is incident on a parallel thin film, a small portion of it gets reflected from the top surface and a major portion is transmitted into the film. Again, a small part of the transmitted component is reflected back into the film by the bottom surface and the rest of it is transmitted from the lower surface of the film. Thin films transmit incident light strongly and reflect only weakly. After two reflections, the intensities of reflected rays drop to a negligible strength. Therefore, we consider the first two reflected rays only (see Fig. 15.2). These two rays are derived from the same incident ray but appear to come from two sources located below the film. The sources are virtual coherent sources. The reflected waves 1 and 2 travel along parallel paths and interfere at infinity. This is a case of *two-beam* interference.

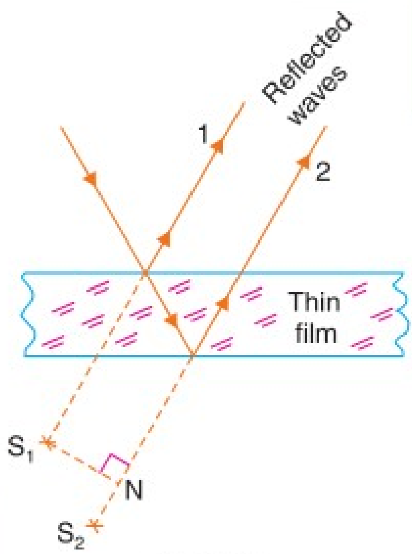


Fig. 15.2

The condition for maxima and minima can be deduced once we have calculated the optical path difference between the two rays at the point of their meeting.

15.2.1. INTERFERENCE DUE TO REFLECTED LIGHT

Let us consider a transparent film of uniform thickness 't' bounded by two parallel surfaces as shown in Fig. 15.3. Let the refractive index of the material be μ. The film is surrounded by air on both the sides. Let us consider plane waves from a monochromatic source falling on the thin film at an angle of incidence *'i'*. Part of a ray such as AB is reflected along BC, and part of it is transmitted

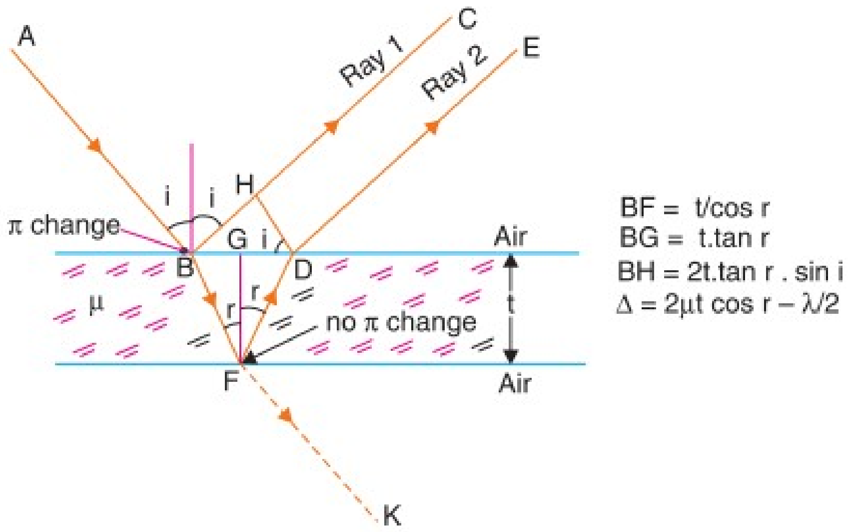


Fig. 15.3

into the film along BF. The transmitted ray BF makes an angle 'r' with the normal to the surface at the point G. The ray BF is in turn partly reflected back into the film along FD while a major part refracts into the surrounding medium along FK. Part of the reflected ray FD is transmitted at the upper surface and travels along DE. Since the film boundaries are parallel, the reflected rays BC and DE will be parallel to each other. The waves travelling along the paths BC and BFDE are derived from a single incident wave AB. Therefore they are coherent and can produce interference if they are made to overlap by a condensing lens or the eye.

(*i*) Geometrical Path Difference:

Let DH be normal to BC. From points H and D onwards, the rays HC and DE travel equal path. The ray BH travels in air while the ray BD travels in the film of refractive index μ along the path BF and FD. The geometric path difference between the two rays is

$$BF+FD - BH.$$

(*ii*) Optical Path Difference:

Optical path difference $\Delta_a = \mu L$

$\therefore$ $$\Delta_a = \mu\,(BF+FD) - 1(BH) \qquad (15.1)$$

$\therefore$ In the ΔBFD, $\angle BFG = \angle GFD = \angle r$

$$BF = FD$$

$$BF = \frac{FG}{\cos r} = \frac{t}{\cos r}$$

$\therefore$ $$BF + FD = \frac{2t}{\cos r} \qquad (15.2)$$

Also, $$BG = GD$$

$\therefore$ $$BD = 2BG$$

$$BG = FG \tan r = t \tan r$$

∴ $$BD = 2t \tan r$$

In the Δ^{le} BHD $\angle HBD = (90 - i)$

$$\angle BHD = 90^0$$

∴ $$\angle BDH = i$$

∴ $$BH = BD \sin i = 2t \tan r \sin i \qquad (15.3)$$

From Snell's law,

$$\sin i = \mu \sin r$$

∴ $$BH = 2t \tan r\,(\mu \sin r) = \frac{2\mu t \sin^2 r}{\cos r} \qquad (15.4)$$

Using the equations (15.2) and (15.4) into eq.(15.1), we get

$$\Delta_a = \mu\left[\frac{2t}{\cos r}\right] - \left[\frac{2\mu t \sin^2 r}{\cos r}\right]$$

$$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

$$= \frac{2\mu t}{\cos r} \cos^2 r$$

∴ $$\Delta_a = 2\mu t \cos r \qquad (15.5)$$

(*iii*) Correction on account of phase change at reflection:

When a ray is reflected at the boundary of a rarer to denser medium, a path change of λ/2 occurs for the ray BC (see Fig.15.3). There is no path difference due to transmission at D. Including the change in path difference due to reflection, the true path difference

$$\Delta_t = 2\mu t \cos r - \frac{\lambda}{2} \qquad (15.6)$$

15.2.2. CONDITIONS FOR MAXIMA (BRIGHTNESS) AND MINIMA (DARKNESS)

Maxima occur when the optical path difference $\Delta = m\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves *interfere constructively*. Thus, when

$$2\mu t \cos r - \frac{\lambda}{2} = m\lambda \qquad (15.7)$$

the reflected rays undergo constructive interference to produce brightness or maxima at the point of their meeting.

$$2\mu t \cos r = m\lambda + \lambda/2$$

or $$2\mu t \cos r = (2m+1)\lambda/2$$ **Condition for Brightness** (15.8)

Minima occur when the optical path difference is $\Delta = (2m+1)\lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave falls on the troughs of the others and *the waves interfere destructively*. Thus, when

$$2\mu t \cos r - \lambda/2 = (2m+1)\lambda/2 \qquad (15.9)$$

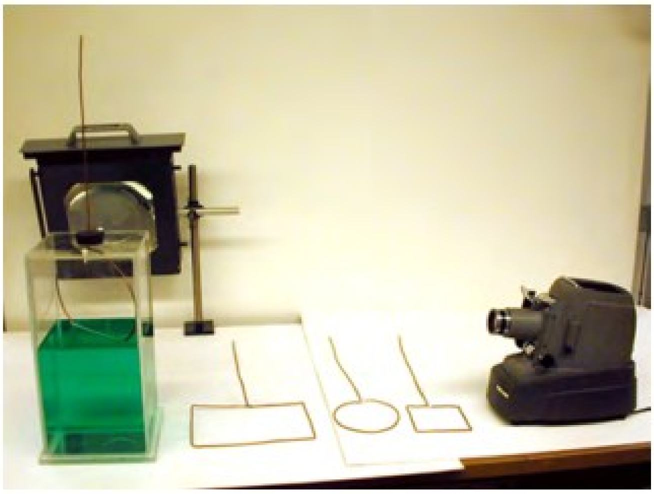
the reflected rays undergo destructive interference to produce darkness. Equ.(15.9) may be rewritten as

$$2\mu\ t\ \cos\ r\ = (m+1)\ \lambda$$

The phase relationship of the interfering waves does not change if one full wave is added to or subtracted from any of the interfering waves. Therefore (m + 1)λ can as well be replaced by mλ for simplicity in expression. Thus,

$$2\mu\ t\ \cos\ r = m\lambda \qquad \textbf{Condition for Darkness} \qquad (15.10)$$

15.2.3. SOME IMPORTANT POINTS



Thin film interference -soap films.

(a) It is seen that the conditions of interference depend on four parameters, namely μ, t, λ and r. In the case of constant thickness (parallel) film, (μt) is constant. When a parallel beam of light is incident on such a film, r also remains constant. Then the interference conditions solely depend on the wavelength λ.

(b) When monochromatic light falls on a parallel beam, the whole film will appear *uniformly* dark or bright. If the condition of constructive interference is satisfied, the film will show intense colour corresponding to the incident light.

(c) If a parallel beam of white light falls on a parallel film, those wavelengths for which the path difference is mλ, will be absent from the reflected light. The other colours will be reflected. Therefore, the film will appear uniformly coloured with one colour being absent.

15.2.4. NARROW LIGHT SOURCE VERSUS EXTENDED LIGHT SOURCE

In case of Fresnel's biprism and Lloyd's mirror, interference fringes were produced by two coherent sources. The initial source is narrow. The fringes obtained on a screen are viewed with an eyepiece. In case of a thin film, a narrow source limits the area of the film that can be viewed. Consider a thin film illuminated by a narrow source of light S (Fig. 15.4). The ray 1 produces

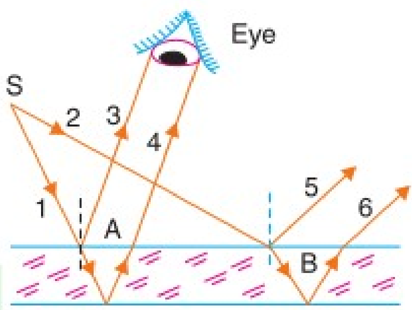


Fig. 15.4

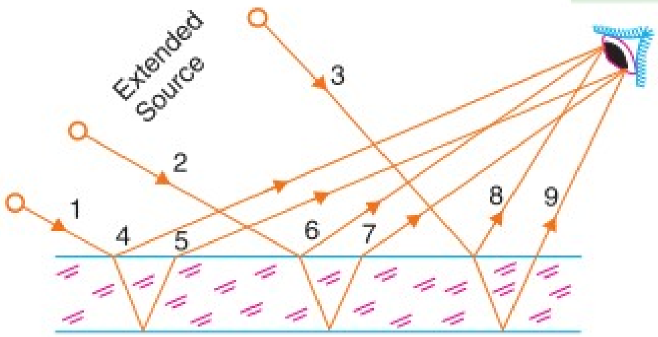


Fig. 15.5

interference fringes because rays 3 and 4 reach the eye. The ray 2 is incident on the surface of the film at a different angle and is reflected along 5 and 6. The rays 5 and 6 do not reach the eye. Similar is the case for other rays incident at different angles on the film surface. The reflected rays do not reach the eye. Thus, *only* the portion A of the film is visible to the eye.

If an extended (or broad) light source is used to illuminate the film, as in Fig.15.5, a larger area of the film surface is observed. The ray 1 after reflection from the upper and the lower surface of the film emerges as rays 4 and 5, which reach the eye. Ray 2 from some other point of the source after reflection from the upper and lower surfaces of the film emerge as rays 6 and 7 which also reach the eye. Also, ray 3 from some other point of the source after reflection from the upper and lower surfaces of the film emerge as rays 8 and 9 which also reach the eye. Therefore, the rays incident at different angles on the film are accommodated by the eye and the field of view is large. Therefore, a broad source of light is required to observe interference in thin films.

15.2.5. RESTRICTION ON THICKNESS OF THE FILM

We know that interference colours are observed only in thin films but not in thick plates such as windowpanes or glass slabs. This is due to the fact that light waves can interfere only when both

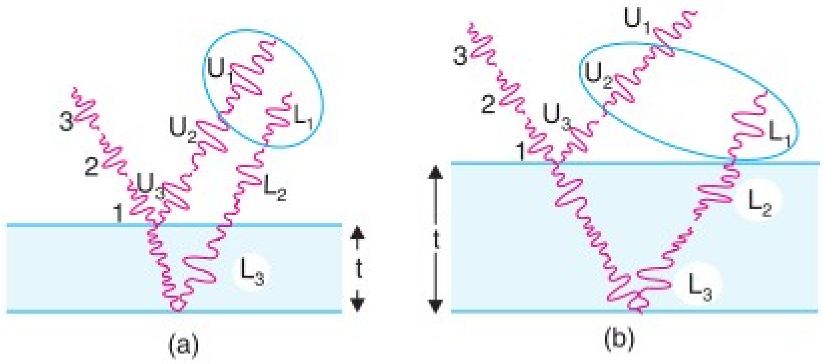


Fig. 15.6

Thick films do not exhibit interference.

the conditions of temporal and spatial coherence are satisfied. In Fig. 15.3 we have assumed that a monochromatic wave of infinite length is incident on the film. In reality, the incident light consists of wave trains of finite length and coherence extends over the length of each wave train only. Interference can occur only when parts of the same group of wave trains overlap. Superposition of different wave trains cannot produce interference because they will be incoherent and do not maintain any constant phase relationship with each other.

Fig. 15.6 shows the real situation. Wave trains 1,2,3 of finite length are incident in succession on a thin film. Portions of each wave train are reflected by the top and bottom surfaces of the film. Each wave train is divided into two reflected wave trains (U_1, L_1, U_2, L_2 and U_3, L_3). In Fig. 15.6 (a) the film is thin and the difference in the optical path lengths of U_1 and L_1 is small compared to the length of the wave train. Their superposition produces interference, as U_1 and L_1 are parts of the same wave train 1 and hence are coherent. In Fig.15.6 (b) the film is thicker and the optical path difference between U_1 and L_1 is large than the coherence length. Consequently, superposition takes place between parts of different wave trains, U_2 and L_1 and U_3 and L_2. Therefore interference does not take place.

It implies that interference occurs *only when* the optical path difference, Δ, between the superposing waves is less than the coherence length (see § 16.3).

i.e., $\Delta << l_{coh}$ (15.11)

∴ $(2\mu t \cos r - \lambda/2) << l_{coh}$ (15.12)

But $$l_{coh} = \frac{\lambda^2}{\Delta\lambda}$$ (Refer to equ. 16.11)

∴ $(2\mu t \cos r - \lambda/2) < \lambda^2/\Delta\lambda$

Rearranging the terms, we obtain

$$t < \frac{\lambda\left[\frac{\lambda}{\Delta\lambda} + \frac{1}{2}\right]}{2\mu \cos r} \quad (15.13)$$

$\lambda/\Delta\lambda >> 1/2$ and for normal incidence $\cos r = 1$.

$$\therefore \quad t < \frac{\lambda^2}{2\mu\Delta\lambda} \quad (15.14)$$

The above equation indicates that *interference in thin film will be observed if the thickness of the film is less than the coherence length of the incident light waves*. Normally, the coherence length of the light from ordinary sources is of the order of a fraction of a millimeter. Therefore, interference is seen with the films of thickness of the order of a few hundred microns only. It is because of this reason that thick films do not exhibit interference.

15.3. INTERFERENCE DUE TO TRANSMITTED LIGHT

Consider a thin transparent film of thickness *t* and refractive index μ. A ray SA after refraction goes along AB. At B it is partly reflected along BC and partly refracted along BR. The ray BC, after reflection at C, finally emerges along DQ. Here at B and C reflection takes place at the rarer medium. Therefore, no phase change occurs. Draw BM normal to CD and DN normal to BR. The optical path difference between DQ and BR is given by

$$\Delta = \mu(BC + CD) - BN$$

Also, $\mu = \frac{\sin i}{\sin r} = \frac{BN}{MD}$ or $BN = \mu.MD$

In Fig.15.7, ∠BPC = *r* and CP = BC = CD

∴ BC + CD = PD

∴ Δ = μ(PD) – μ (MD) = μ(PD – MD) = μ.PM

In ΔBPM, $\cos r = \frac{PM}{BP}$ or PM = BP. *cos r*

But BP = 2 *t*

∴ PM = 2 *t cos r*

∴ Δ = μ.PM = 2 μ *t cos r* (15.15)

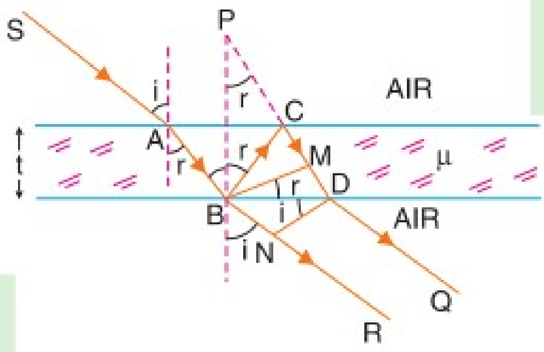


Fig. 15.7

Bright Fringes:

When the optical path difference Δ = *m* λ, bright fringe occurs.

∴ $2\mu t \cos r = m\lambda$ (15.16)

where m = 0,1,2,3,….

Dark Fringes:

When the optical path difference $\Delta = (2m + 1)\lambda / 2$, dark fringe occurs.

$$\therefore \qquad 2\mu\ t\ \cos\ r = \frac{(2m+1)}{2}\lambda \qquad (15.17)$$

where m = 0,1,2,3,….

In case of transmitted light, the fringes are less distinct because the difference in amplitudes of BR and DQ is very large. However, when the angle of incidence is nearly 45° the fringes are more distinct.

15.4. HAIDINGER FRINGES

In thin films interference fringes are produced due to the path difference $2\mu\ t \cos r$ between the overlapping rays. For a given film the path difference may arise due to (*i*) the angle of refraction r inside the film or (*ii*) the change in thickness. We can express the change in path difference by differentiating the expression $2\ \mu\ t \cos r$.

$$\text{Change in path difference, } \delta(\Delta) = 2\mu\ t \cdot \delta(\cos\ r) + 2\mu\ \cos\ r\ (\delta\ t) \qquad (15.18)$$

When the film is of **uniform** (constant) thickness, the change in path difference is only due to the change in r. If the thickness of the film is large, the path difference will change appreciably

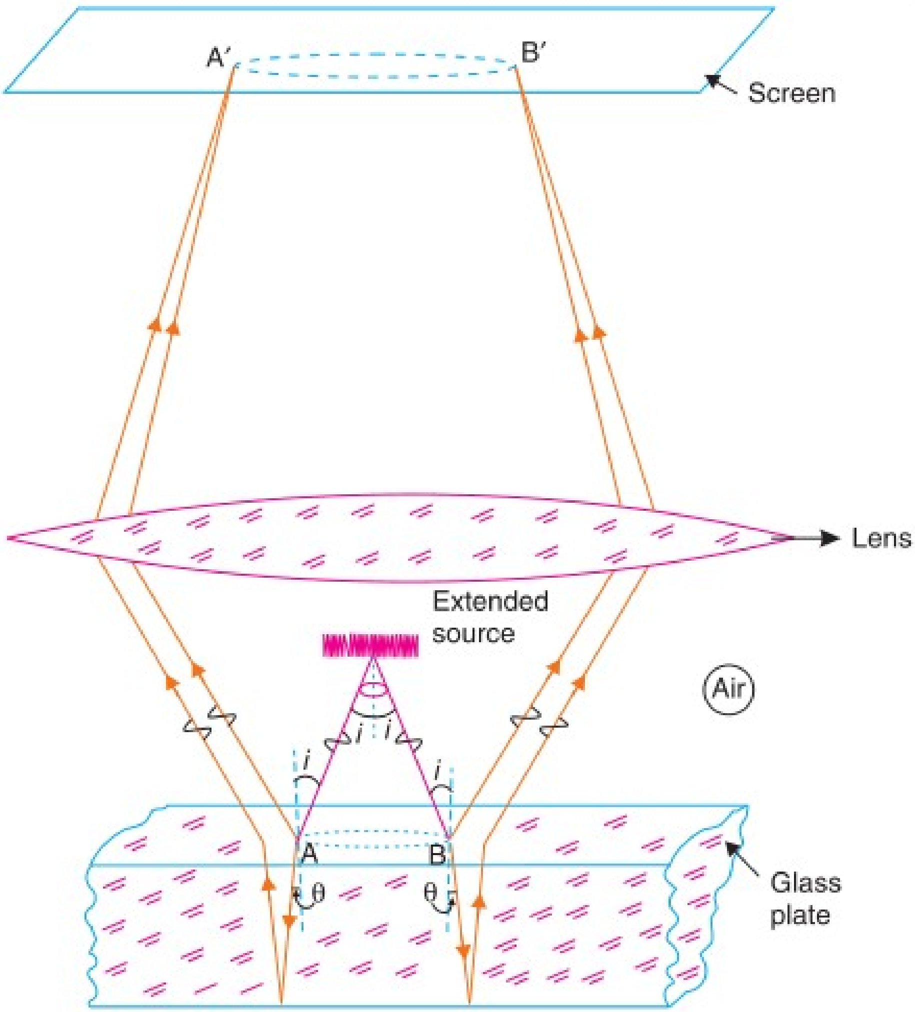


Fig. 15.8

even when r changes in a small way. Fringes are produced in this case due to the superposition of rays, which are equally inclined to the normal. These fringes are called **fringes of equal inclination**. The fringes of equal inclination are known as **Haidinger fringes.** In this case all the pairs of interfering rays of equal inclination pass through the plate as a parallel beam and hence meet at infinity. The other pairs of differrent inclination meet at different points at infinity. Therefore, they can be located with a telescope focussed to infinity. The fringes are therefore said to be *localized at infinity*.

To produce Haidinger fringes, the source must be an extended source, the film thickness must be appreciably large and the observing instrument is to be focussed for parallel rays.

Fig. 15.8 shows the formation of Haidinger fringes. Let us consider that a thin plate is illuminated by an extended monochromatic light source. A lens is arranged parallel to the plate and a screen is kept in the focal plane of the lens. Light from the extended source is incident on the plate in diverse directions. The waves propagating parallel to the plane of the page and falling on the plate at an angle i at points A and B get reflected from the top and bottom surfaces of the plate. The reflected pairs of waves will meet at points A′ and B′ respectively on the screen due to the focussing action of the lens. Depending on their path difference, the reflected waves produce either brightness or darkness on the screen. In fact the waves incident at the top surface of the plate at an angle i travel along the generators of a cone as shown in Fig. 15.8. Each pair of parallel reflected waves interfere at diametrically opposite points. Thus, a circular fringe is produced. Similarly, the waves incident at a different angle will produce a collection of identical points arranged along a circle of another radius. As a result, a system of alternating bright and dark circular fringes with a common centre will be observed on the screen.

Each fringe is characterized by a particular value of m. Bright fringes are produced when the condition $2\,\mu\,t\cos r = m\lambda$ is satisfied; and dark fringes are produced where the condition

$2\,\mu\,t\cos r = (2m+1)\,\lambda/2$ is satisfied. The parallel pairs of reflected rays meet only at infinity; therefore a lens is used to focus them. Accordingly, these fringes of equal inclination are said to be localized at infinity.

15.5. VARIABLE THICKNESS (WEDGE-SHAPED) FILM

Let us now study the interference of light in a film of varying thickness. *A thin film having zero thickness at one end and progressively increasing to a particular thickness at the other end is called a wedge.* A thin wedge of air film can be formed by two glass slides resting on each other at one edge and separated by a thin spacer at the opposite edge.

The arrangement for observing the interference pattern in a wedge shaped film is shown in Fig. 15.9(a). The wedge angle is usually very small and of the order of a fraction of a degree. When a parallel beam of *monochromatic* light illuminates the wedge from above, the rays reflected from its two bounding surfaces will not be parallel. They appear to diverge from a point near the film. The path difference between the rays reflected from the upper and lower surfaces of the air film varies along its length due to variation in film thickness. Therefore, alternate bright and dark fringes are observed on its top surface (see Fig. 15.9b). The fringes are localized at the top surface of the film.

When the light is incident on the wedge from above, it gets partly reflected from the glass-to-air boundary at the top of the air film. Part of the light is transmitted through the air film and gets reflected partly at the air-to-glass boundary, as shown in Fig. 15.10. The two rays BC and DE, thus reflected from the top and bottom of the air film, are coherent as they are derived from the same ray AB through *division of amplitude*. The rays are close enough if the thickness of the film is of the order of a wavelength of light. For small film thickness the rays interfere producing darkness or brightness depending on the phase difference. The thickness of the glass plates is large compared

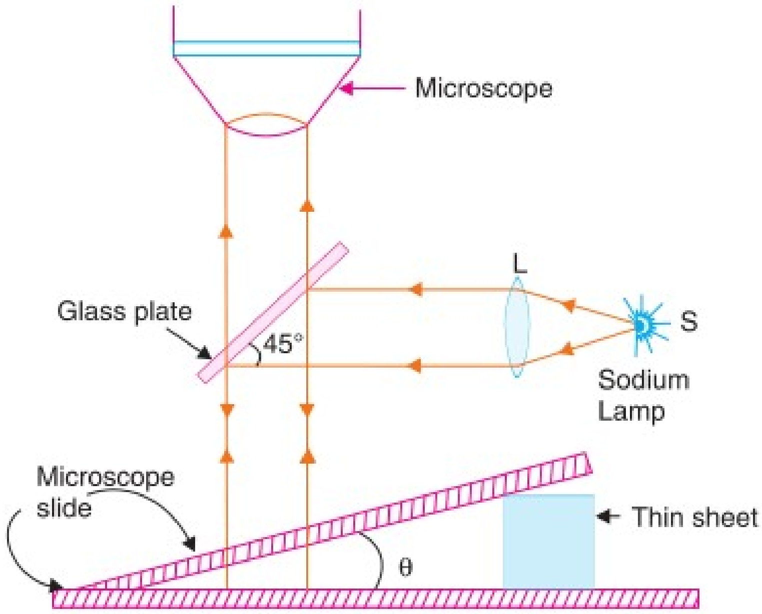


Fig. 15.9 (a)

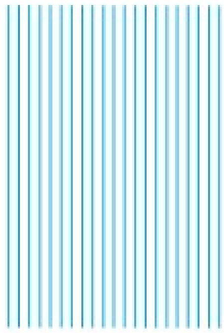


Fig. 15.9 (b)

with the wavelength of the incident light. Hence, the observed interference effects are entirely due to the wedge-shaped air film.

The optical difference between the two rays BC and DE is given by

$$\Delta = 2\mu\, t \cos r - \lambda/2$$

where $\lambda/2$ takes account the gain of half–wave due to the abrupt jump of π radians in the phase of the wave reflected from the bottom boundary of air – to – glass.

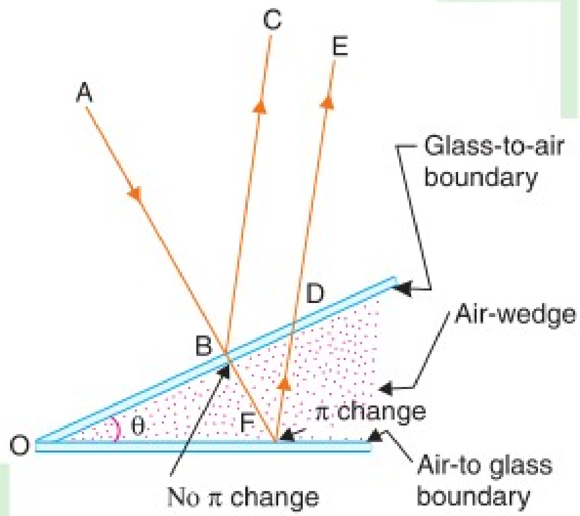


Fig. 15.10

Maxima occur when the optical path difference $\Delta = m\,\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves *interfere constructively*. This needs that

$$\Delta = 2\mu\, t \cos r - \lambda/2$$

Minima occur when the optical path difference is $\Delta = (2\,m + 1)\,\lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave falls on the troughs of the others and *the waves interfere destructively*. It needs that

$$2\mu\, t \cos r = m\lambda.$$

Referring to Fig.15.11, let us say a dark fringe occurs at A where the relation

$$2\mu\, t \cos r = m\lambda$$

is satisfied. If normal incidence is assumed, *cos r = 1* and if the thickness of air film at A is denoted by t_1, then at A

$$2\mu\, t_1 = m\lambda \qquad (15.19)$$

The next dark fringe will occur, say, at C where the thickness CL = t_2. Then at C

$$2\mu\, t_2 = (m+1)\,\lambda \qquad (15.20)$$

Subtracting eq. (15.19) from eq. (15.20), we get

$$2\mu\,(t_2 - t_1) = \lambda \qquad (15.21)$$

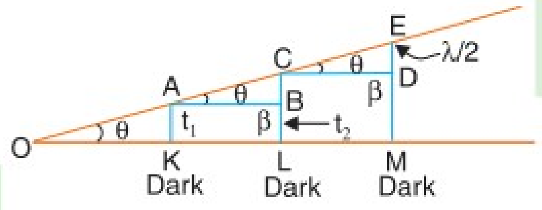


Fig. 15.11

But $(t_2 - t_1) = BC$

$\therefore \quad 2\mu(BC) = \lambda$

or $$BC = \frac{\lambda}{2\mu} \qquad (15.22)$$

From the Δ^{le}ABC, ∠CAB = θ and BC = AB tan θ

$$\therefore \quad (AB)\ \tan\theta = \frac{\lambda}{2\mu} \qquad (15.23)$$

AB is the distance between successive dark fringes and it also equals the separation of the successive bright fringes. It is, therefore, called the **fringe width, β**. That is AB = β. We may write eq. (15.23) as

$$\beta = \frac{\lambda}{2\mu\ \tan\theta} \qquad (15.24)$$

For small values of θ, tan θ ≈ θ.

$$\therefore \quad \beta = \frac{\lambda}{2\mu\theta} \qquad (15.25)$$

As the quantities on the right side of the above equation are all constant, β is constant for a given wedge angle. According to equ.(15.25), an increase in the angle θ makes the fringes move closer. At an angle θ ≈ 1°, the interference pattern vanishes. On the other hand, if θ is gradually decreased, the fringe separation increases, and ultimately the fringes disappear as the faces of the film become parallel.

The interference pattern has the following salient features.

(*i*) Fringe at the apex is dark.
(*ii*) Fringes are straight and parallel.
(*iii*) Fringes are equidistant.
(*iv*) Fringes are localized.
(*v*) Fringes are of equal thickness.

(*i*) Fringe at the apex is dark: At the apex, the two glass slides are in contact with each other. Therefore, the thickness of the air film at the contact edge is negligible ($t \cong 0$). The optical path difference there becomes

$$\Delta = 2\mu t - \lambda/2 = 0 - \lambda/2 = -\lambda/2 \qquad (15.26)$$

It implies that a path difference of λ/2 or a phase difference of π occurs between the reflected waves at the edge. The two waves interference destructively. Therefore, the fringe at the apex is always dark (See Fig. 15.12).

(*ii*) Straight and parallel fringes: Each fringe in the pattern is produced by the interference of rays reflected from sections of the wedge having the same thickness. The locus of points having the same thickness lie along lines parallel to the contact edge. Therefore, the fringes are straight. Since the fringes are equidistant [see (iii)], they will be parallel (See Fig. 15.12).

***(iii)* Equidistant fringes:** The fringe width β is given by

$$\beta \approx \lambda/2\theta \quad (15.27)$$

where λ is the wavelength of the incident light and θ is the angle of the wedge. As the quantities λ and θ are constants, β is constant for a given wedge angle. Therefore, the fringes are equidistant (see Fig.15.12).

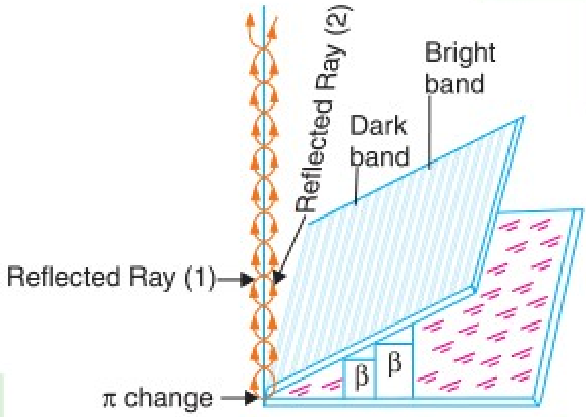


Fig. 15.12

***(iv)* Localized fringes:** The fringes form very close to the top surface of the wedge and can be seen with a microscope.

***(v)* Fringes of equal thickness:** In thin films of thickness of the order of a few λ, the rays from various parts of the film have almost the same inclination and hence the path difference between the overlapping waves changes mainly due to change of thickness. The fringes produced in such cases are mainly due to the variation in thickness of the film. Each fringe will be the locus of points of the same thickness. Such fringes are called **fringes of equal thickness**.

15.5.1. DETERMINATION OF THE WEDGE ANGLE

The wedge angle θ can be experimentally determined with the help of a travelling microscope. Using the microscope the positions of dark fringes at two distant points Q and R are noted (Fig. 15.13). Let the distance OQ be x_1 and OR be x_2. Let the thickness of the wedge be t_1 at Q and t_2 at R.

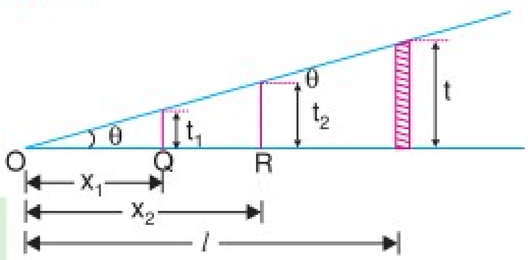


Fig. 15.13

The dark fringe at Q is given by

$$2\mu t_1 = m\lambda \quad (15.28)$$

But as θ is very small, we can write

$$t_1 = x_1 \tan\theta \cong x_1\theta$$

$$\therefore \quad 2\mu\, x_1\, \theta = m\lambda \quad (15.29)$$

We can write similarly for the dark fringe at R as

$$2\mu\, x_2\, \theta = (m+N)\lambda \quad (15.30)$$

where N is the number of dark fringes lying between the positions Q and R. Subtracting equ.(15.29) from equ.(15.30), we get

$$2\mu(x_2 - x_1)\theta = N\lambda$$

$$\therefore \quad \theta = \frac{N\lambda}{2\mu(x_2 - x_1)} \quad (15.31)$$

In case of air $\mu = 1$ and the above relation reduces to

$$\theta = \frac{N\lambda}{2(x_2 - x_1)} \quad (15.32)$$

15.5.2. DETERMINATION OF THE THICKNESS OF THE SPACER

The thickness of the spacer used to form the wedge shaped air film between the glass slides can be determined from the above measurements. If 't' is the thickness of the spacer (foil or wire) used, we can write from Fig.15.13 that

$$t = l \tan \theta \cong l\,\theta \tag{15.33}$$

where l is the length of the air wedge. Using the equ.(15.32) into equ.(15.33), we obtain

$$\therefore \qquad t = \frac{lN\lambda}{2(x_2 - x_1)} \tag{15.34}$$

15.5.3. FIZEAU FRINGES

If a parallel beam of light is incident perpendicularly or nearly perpendicular on a variable thickness film, then dark and bright fringes are seen in reflected light. These fringes are fringes of equal thickness, because each fringe corresponds to lines of equal optical thickness. These fringes or localized fringes and are observed at the top of the film. These localized fringes of equal thickness are known as *Fizeau fringes*. Contours following lines of equal optical thickness are seen if the area is large. The fringes may be obtained in case of thick films also if the source is small.

15.5.4. COLOURS IN THIN FILMS

Colours in thin films of oil.

The colours exhibited in reflection by thin films of oil, mica, soap bubbles and coatings of oxides on heated metals etc are due to interference of light from an extended source such as sky. Thomas Young explained the origin of colours in thin films. It may be understood as follows. The films are usually observed by reflected light. The eye looking at the thin film receives light waves reflected from the top and bottom surfaces of the film. The reflected rays are very close to each other and are in a position to interfere. The optical path difference between the interfering rays is $\Delta = 2\mu t \cos r - \lambda/2$. It is seen that the path difference depends upon the thickness t of the film, the wavelength λ and the angle r, which is related to the angle of incidence of light on the film. White light consists of a range of wavelengths and for specific values of t and r, waves of only certain wavelengths (colours) constructively interfere. Therefore, only those colours are present in the reflected light. The other wavelengths interfere destructively and hence are absent from the reflected light. Hence, the film at a particular point appears coloured. As the thickness and the angle of incidence vary from point to point, different colours are intensified at different places. The colours seen are not isolated colours, as at each place there is a mixture of colours. The composition of colours is different at different places and contours of impressive hues are observed over the entire surface of the film.

15.6. NEWTON'S RINGS

Newton's rings are an example of fringes of equal thickness. Newton's rings are formed when a plano-convex lens P of a large radius of curvature placed on a sheet of plane glass AB is illuminated from the top with monochromatic light (Fig. 15.14). The combination forms a thin circular air film of variable thickness in all directions around the point of contact of the lens and the glass plate. The locus of all points corresponding to specific thickness of air film falls on a circle whose centre is at O. Consequently, interference fringes are observed in the form of a series of concentric rings with their centre at O. Newton originally observed these concentric circular fringes and hence they are called **Newton's rings**.

The experimental arrangement for observing Newton's rings is shown in Fig. 15.14.

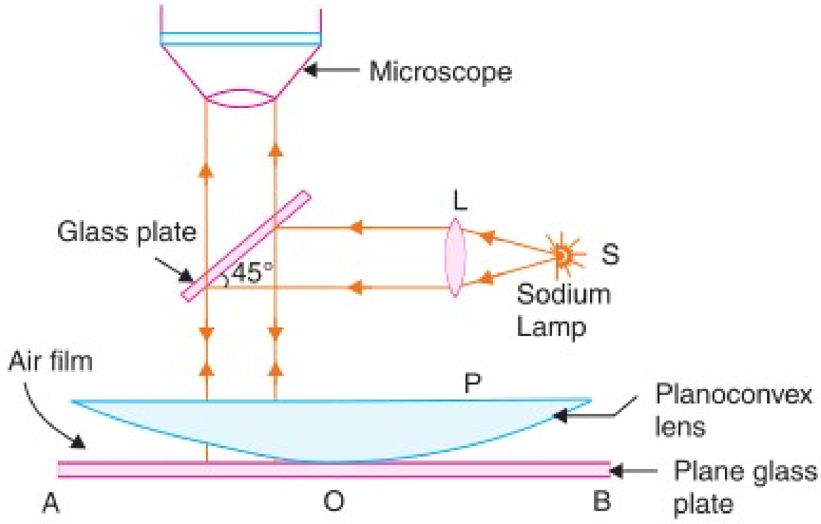


Fig. 15.14

Monochromatic light from an extended source S is rendered parallel by a lens L. It is incident on a glass plate inclined at 45° to the horizontal, and is reflected normally down onto a plano-convex lens placed on a flat glass plate. Part of the light incident on the system is reflected from the glass-to-air boundary, say from point D (Fig. 15.15). The remainder of the light is transmitted through the air film. It is again reflected from the air-to-glass boundary, say from point J. The two rays reflected from the top and bottom of the air film are derived through division of amplitude from the same incident ray CD and are therefore coherent. The rays 1 and 2 are close to each other and interfere to produce darkness or brightness. The condition of brightness or darkness depends on the path difference between the two reflected light rays, which in turn depends on the thickness of the air film at the point of incidence.

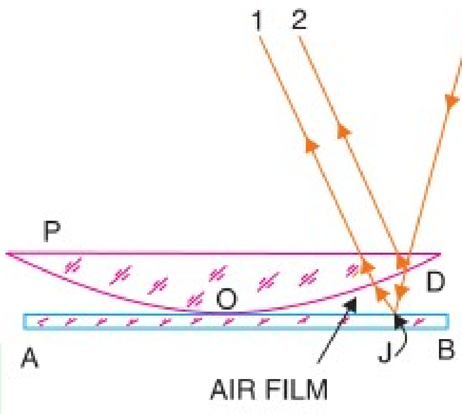


Fig. 15.15

15.6.1. CONDITION FOR BRIGHT AND DARK RINGS

The optical path difference between the rays is given by $\Delta = 2\mu t \cos r - \lambda/2$. Since $\mu = 1$ for air and $\cos r = 1$ for normal incidence of light,

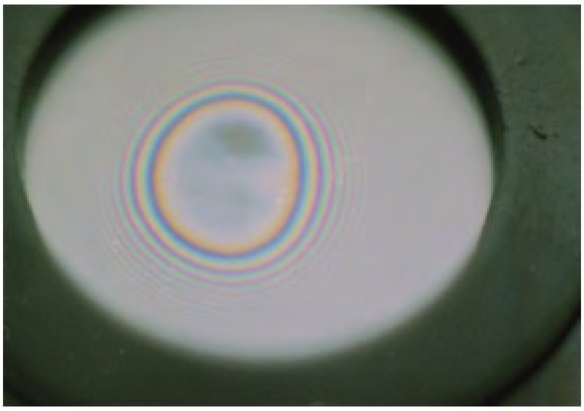
$$\Delta = 2t - \lambda/2 \tag{15.35}$$

Intensity maxima occur when the optical path difference $\Delta = m\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the other and the waves *interfere constructively*. Thus, if $2t - \lambda/2 = m\lambda$

$$\boxed{2t = (2m+1)\lambda/2} \qquad (15.36)$$

bright fringe is obtained.

Intensity minima occur when the optical path difference is $\Delta = (2\ m + 1)\ \lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave fall on the troughs of the other and *the waves interfere destructively*.



Circular fringes.

Hence, if $\quad 2t - \lambda/2 = (2m+1)\ \lambda/2$

$$\boxed{2t = m\lambda} \qquad (15.37)$$

dark fringe is produced.

15.6.2. CIRCULAR FRINGES

In Newton's ring arrangement, a thin air film is enclosed between a plano-convex lens and a glass plate. The thickness of the air film at the point of contact is zero and gradually increases as we move outward. The locus of points where the air film has the same thickness then fall on a circle whose centre is the point of contact. Thus, the thickness of air film is constant at points on any circle having the point of lens–glass plate contact as the centre. The fringes are therefore circular.

Fig. 15.16

15.6.3. RADII OF DARK FRINGES

Let R be the radius of curvature of the lens (Fig.15.17). Let a dark fringe be located at Q. Let the thickness of the air film at Q be PQ = t. Let the radius of the circular fringe at Q be OQ= r_m. By the Pythagorus theorem,

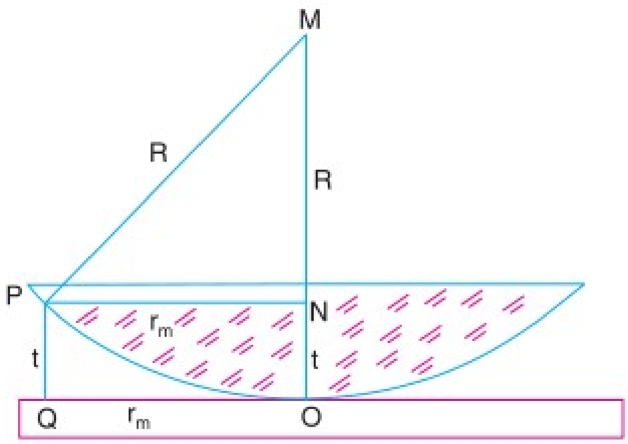


Fig. 15.17

$$PM^2 = PN^2 + MN^2$$

$$\therefore \qquad R^2 = r_m^2 + (R-t)^2$$

or $$r_m^2 = 2Rt - t^2 \qquad (15.38)$$

As R >> t, 2Rt >> t^2.

$$\therefore \qquad r_m^2 \cong 2Rt \qquad (15.39)$$

The condition for darkness at Q is that

$$2t = m\lambda$$

$$\therefore \qquad r_m^2 \cong m\lambda R$$

$$r_m = \sqrt{m\lambda R} \qquad (15.40)$$

The radii of dark fringes can be found by inserting values 1,2,3, ...for m. Thus,

$$r_1 = \sqrt{1\lambda R} \quad \text{or} \quad r_1 \propto \sqrt{1}$$

$$r_2 = \sqrt{2\lambda R} \quad \text{or} \quad r_2 \propto \sqrt{2}$$

$$r_3 = \sqrt{3\lambda R} \quad \text{or} \quad r_3 \propto \sqrt{3} \quad \text{and so on}$$

It means that *the radii of the dark rings are proportional to under root of the natural numbers.*

The above relation also implies that

$$r_m \propto \sqrt{\lambda}$$

Thus, *the radius of the m^{th} dark ring is proportional to under root of wavelength.*

Ring Diameter:

Diameter of m^{th} dark ring $D_m = 2r_m$

$$D_m = 2\sqrt{2Rt}$$

$$D_m = 2\sqrt{m\lambda R} \qquad (15.41)$$

15.6.4. SPACING BETWEEN FRINGES

It is seen that the diameter of dark rings is given by

$$D_m = 2\sqrt{m\lambda R}$$

where m = 1,2,3, ...

The diameters of dark rings are proportional to the square root of the natural numbers. Therefore, the diameter of the ring does not increase in the same proportion as the order of the ring, for example, if m increases as 1,2,3,4, ...the diameters are

$$D_1 = 2\sqrt{\lambda R}$$

$$D_2 = 2\,(1.4)\sqrt{\lambda R}$$

$$D_3 = 2\,(1.7)\sqrt{\lambda R}$$

$$D_4 = 2\,(2)\sqrt{\lambda R} \quad \text{and so on.}$$

Therefore, the rings get closer and closer, as m increases. This is why the rings are not evenly spaced.

Newton's rings arrangement.

15.6.5. FRINGES OF EQUAL THICKNESS

Newton's rings are formed as result of interference between light waves reflected from the top and bottom surfaces of a thin air film enclosed between a plano-convex lens and a plane glass plate.

The occurrence of alternate bright and dark rings depends on the optical path difference arising between the reflected rays. If the light falls normally on the air film the optical path difference between the waves reflected from the two surfaces of the film is

$$\Delta = 2t - \lambda/2$$

It is seen that the path difference between the reflected rays arises due to the variation in the thickness 't' of the air film. Reflected light will be of minimum intensity for those thickness for which the path difference is m λ and maximum intensity for those thickness for which the path difference is $(2m+1)\lambda/2$. Thus, each maxima and minima is a locus of constant film thickness. Therefore, the fringes are known as fringes of equal thickness.

15.6.6. DARK CENTRAL SPOT

The central spot is dark as seen by reflection. Newton's rings are produced due to superposition of light rays reflected from the top and bottom surfaces of a thin air film enclosed between a plano-convex lens and a plane glass plate. The occurrence of brightness or darkness depends on the optical path difference arising between the reflected rays. The optical path difference is given by $\Delta = 2t - \lambda/2$.

At the point of contact 'O' of the lens and glass plate (Fig.15.18), the thickness of air film is negligibly small compared to a wavelength of light.

$$\therefore \quad t \cong 0$$

$$\therefore \quad \Delta \cong \lambda/2$$

The wave reflected from the lower surface of the air film suffers a phase change of π while the wave reflected from the upper surface of the film does not suffer such change.

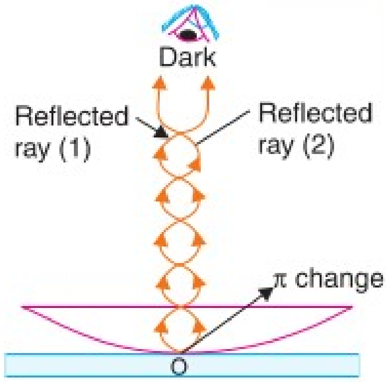


Fig. 15.18

Thus, the superposing waves are out of step by $\lambda/2$ which is equivalent to a phase difference of 180° (or π rad). Thus the two interfering waves at the centre are opposite in phase and produce a dark spot.

15.6.7. DETERMINATION OF WAVELENGTH OF LIGHT

A plano-convex lens of large radius of curvature (about 100 cm) and a flat glass plate are cleaned. The lens is kept with its convex face on the glass plate and they are held in position with the help of a metal ring arrangement. The system is held under a low power travelling microscope kept before a sodium vapour lamp. It is arranged that the yellow light coming from the sodium lamp falls on a glass plate held at 45° light beam. The light is turned through 90° and is incident normally on the lens-plate system. The microscope is adjusted till the circular rings came into focus. The centre of the cross-wire is made to come into focus on the centre of the dark spot, which is at the centre of the circular ring system. Now, turning the screw the microscope is moved on the carriage slowly towards one side, say right side. As the cross-wires move in the field of view, dark rings are counted. The movement is stopped when the 22nd dark ring is reached. Then the microscope is moved in the opposite direction and stopped at the 20th or 19th dark ring. The vertical cross-wire is made tangential to

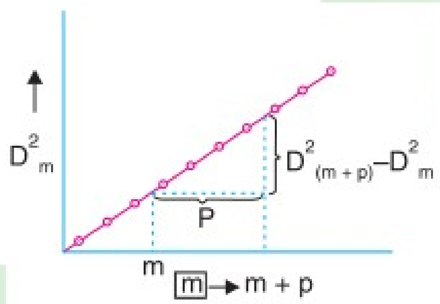


Fig. 15.19

the 19th ring and the reading is noted with the help of the scale graduated on the carriage. Thus, starting from the 19th ring, the tangential positions of the 18th, 17th, 16th,…..,5th dark rings are noted down. Now, the microscope is moved quickly to the left side of the ring system and it is stopped at the 5th dark ring. The cross-wire is again made tangential to the 5th dark ring and its position is noted. The difference between the readings on right and left sides of the 5th dark ring gives its diameter value. The procedure is repeated till 19th ring is reached and its reading is noted. From the value of the diameters the squares of the diameters are calculated. A graph is plotted between D_m^2 and the ring number '*m*'. A straight line would be obtained, as shown in Fig. 15.19.

We have

$$D_m^2 = 4m\lambda R \tag{15.42}$$

For the (m+p)th ring,

$$D_{m+p}^2 = 4(m+p)\lambda R \tag{15.43}$$

$$\therefore \quad D_{m+p}^2 - D_m^2 = 4p\lambda R$$

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR} \tag{15.44}$$

The slope of the straight line (Fig.15.18) gives the value of 4λR. Thus,

$$\lambda = \frac{Slope}{4R} \tag{15.45}$$

The radius of curvature R of the lens may be determined using a spherometer and λ is computed with the help of the above equation.

15.6.8. REFRACTIVE INDEX OF A LIQUID

The liquid, whose refractive index is to be determined, is filled in the gap between the lens and plane glass plate. Now the liquid film substitutes the air film. The condition for interference may then be written as

$$2\mu t \cos r = m\lambda \qquad \text{Darkness}$$

where μ is the refractive index of the liquid. For normal incidence the equation becomes

$$2\mu t = m\lambda$$

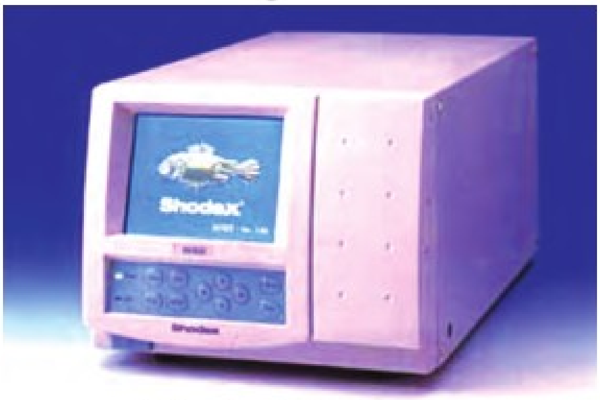
As $t = \frac{r^2}{2R}$,

$$\frac{2\mu\, r^2}{2R} = m\lambda$$

Or

$$r^2 = \frac{m\lambda R}{\mu}$$

$$\therefore \quad D^2 = \frac{4m\lambda R}{\mu}$$



Refractive index detector.

Following the above relation, the diameter of mth dark ring may be expressed as

$$\left[D_m^2\right]_L = \frac{4m\lambda R}{\mu} \tag{15.46}$$

Similarly, the diameter of the (m+p)th ring is given by

$$\left[D_{m+p}^2\right]_L = \frac{4(m+p)\lambda R}{\mu} \tag{15.47}$$

Subtracting eq. (15.46) from eq. (15.47), we get

$$\left[D_{m+p}^2\right]_L - \left[D_m^2\right]_L = \frac{4\,p\lambda R}{\mu} \qquad (15.48)$$

But we know that

$$\left(D_{m+p}^2\right)_{air} - \left(D_m^2\right)_{air} = 4\,p\lambda R \qquad (15.49)$$

$$\therefore \qquad \mu = \frac{\left(D_{m+p}^2\right)_{air} - \left(D_m^2\right)_{air}}{\left(D_{m+p}^2\right)_{liq} - \left(D_m^2\right)_{liq.}} \qquad (15.50)$$

15.6.9. NEWTON'S RINGS IN TRANSMITTED LIGHT

Newton's rings in transmitted light may be observed with the arrangement made as in Fig. 15.20. The condition for maxima or bright rings is

$$2\mu\,t\,\cos r = m\lambda$$

and for dark rings $\quad 2\mu\,t\,\cos r = (2m+1)\lambda/2$

As $\mu = 1$ for air and r = 0 for normal observation, the above expressions may be simplified to

For bright fringes $\quad 2t = \mu\lambda$

and for dark rings $\quad 2t = (2m+1)\lambda/2$

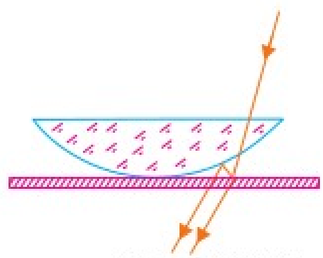


Fig. 15.20

As $t = \dfrac{r^2}{2R}$, the radius for the bright ring is given by $r_m^2 = m\lambda R \qquad (15.51)$

and the radius for dark rings is given by $r_m^2 = (2m+1)\lambda R/2 \qquad (15.52)$

15.6.10. NEWTON'S RINGS FORMED BY TWO CURVED SURFACES

Case 1: Lower surface concave:

Let us consider two curved surfaces of radii of curvature R_1 and R_2 in contact at the point O. A thin air film is enclosed between the two surfaces. The dark and bright rings are formed and can be viewed with a travelling microscope. Suppose the radius of the m^{th} dark ring is r. The thickness of the air film at P is

$$PQ = PT - QT$$

From geometry, $\quad PT = \dfrac{r^2}{2R_1} \quad$ and $\quad QT = \dfrac{r^2}{2R_2}$

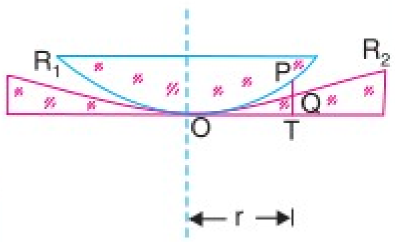


Fig. 15.21

$$\therefore \qquad PQ = \frac{r^2}{2R_1} - \frac{r^2}{2R_2} \qquad (15.53)$$

But PQ = t. The condition for dark rings in reflected light is given by $2\mu\,t\,\cos r = m\lambda$. As $\mu = 1$ and $\cos r = 1$ for normal incidence, the above condition reduces to $2t = m\lambda$.

$$\therefore \qquad 2\left(\frac{r^2}{2R_1} - \frac{r^2}{2R_2}\right) = m\lambda$$

$$r^2\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=m\lambda \qquad \text{where } m = 0,1,2,3,\ldots.. \qquad (15.54)$$

For bright fringes the condition is $2\mu\ t\ \cos\ r\ =(2m+1)\lambda/2$

which reduces to $2t=(2m+1)\lambda/2$

or $$r^2\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{(2m+1)\lambda}{2} \qquad \text{where } m = 0,1,2,3,\ldots.. \qquad (15.55)$$

Case 2: Lower surface convex:

Let us consider two curved surfaces of radii of curvature R_1 and R_2 in contact at the point O. A thin air film is enclosed between the two surfaces. The dark and bright rings are formed and can be viewed with a travelling microscope. Suppose the radius of the m[th] dark ring is r. The thickness of the air film at P is

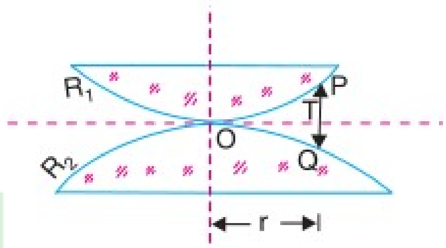


Fig. 15.22

$$PQ = PT + QT$$

From geometry $$PT=\frac{r^2}{2R_1} \quad \text{and} \quad QT=\frac{r^2}{2R_2}$$

$$\therefore \qquad PQ=\frac{r^2}{2R_1}+\frac{r^2}{2R_2}$$

But PQ = t. The condition for dark rings in reflected light is given by $2\mu\, t \cos r = m\lambda$.

As $\mu = 1$ and $\cos r = 1$ for normal incidence, the above condition reduces to $2\,t = m\lambda$

$$\therefore \qquad 2\left(\frac{r^2}{2R_1}+\frac{r^2}{2R_2}\right)=m\lambda$$

$$r^2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=m\lambda \qquad \text{where } m = 0,1,2,3,\ldots(15.56)$$

For bright fringes the condition is $2\mu\ t\ \cos\ r\ =(2m+1)\lambda/2$

which reduces to $2t=(2m+1)\lambda/2$

or $$r^2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{(2m+1)\lambda}{2} \qquad \text{where } m = 0,1,2,3,\ldots(15.57)$$

15.7. MICHELSON'S INTERFEROMETER

An interferometer is an instrument in which the phenomenon of interference is used to make precise measurements of wavelengths or distances.

Michelson's interferometer.

15.7.1. PRINCIPLE

In Michelson interferometer, a beam of light from an extended source is divided into two parts of equal intensities by partial reflection and refraction. These beams travel in two mutually perpendicular directions and come together after reflection from plane mirrors. The beams overlap on each other and produce interference fringes.

15.7.2. CONSTRUCTION

The schematic of a simple Michelson interferometer is shown in Fig.15.23. It consists of a beam splitter G_1, a compensating plate G_2, and two plane mirrors M_1 and M_2. The beam splitter G_1 is a partially silvered plane parallel glass plate. The compensating plate G_2 is a simple plane parallel glass plate having the same thickness as G_1. The two plates G_1 and G_2 are held parallel to each other and are inclined at an angle of 45° with respect to the mirror M_2. The mirror M_1 is mounted on a carriage and can be moved exactly parallel to itself with the help of a micrometer screw. The distance through which the mirror M_1 is moved can be read with the help of a graduated drum attached to the screw. Displacements of the order of 0.1 μm (1000 Å) can be easily read. The plane mirrors M_1 and M_2 can be made perfectly perpendicular with the help of the fine screws attached to them. The interference bands are observed in the field of view of the telescope T.

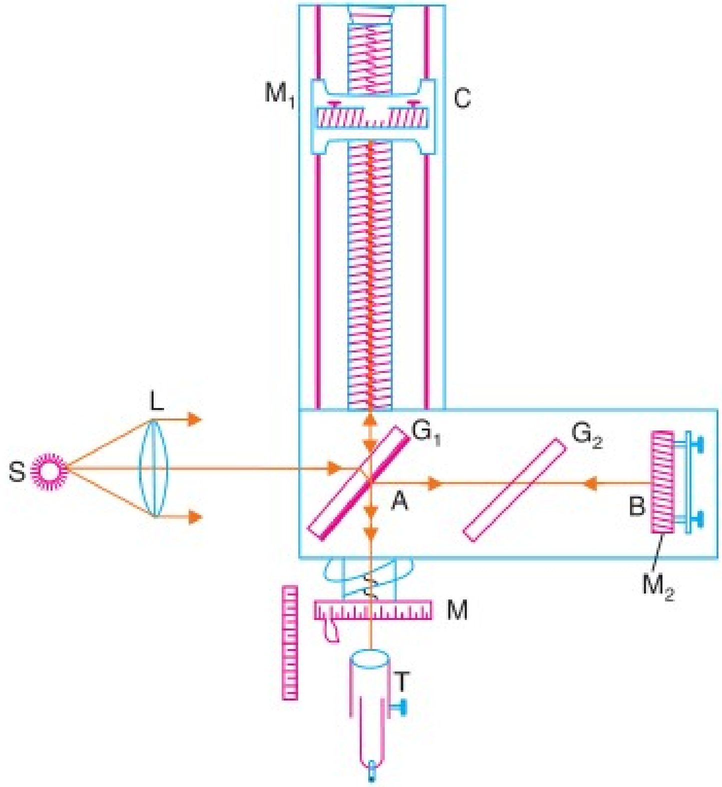


Fig. 15.23

15.7.3. WORKING

Monochromatic light from an extended source S is rendered parallel by means of a collimating lens L and is made incident on the beam splitter G_1. It is partly reflected at the back surface of G_1 along AC and partly transmitted along AB. The beam AC travels normally towards the plane mirror M_1 and is reflected back along the same path and comes out along AT. The transmitted beam travels toward the mirror M_2 and is reflected along the same path. It is reflected at the back surface of G_1 and proceeds along AT. The two beams received along AT are produced from a single source through division of amplitude and are hence *coherent.* The superposition of these beams leads to interference and produces interference fringes.

From the Fig. 15.23 it is clearly seen that a light ray starting from the source S and undergoing reflection at the mirror M_1 passes through the glass plate G_1 three times. On the other hand, in the absence of plate G_2, the ray reflected at M_2 travels through the glass plate G_1 only once. For compensating this path difference, a compensating plate G_2 of the same thickness is inserted into the path AB and is held exactly parallel to G_1.

If we look into the instrument from T, we see mirror M_1 and in addition we see a virtual image, M'_2, of mirror M_2. Depending on the positions of the mirrors, image M'_2 may be in front of, or behind, or exactly coincident with mirror M_1.

15.7.4. CIRCULAR FRINGES

Circular fringes are produced with monochromatic light when the mirrors M_1 and M_2 are exactly perpendicular to each other. The origin of the circular fringes can be understood as follows.

If we look into the instrument from T, we see mirror M_1 directly, and in addition we will see the virtual image M'_2 of mirror M_2 formed by reflection in the glass plate G_1 (Fig 15.24). It means that one of the interfering beams come from M_1 and the other beam appears to come from the virtual image M'_2. The situation is similar to an air film enclosed between mirrors M_1 and M'_2 with the difference that in case of a real film between two surfaces, multiple reflections take place, whereas in this case only two reflections take place.

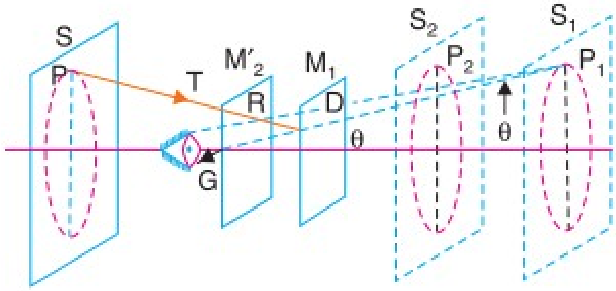


Fig. 15.24

If the two arms of the interferometer are equal in length, image M'_2 coincides with mirror M_1. If M'_2 and M_1 do not coincide, the distance between them is finite, $M'_2 M_1 = d$. Now if a light ray comes from a point S and is reflected by both M'_2 and M_1, the observer will see two virtual images- S_1 due to reflection at M'_2 and S_2 due to reflection at M_1. The virtual images are separated by a distance 2d. If the observer looks into the system at an angle θ, the path difference between the two beams will be 2d cos θ. The light that comes from M_2 and goes to T undergoes rare-to-dense reflection and therefore a π-phase change occurs. In view of this, the total path difference between the two beams is given by

$$\Delta = 2d \cos\theta + \lambda / 2.$$

The condition for obtaining brightness

$$2d \cos\theta + \lambda / 2 = m\lambda$$

where m = 0,1,2, ...

For a given mirror separation d, a given wavelength λ and order m, angle θ is constant. This means that the fringes are of circular shape. They are called fringes of equal inclination.

In case the mirror M_1 coincides with the virtual image M'_2, d = 0. The path difference between the interfering beams will be λ / 2. Consequently, we obtain a minimum at the coincidence position and the centre of the field will be dark, as shown in Fig. 15.25 (a).

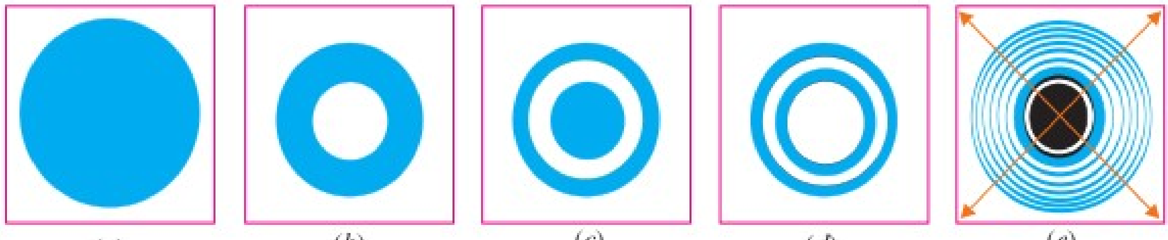

(*a*) (*b*) (*c*) (*d*) (*e*)

Fig. 15.25

If one of the mirrors is now moved through a distance λ/4, the path difference changes by λ/2 and therefore a maximum is obtained. By moving the mirror through another λ/4, a minimum is obtained; moving it by another λ/4 again a maximum is obtained and so on. Therefore, a new ring appears in the centre of the field each time the mirror is moved through λ/2. As d increases new rings appear in the centre faster than rings already present disappear in the periphery; and the field becomes more crowded with thinner rings (Fig. 15.25e). Conversely, as d is made smaller, the rings contract and disappear in the centre.

15.7.5. LOCALIZED FRINGES

When the two mirrors are tilted, they are not exactly perpendicular to each other and therefore the mirror M_1 and the virtual image M'_2 are not parallel. In this case the air path between them is wedge-shaped and the fringes appear to be straight. If one of the mirrors is moved, the fringes move across the field. The position of any particular bright fringe is taken up by the one next to it. The fringes can be counted as they pass a reference mark. If m fringes move across the field of view when M_1 moves through a distance d, then

$$d = m\,\lambda/2$$

or

$$\lambda = \frac{2d}{m} \tag{15.58}$$

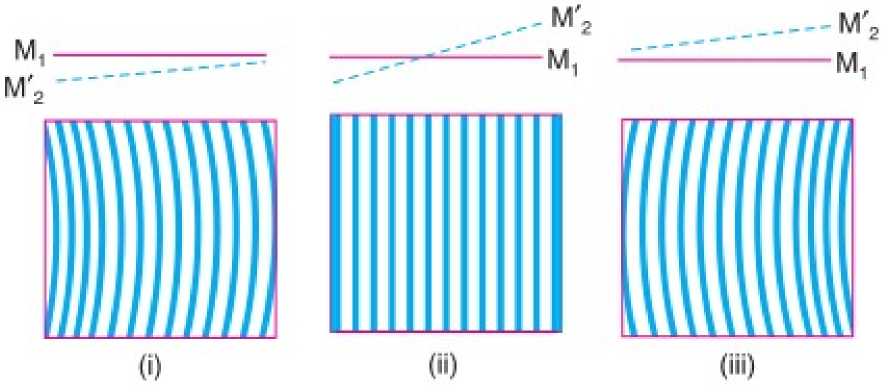
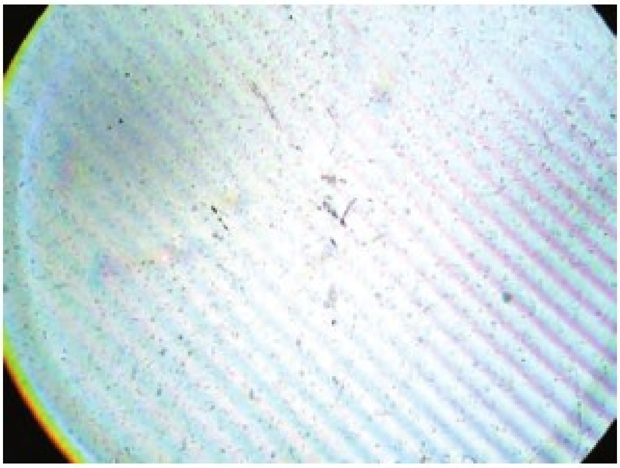


Fig. 15.26

15.7.6. WHITE LIGHT FRINGES

Instead of a monochromatic source, if a white light source is used, a few coloured fringes with a central dark fringe can be observed. In observing these fringes, the mirrors are slightly tilted as for localised fringes and position of M_1 is found where it intersects M'_2. This position is often difficult to find with white light. The position can best be located with monochromatic light when the fringes become straight. Then a very slow motion of M_1 in this region using white light will bring these fringes into view, when a central dark fringe is surrounded by 8 to 10 coloured fringes on either side are observed. These fringes are useful for the determination of zero path difference.



White light fringes.

15.7.7. VISIBILITY OF FRINGES

In case of Michelson interferometer, the intensity is given by

$$I = 4A^2 \cos^2 \frac{\delta}{2}$$

Here $$\delta = \frac{2\pi}{\lambda}(2d \cos \theta)$$

where d is the distance between M_1 and M_2'. The intensity is maximum when δ is an integral multiple of 2π. The intensity is zero when δ is an odd multiple of π. When a monochromatic source of light is used, the minimum intensity of the fringes is zero. The visibility of fringes in the case of a Michelson interferometer is

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

for monochromatic light, $I_{min} = 0$ and therefore, $V = 1$

However, if the source of light is not strictly monochromatic, but contains two nearby wavelengths, the condition for maximum intensity for both the wavelengths is satisfied only for particular values of path difference ($2d \cos \theta$).

As the value of d is altered, the two wavelengths do coincide over a considerable range and here the fringe visibility is a maximum. For values of d other than maximum intensity positions for both wavelengths, the two fringe patterns will be complementary, provided the intensities for both the wavelengths are equal. If intensities are not equal, the minimum visibility will not be zero. The minimum visibility will be

$$V_{min} = \frac{A_1^2 - A_2^2}{A_1^2 + A_2^2}$$

where A_1 and A_2 are the amplitudes. Hence the source will be perfectly monochromatic if visibility is maximum and constant for different values of $2d \cos \theta$. If the visibility changes with the change of $2d \cos \theta$, the source is not strictly monochromatic.

15.8. APPLICATIONS OF MICHELSON INTERFEROMETER

Michelson interferometer can be used to determine (i) the wavelength of a given monochromatic source of light (ii) the difference between the two neighbouring wavelengths or resolution of the spectral lines, (iii) refractive index and thickness of various thin transparent materials and (iv) for measurement of the standard metre in terms of the wavelength of light.

15.8.1. MEASUREMENT OF WAVELENGTH

Michelson interferometer is used to determine the wavelength of light from a monochromatic source. The monochromatic source is kept at S. If the mirrors M_1 and M_2 are exactly perpendicular, circular fringes are obtained. If the mirror M_1 is moved forward or backward, the circular fringes appear or disappear at the centre. Now, as the mirror is moved through a known distance d and the number of fringes disappearing at the centre is counted. Suppose d_1 is the initial thickness of the air film between the mirror M_1 and the image of M_2 corresponding to the bright fringe of order m_1 and d_2 is the final thickness of the air film corresponding to a bright fringe of order m_n in the same position. Then,

$$2d_1 = m_1\lambda$$

and $$2d_2 = m_n\lambda$$

By subtraction, we get $2(d_2 - d_1) = (m_n - m_1)\lambda$

$\therefore \quad 2d = N\lambda$ where $(d_2 - d_1) = d$ and $(m_n - m_1) = N$

$$\therefore \quad \lambda = \frac{2d}{N} \tag{15.59}$$

15.8.2. DETERMINATION OF THE DIFFERENCE IN THE WAVELENGTH OF TWO WAVES

If a source of light consists of two wavelengths λ_1 and λ_2, which differ slightly, then two sets of fringes corresponding to the two wavelengths are produced in a Michelson interferometer. By adjusting the position of the mirror M_1 of the interferometer, the position is found when the fringes are very bright. In this position, the bright fringe due to λ_1 coincides with the bright fringes due to λ_2. When the mirror M_1 is moved, the two sets of fringes get out of step because their wavelengths are different. When the mirror M_1 has been moved through a certain distance, the bright fringe due to one set will coincide with the dark fringe due to the other set and no fringes will be seen in this case. Again by moving the mirror M_1, a position is reached when a bright fringe of one set falls on the bright fringe of the other and the fringes are again distinct. This is possible when the mth order of the longer wavelength coincides with the (m + 1)th order of the shorter wavelength.

Let m_1 and m_2 be the changes in the order at the centre of the field when the mirror M_1 is displaced through a distance d between two consecutive positions of maximum distinctness of the fringes.

$$\therefore \quad 2d = m_1 \lambda_1 = m_2 \lambda_2$$

If λ_1 is greater than λ_2,

$$m_2 = m_1 + 1$$

$$\therefore \quad 2d = m_1 \lambda_1 = (m_1 + 1) \lambda_2$$

$$\therefore \quad m_1 = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$\therefore \quad 2d = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

$$\text{or} \quad \lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2d}$$

Taking λ as the mean wavelength of the two wavelengths λ_1 and λ_2, the small difference $\Delta\lambda$ is given by

$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2d} \tag{15.60}$$

15.8.3. THICKNESS OF A THIN TRANSPARENT SHEET

Let a transparent sheet of thickness t and refractive index μ be inserted in the path of one of the interfering beams of Michelson interferometer. The optical path of that beam increases because of the sheet. It becomes μt instead of t. The increase in the optical path is $(\mu t - t)$ or $(\mu - 1)t$. Since the beam traverses the medium twice, the extra path difference between the two interfering beams is $2(\mu - 1)t$. If m is the number of fringes by which the fringe system is displaced, then

$$2(\mu - 1)t = m\lambda$$

When monochromatic light is used, it is difficult to distinguish the sudden shift of fringes when the thin sheet is inserted. It is also not possible to count the number of fringes shifted. The difficulty is overcome by using white light first to locate the central dark fringe and it is made to coincide with the cross-wire of the telescope. The thin sheet is then introduced into the path of the beam. Position of mirror M_1 is adjusted till again a dark fringe of zero path difference coincides with the cross-wire of the telescope. The distance d through which the mirror is moved is noted. The white light is now replaced with the monochromatic light and the mirror M_1 is moved back slowly and the number of fringes contained in d is found. The thickness t is obtained from the relation

$$t = \frac{m\lambda}{2(\mu - 1)} \qquad (15.61)$$

15.8.4. DETERMINATION OF THE REFRACTIVE INDEX OF GASES

When a tube containing a gas is introduced in the path of the beam going towards M_1, a path difference equal to $2(\mu - 1)\,l$ is introduced between the two interfering beams. Here, μ is the refractive index of the gas and l is the length of the tube. If m fringes cross the centre of the field of view, then $2(\mu - 1)\,l = m\lambda$. Knowing l, m, and λ, μ an be calculated.

In the path of the rays going towards M_1, a tube containing air at atmospheric pressure is introduced and the fringes are obtained in the centre of the field of view. In that case, refractive index of the air at various pressures can be determined. Let the length of the tube be l and let it contain air at atmospheric pressure. The tube is completely evacuated and m fringes cross the centre of the field of view. The path difference introduced between the two interfering beams is $2(\mu - 1)\,l$.

$$\therefore \quad 2(\mu - 1)\,l = m\lambda$$

$$\therefore \quad \mu = \frac{m\lambda}{2l} + 1$$

15.8.5. STANDARDISATION OF THE METRE

The experiment to measure the standard metre in terms of the wavelength of the cadmium red line was first performed by Michelson and Benoit in 1895. It is not possible to count the number of fringes which cross the field of view when one of the mirrors of the Michelson interferometer is moved through whole length of one metre. Moreover, for a path difference of more than 20 cm, it is not possible to obtain the fringes. Therefore, the mirror must not be moved through a distance of more than 10cm. In practice nine **etalons** were used, each being twice the length of the preceding etalon. The length of the shortest etalon used is 0.390625 mm and of the longest was 10 cm. The experiment is divided into two main parts.

(*i*) The number of wavelengths of the monochromatic cadmium light is counted for the shortest etalon.

(*ii*) The length of the second etalon is compared with the shorter etalon and the process is repeated until the number of wavelengths for a length of 10 cm-etalon is known. From this 10 cm-etalon, the number of wavelengths for a length of one metre in terms of the wavelength of cadmium red line is known. This acts as a standard metre because, even if the original standard metre is destroyed, the standard metre can be formed again from the knowledge of the number of wavelengths. The standard metre is represented in terms of the wavelengths of red, green and blue lines of cadmium.

Etalon

An etalon is a substandard for length. It consists of two mirrors, which are plane-parallel and silvered on their front faces. The distance between their surfaces is l (Fig. 15.27). The mirrors can be made perfectly parallel by means of screws attached to them.

Experiment:

(*i*) The Michelson interferometer is used as shown in Fig. 15.28. Light from the source S

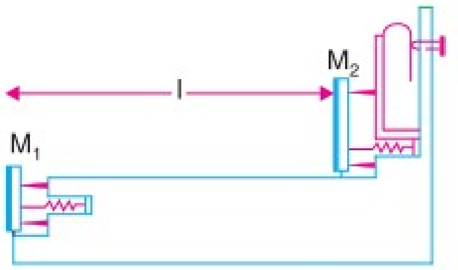


Fig. 15.27

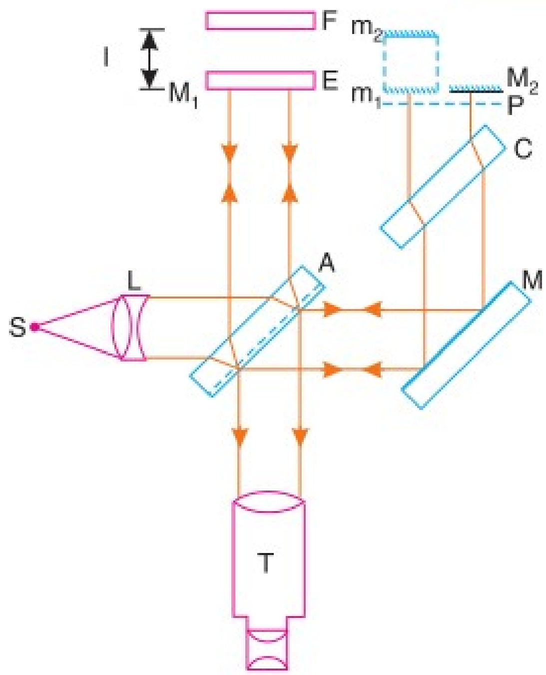


Fig. 15.28

after passing through the lens L is incident on the glass plate A. One portion of the beam is reflected towards M_1 while the other part after reflection from M falls on the mirrors m_1 and m_2 of the shortest etalon and the fixed mirror M_2. The centre of m_2 lies in a horizontal plane parallel and above the plane containing the centres of m_1 and M_2. P is the reference plane and is the image of M_1. The mirrors m_1 and m_2 are adjusted such that their planes are parallel to the reference plane P. Circular fringes are visible in the field of view of both the mirrors m_1 and m_2 when seen through the telescope T having a small aperture. Then the mirror M_1 is adjusted such that the reference plane P makes a small angle with m_1 and m_2. When the reference plane P intersects the plane of m_1 just in the middle, straight-line fringes are obtained with white light, as shown in Fig.15.29 (*a*). The fringes are due to the wedge-shaped film between P and m_1.

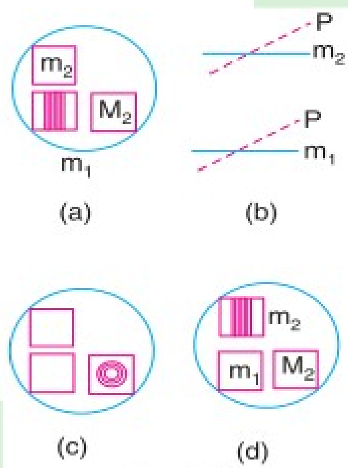


Fig. 15.29

White light is replaced by monochromatic cadmium light and the fixed mirror M_2 is adjusted to be perfectly parallel to M_1 so that circular fringes are visible in the field of view. It should be remembered that with white light, when the mirror M_1 is at E, straight-line fringes are visible in m_1 and with monochromatic cadmium red light circular fringes are visible in M_2 in the field of view of the telescope. The circular fringes are formed at infinity.

The mirror M_1 is moved and the number of circular fringes that cross the field of view are counted. When the plane of reference P intersects the middle of m_2, straight-line fringes will be seen in m_1 with white light (See Fig. 15.29d). White light is used only to note the initial and the final positions of the reference plane P intersecting m_1 and m_2, whereas cadmium light is used to count the number of fringes. Suppose the mirror M_1 has moved from E to F through a distance l and n fringes have crossed the field of view. Then, for a length l of the etalon, the number of fringes crossing the field of view is n and $l = \frac{n\lambda}{2}$.

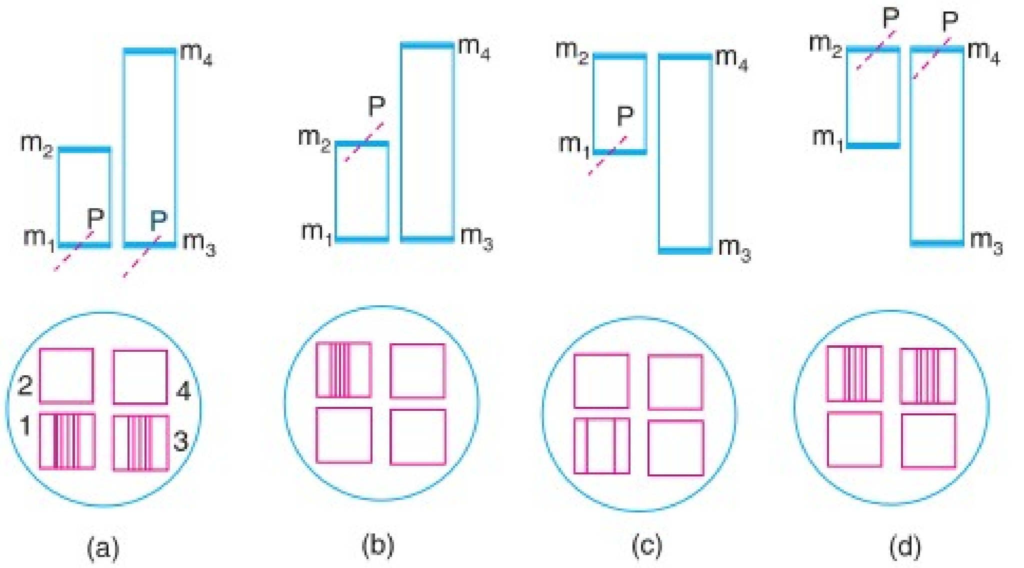


Fig. 15.30

(ii) The next step is to compare the shortest etalon with the next etalon. These two etalons are arranged side by side. First the mirror M_1 is adjusted such that the reference plane P intersects the middle of the mirrors m_1 and m_3. Here m_1 and m_3 are the two lower mirrors of the two etalons whereas m_2 and m_4 are the upper mirrors. With white light straight line fringes are visible in m_1 and m_3, as shown in Fig. 15.30 (a). It shows that m_1 and m_3 are coplanar. After this, the mirror M_1 is moved such that straight-line fringes are visible in the field of view of the upper mirror m_2 and the central dark fringe is in the middle. Keeping M_1 fixed, the etalon m_1m_2 is moved backwards until m_1 intersects P (see Fig. 15.30c). The etalon is adjusted such that the central dark fringe is in the middle of m_1. Therefore, the etalon has moved through a distance equal to its length.

If the second etalon m_3m_4 is exactly twice the length of the etalon m_1m_2, the mirrors m_2 and m_4 should be coplanar. When the mirror M_1 is moved such that the reference plane P just intersects the middle of m_2 and m_4, straight line fringes are produced (see Fig. 15.30d). If the fringes are not visible in m_2 and m_4 simultaneously, the mirrors m_2 and m_4 are not coplanar and the etalon m_3 m_4 is not exactly twice the length of the etalon m_1m_2. The etalon m_3m_4 is compared with etalon m_1m_2. Similarly, the etalon m_3m_4 is compared with the next etalon and the process is repeated. The 10 cm etalon is taken as a substandard and is compared directly with a prototype standard metre.

The final results with cadmium red, green and blue lines are:

a. Cadmium red line, λ_R = 6438.4722 Å and 1 metre = 1,553,163.5 λ_R.
b. Cadmium green line, λ_G = 5085.8240 Å and 1 metre = 1,966,249.7 λ_G.
c. Cadmium blue line, λ_B = 4799.9107 Å and 1 metre = 2,083,372.1 λ_B.

The green spectral line of mercury as emitted by the single isotope Hg (198) is considerably sharper than the cadmium red line. Its wavelength is 5460.7532 Å. It is also used to calibrate the standard metre.

15.9. TWYMAN AND GREEN INTERFEROMETER

Twyman and Green interferometer has been designed for testing the optical homogeneity of prisms, lenses and glass plates. It is also used for testing the rulings of gratings for the absence of ghosts.

Twyman and Green Interferometer

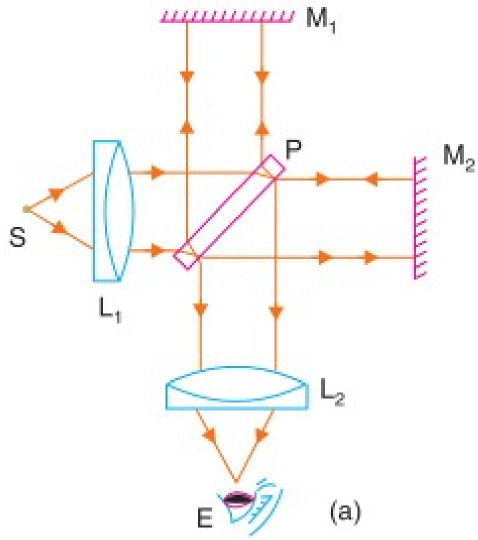
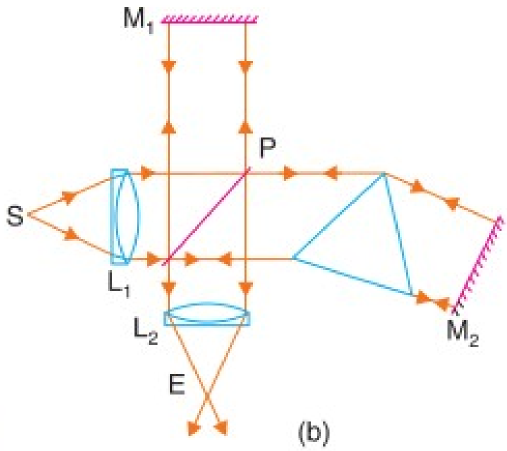


Fig. 15.31

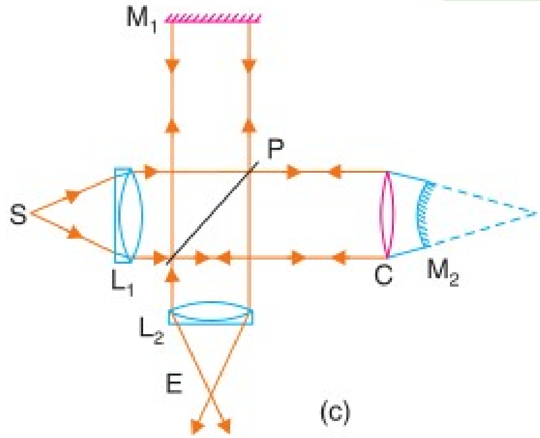
This interferometer resembles the Michelson interferometer in which the extended source is replaced by a monochromatic point source S at the focus of a well-corrected lens L_1. Single plane wavefront emerging from L_1, after partial reflection at the half-silvered plate P set at 45°, gives rise to two plane wavefronts which fall normally on plane mirrors M_1 and M_2 set perpendicular to each other. The plane wavefronts reflected at M_1 and M_2 are superposed at P and focussed by a second well-corrected lens L_2 at E where the observer's eye is situated. When M_1 and M_2 are exactly perpendicular, the superposed plane wavefronts are exactly parallel. Therefore, the phase difference between the superposed disturbances is the same at every point in the field of view. The field is therefore, of uniform intensity depending on the difference between the paths PM_1P and PM_2P. The intensity is a maximum when this path difference is $n\lambda$, where $n = 0,1,2,...$

In order to adjust the instrument, an illuminated pinhole in a screen is placed at S in the focal plane of L_1, and also a screen is placed at E in the focal plane of L_2. The distance PM_1 is made equal to PM_2. Now, two images of the pinhole due to reflections at M_1 and M_2 are received on the screen alongside the pinhole itself. Similarly two images are seen on the screen at E. The mirrors M_1 and M_2 adjusted by tilting screws until the two images at S coincide with pinhole and at the same time the two images at E coincide. On removing the screen and using a monochromatic point source at S, no fringes should be seen. If otherwise, the mirrors are further adjusted until all types of fringes disappear.

Now, suppose after adjusting the interferometer the plane mirror M_2 is replaced by an optically perfect prism plus a plane mirror adjusted as in Fig. 15.31(b) or a perfect lens plus a spherical mirror adjusted as in Fig. 15.31(c). The field of view will remain of uniform intensity. If however the prism or the lens is not optically perfect, the wavefront returning to P will no longer be plane and the phase difference between the superposed disturbances will vary across the field of view and so fringes will be seen. Since fringes are foci of equal phase difference, the observed fringes are contours of the deformed wavefront. Thus, the imperfections of the prism or the lens are found in terms of the wavefront deformation.



Arrangement for testing a prism.



Arrangement for testing a camera lens.

Fig. 15.31

In fact, the fringes are not in focus in any particular plane but if the eye is focussed on a prism face as seen through the lens L_2, the fringes are observed coincident with the prism face. These fringes are marked on the prism face by a pencil. The prism face is then perfected by local figuring and polishing, until the field of view in the interferometer is of uniform intensity.

In case of a lens, the lens under test is mounted on a nodal slide, which automatically maintains the centre of curvature of M_2 at the focus of the lens. If the lens is not perfect, the fringes will be seen which can be marked and corrected as in case of prism.

15.10. MACH-ZEHNDER INTERFEROMETER

This interferometer is used to study slight changes in refractive index of various gases over a considerable region. Its principle is similar to that of Jamin's interferometer. It consists of two beam splitters B_1 and B_2 and two totally reflecting mirrors M_1 and M_2 (Fig. 15.32). Two similar tubes T_1 and T_2 are placed in the two arms of interferometer and are evacuated. While T_1 remains evacuated, gas is admitted slowly into T_2. The number of fringes, that cross through the cetre of the field of view of the telescope, is counted. If the length of the tube is l and N fringes cross the field of view when the refractive index changes from μ_1 to μ_2, then

Mach-Zehnder Interferometer.

$$(\mu_2 l - \mu_1 l) = N\lambda$$

$$(\mu_2 - \mu_1)\ l = N\lambda$$

or
$$\Delta\mu = \frac{N\lambda}{l} \qquad (15.62)$$

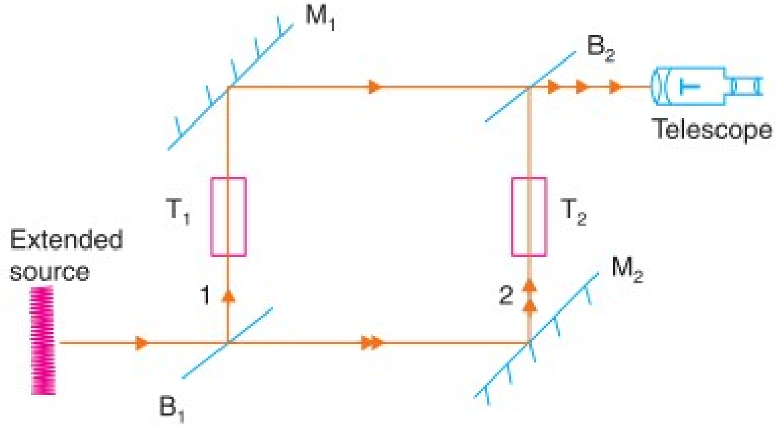


Fig. 15.32

Using the above expression the change in refractive index can be determined. Since the tube can be very long, a small change in refractive index can be measured accurately.

This interferometer is particularly useful in studying the flow pattern in wind tunnels. In this application only one tube is used, which is the test chamber, such as a wind tunnel or shock tube. Since some wind tunnels are several meters in diameter, the interferometer is usually very large. It has been found that the value of (μ – 1) is directly proportional to the air pressure P at a given temperature.

$$(\mu - 1) = \frac{2}{3}P \tag{15.63}$$

The variation in air density in the tunnel (test chamber) therefore causes nonuniform distribution of refractive index. The direct beam and the beam passing through test chamber on combining produce fringe pattern.

15.11. MULTIPLE BEAM INTERFERENCE

We assumed in § 15.1 that the high order reflections occurring at interfaces of thin film are negligible. However, if for any reason the reflectance of the interfaces is not negligible, then the higher order reflections are to be taken into account. When the reflected or transmitted beams meet, multiple beam interference takes place. We are specifically interested in the fringes associated with an air space between two reflecting surfaces. Usually, these surfaces consist of metal films deposited on glass plates.

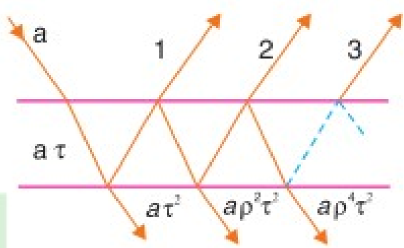


Fig. 15.33

Let us consider the reflected rays 1,2,3, etc as shown in Fig. 15.33. The amplitude of the incident ray is a. Let ρ be the reflection coefficient, τ the transmission coefficient.

The amplitude coefficient of reflection is

$$\rho = \frac{\text{amplitude of the reflected wave}}{\text{amplitude of the incident wave}} \tag{15.64}$$

If the film does not absorb light, the amplitudes of the reflected and transmitted waves are $a\rho$ and $a(1 - \rho)$ respectively.

15.11.1. INTENSITY DISTRIBUTION

Let a be the amplitude of the light incident on the first surface. A certain fraction of this light, $a\rho$, is reflected and another fraction, $a\tau$ is transmitted (Fig. 15 33). The factors ρ and τ are known as the *amplitude reflection coefficient* and *amplitude transmission coefficient* respectively. Again, at the second surface, part of the light is reflected with amplitude $a\rho^2$ and part is transmitted with amplitude $a\tau^2$. The next ray is transmitted with an amplitude $a\rho^2\tau^2$, the next one with after that with $a\rho^4\tau^2$ and so on. If T and R be the fractions of the incident light intensity which are respectively transmitted and reflected at each silvered surface, then, $\tau^2 = T$ and $\rho^2 = R$. Therefore, the amplitudes of the successive rays transmitted through the pair of plates will be

$$a\,T,\ \ a\,T\,R,\ \ a\,T\,R^2, ...$$

In complex notation, the incident amplitude is given by $E = ae^{i\,\omega t}$.

Then the waves reaching a point on the screen will be

$$E_1 = a\,T\,e^{i\omega t}$$

$$E_2 = a\,T\,R\,e^{i(\omega t-\delta)}$$

$$E_3 = a\,T\,R^2\,e^{i(\omega t-2\delta)}, \text{ and so on.}$$

$$\therefore \qquad E_N = aR^{(N-1)}\,T\,e^{i\left[\omega t-(N-1)\delta\right]}$$

By the principle of superposition, the resultant amplitude is given by

$$A = a\,T + a\,T\,R\,e^{-i\delta} + a\,T\,R^2 e^{-2i\delta} + a\,T\,R^3 e^{-3i\delta} + ...$$

$$= a\,T\left[1 + Re^{-i\delta} + R^2 e^{-2i\delta} + R^3 e^{-3i\delta} + ...\right]$$

Using the expression for sum of the terms of a geometrical progression, we get

$$A = a\frac{1 - R^N e^{-iN\delta}}{1 - \text{Re}^{-i\delta}}$$

When the number of terms in the above expression approaches infinity, the term $R^N e^{-iN\delta}$ tends to zero, and the transmitted amplitude reduces to

$$A = a\,T\left[\frac{1}{1 - Re^{-i\delta}}\right] \qquad (15.65)$$

The complex conjugate of A is given by

$$A^* = a\,T\left[\frac{1}{1 - Re^{+i\delta}}\right] \qquad (15.66)$$

The transmitted energy $I_T = AA^*$

$$= \frac{a^2T^2}{\left(1 - Re^{-i\delta}\right)\left(1 - Re^{+i\delta}\right)} = \frac{a^2T^2}{1 + R^2 - R\left(e^{i\delta} + e^{-i\delta}\right)}$$

$$= \frac{a^2T^2}{1 + R^2 - 2R\cos\delta} = \frac{a^2T^2}{(1-R)^2 + 2R(1-\cos\delta)}$$

$$= \frac{a^2T^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}} = \frac{a^2T^2}{(1-R)^2}\left[\frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}\right] \qquad (15.67)$$

The intensity will be maximum when $\sin^2\frac{\delta}{2} = 0$, *i.e.*, $\delta = 2m\pi$, where m = 0,1,2,3,—Thus,

$$I_{max} = \frac{a^2T^2}{(1-R)^2} \tag{15.68}$$

The intensity will be a minimum, when $\sin^2\frac{\delta}{2} = 1$ .i.e., $\delta = (2m+1)\pi$ where m = 0,1,2,3…Thus,

$$I_{min} = \frac{a^2T^2}{(1-R)^2} \cdot \frac{1}{1+\frac{4R}{(1-R)^2}} = \frac{a^2T^2}{(1+R)^2} \tag{15.69}$$

We can now rewrite the equ. (15.67) as

$$I_T = \frac{I_{max}}{1+\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}} \tag{15.70}$$

Similarly, the interference intensity from the reflected light beams can be shown to be

$$I_R = \frac{4R\sin^2\left(\frac{\delta}{2}\right)I_{max}}{(1-R^2)+4R\sin^2\left(\frac{\delta}{2}\right)} \tag{15.71}$$

15.11.2. COEFFICIENT OF FINESSE

We now introduce a quantity F, which is called the *coefficient of finesse*. It is defined as

$$F = \frac{4R}{(1-R)^2} \tag{15.72}$$

Then the relative inteference intensity distribution can be expressed as

$$\frac{I_T}{I_{max}} = \frac{1}{1+F\ \sin^2\left(\frac{\delta}{2}\right)} \tag{15.73}$$

and

$$\frac{I_R}{I_{max}} = \frac{F\sin^2\left(\frac{\delta}{2}\right)}{1+F\ \sin^2\left(\frac{\delta}{2}\right)} \tag{15.74}$$

15.11.3. VISIBILITY OF FRINGES

The visibility of fringes is given by $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Substituting the values of I_{max} and I_{min}, we get $V = \frac{2R}{1+R^2}$ (15.75)

Equ. (15.75) shows that the visibility of fringes is a function of reflectivity only. The visibility of fringes increases with increase in the value of R. V reaches the value 0.8 when R = 0.5 and approaches unity as R approaches 1. Thus the higher the reflectivity, the greater is the contrast of the fringes.

15.11.4. SHARPNESS OF THE FRINGES

If a plot is drawn for I against δ at different values of R, we obtain a set of curves, as shown in Fig. 15.34.

It is noted from the graphs that the intensity falls off on both sides of the maximum at higher values of R. The sharpness of a fringe is measured by the half-width of the curve. The **half-width** is the width of the I-δ curve at the position where $I = \frac{1}{2} I_{max}$. We have that

$$I = \frac{I_{max}}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

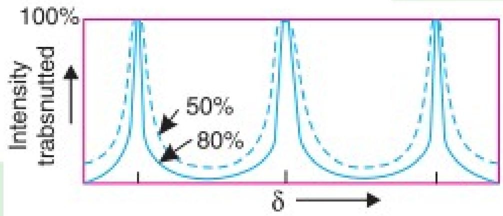


Fig. 15.34

$$\therefore \qquad \frac{1}{2} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

$$\text{or} \qquad 1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2} = 2$$

$$\text{or} \qquad \sin^2\frac{\delta}{2} = \frac{(1-R)^2}{4R}$$

$$\therefore \qquad \delta = 2\sin^{-1}\left(\frac{1-R}{2\sqrt{R}}\right) \tag{15.76}$$

15.12. FABRY-PEROT INTERFEROMETER AND ETALON

The Fabry-Perot interferometer is a high resolving power instrument, which makes use of the 'fringes of equal inclination', produced by the transmitted light after multiple reflections in an air film between two parallel highly reflecting glass plates.

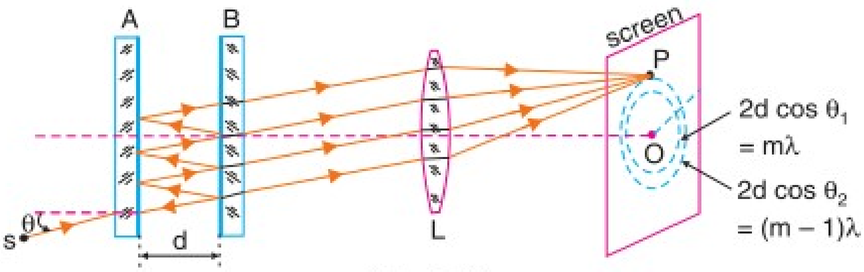


Fig. 15.35

The interferometer consists of two optically plane glass plates A and B with their inner surfaces silvered, and placed accurately parallel to each other. Screws are provided to secure parallelism if disturbed. This system is difficult to manufacture and is no more in use. Instead an etalon which is much more easily manufactured is used. The etalon consists of two semi-silvered plates rigidly held parallel at a fixed distance apart. The reflectance of the two surfaces can be as high as 90 to 99.9%. Although both reflected and transmitted beams interfere

Fabry-Perot Interferometer.

with each other, the Fabry-Perot interferometer is usually used in the transmissive mode.

S is a broad source of monochromatic light and L_1 a convex lens (not shown in Fig. 15.35) which makes the rays parallel. An incident ray suffers a large number of internal reflections successively at the two silvered surfaces, as shown in Fig.15.35. At each reflection a small fraction of the light is transmitted also. Thus, each incident ray produces a group of coherent and parallel, transmitted rays with a constant path difference between any two successive rays. A second convex lens L brings these rays together to a point in its focal plane where they interfere. Hence the rays from all points of the source produce an interference pattern on a screen placed in the focal plane of L.

15.12.1. FORMATION OF FRINGES

Let d be the separation between the two silvered surfaces and θ the inclination of a particular ray with the normal to the plates. The path difference between any two successive transmitted rays corresponding to the incident ray is $2d \cos \theta$. The condition for these rays to produce maximum intensity is given by

$$2d \cos \theta = m \lambda$$

where m is an integer. The locus of points in the source, which give rays of constant inclination, θ is a circle. Hence, with an extended source, the interference pattern consists of a system of bright concentric rings on a dark background, each ring corresponding to a particular value of θ.

15.12.2. DETERMINATION OF WAVELENGTH

When the reflecting surfaces A and B of the interferometer are adjusted exactly parallel, circular fringes are obtained. Let m be the order of the bright fringe at the centre of the fringe system. As at the centre $\theta = 0$, we have

$$2t = m\lambda$$

If the movable plate is moved a distance $\lambda/2$, 2t changes by λ and hence a bright fringe of the next order appears at the centre. If the movable plate is moved from the position x_1 to x_2 and the number of fringes appearing at the centre during this movement is N, then

$$N \cdot \frac{\lambda}{2} = x_2 - x_1$$

or

$$\lambda = \frac{2(x_2 - x_1)}{N} \tag{15.77}$$

Measuring x_2, x_1 and N, we can determine the value of λ.

15.12.3. MEASUREMENT OF DIFFERENCE IN WAVELENGTH

The light emitted by a source may consist of two or more wavelengths, as D_1 and D_2 lines in case of sodium. Separate fringe patterns corresponding to the two wavelengths are not produced in Michelson interferometer. Hence, Michelson interferometer is not suitable to study the *fine structure* of spectral lines. On the other hand, in Fabry-Perot interferometer, each wavelength produces its own ring pattern and the patterns are separated from each other. Therefore, Fabry-Perot interferometer is suitable to study the fine structure of spectral lines.

Difference in wavelengths can be found using coincidence method. Let λ_1 and λ_2 be two very close wavelengths in the incident light. Let us assume that $\lambda_1 > \lambda_2$. Initially, the two plates of the interferometer are brought into contact. Then the rings due to λ_1 and λ_2 coincide partially. Then the movable plate is slowly moved away such that the ring systems separate and maximum discordance occurs. Then the rings due to λ_2 are half way between those due to λ_1. Let t_1 be the separation between the plates when maximum discordance occurs. At the centre

$$2t_1 = m_1\lambda_1 = \left(m_1 + \tfrac{1}{2}\lambda_2\right) \tag{15.78}$$

or $$m_1(\lambda_1 - \lambda_2) = \lambda_2/2$$

$\therefore$ $$m_1 = \frac{\lambda_2}{2(\lambda_1 - \lambda_2)} \tag{15.79}$$

Using the value of m_1 in equ.(15.78), we get $2t_1 = \frac{\lambda_2}{2(\lambda_1 - \lambda_2)}\lambda_1$

$\therefore$ $$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{4t_1} \cong \frac{\lambda_{mean}^2}{4t_1} \tag{15.80}$$

(since $\lambda_1\lambda_2 = \lambda_{mean}^2$ as $\lambda_1 - \lambda_2$ is very small).

When the separation between the plates is further increased, the ring systems coincide again and then separate out and maximum discordance occurs once again. If t_2 is the thickness now,

$$2t_2 = m_2\lambda_1 = \left(m_2 + \tfrac{3}{2}\lambda_2\right) \tag{15.81}$$

From equ.(15.81) and (15.78), we get

$$2(t_2 - t_1) = (m_2 - m_1)\lambda_1 = (m_2 - m_1)\lambda_2 + \lambda_2 \tag{15.82}$$

or $$(m_2 - m_1) = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

Using the above expression into equ.(15.82), we obtain

$$2(t_2 - t_1) = \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)}$$

or $$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2(t_2 - t_1)} \cong \frac{\lambda_{mean}^2}{2(t_2 - t_1)} \tag{15.83}$$

15.13. LUMMER AND GEHRCKE PLATE

It consists of a plane parallel glass plate of about 10 cm long and a few mm thick. A prism C is cemented at one end. A beam of parallel light enters the prism C and after reflection falls on the plate (Fig. 15.36). The angle of incidence at which the ray strikes the face A of the plate is slightly less than the critical angle for the material of the plate. The beam is reflected up and down between the faces A and B of the plate. At each reflection, a beam leaves the plate. The path difference is the same between the beams. They are received by the telescope and interference fringes are viewed in the field of view. It is used for observing the fine structure of spectral lines and Zeeman effect.

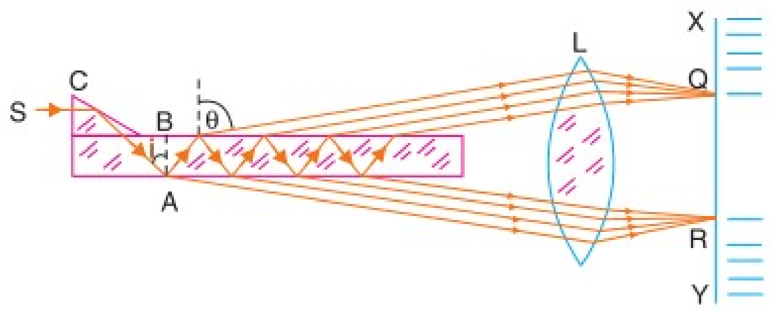


Fig. 15.36

15.14. APPLICATIONS OF THIN FILM INTERFERENCE

The application of interference phenomenon is wide and varied. Interference is used for making precision measurements. For example, the wavelength of light can be measured with accuracy up to eight significant digits. Therefore, interferometers are used to determine and redefine the length of a standard metre. Standard metre was defined formerly as the distance between two marks on a platinum-iridium bar. According to the modern definition the standard metre is a length which contains exactly 1,650,763.73 wavelengths of orange-red light emitted by krypton-86. Another interesting application is in astronomy where double slit interference is used to determine the angular separation of double stars and the diameter of fixed stars. We discuss here a few selected applications.

15.14.1. MEASUREMENT OF SMALL DISPLACEMENTS

The interference phenomenon is used to determine small displacements such as those produced by compression or elongation of a metal rod, and thermal expansion of crystals etc. The order of crystal expansion is quite small and it can be conveniently measured by the interference methods. We study here how Fizeau adopted Newton's rings to study the thermal expansion of a crystal.

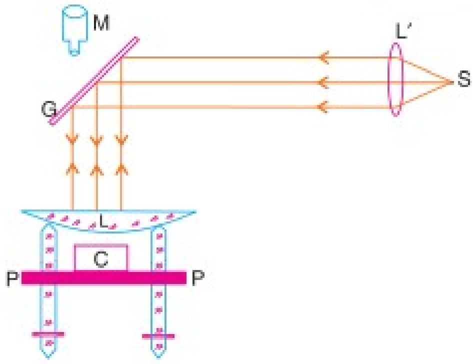


Fig. 15.37

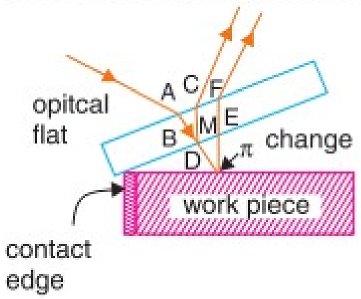
The crystal under test is placed on a metal disc supported by three screws, and a lens is kept on the ends of the screws, as shown in Fig. 15.37. A thin film of air is left between the crystal and the lens. The position of the lens can be adjusted with the help of the screws. When monochromatic light is made incident on the lens crystal combination, Newton's rings are produced which can be examined by means of a travelling microscope. When the crystal is heated, the thickness of the air film between the lens decreases due to thermal expansion of the crystal. As a result, Newton's rings undergo expansion. The expanding rings are counted with the help of the microscope. The distance x between the lens and the plate at any instant is given by

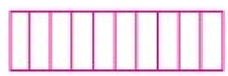
$$2\mu\left(x+\frac{r_m^2}{2R}\right)=m\lambda \tag{15.84}$$

where r_m is the radius of the m^{th} dark ring seen by reflected light, R the radius of curvature of the lower surface of the lens and μ is the refractive index of air between the lens and the plate.

15.14.2. TESTING OF SURFACE FINISH

In modern technology interference is widely used for estimating the quality of a surface finish. Machine components retain surface irregularities left after machining. The extent of suitability of the component for a particular application depends on the irregularities which act as sources of stress leading to fatigue cracks. The surfaces of components, which are going to be subjected to high stresses and load reversals, are therefore required to have a high surface finish. The smoothness of the surface may be quickly inspected visually by keeping an optical flat on the component at an angle and illuminating it with monochromatic light. The air wedge formed therein produces straight and equidistant bands if the surface of the component is smooth. If the bands are curved towards the contact edge the surface is concave and if the fringes curve away, it is convex, as shown in Fig.15.38.




(a) Flat

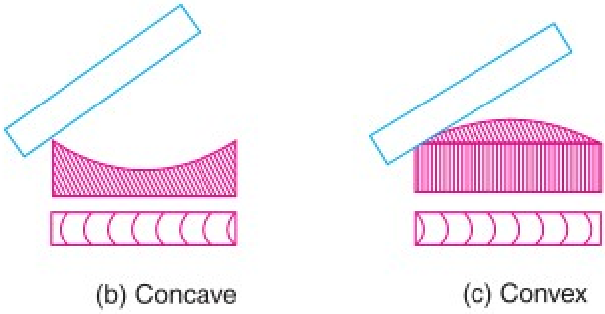


Fig. 15.38

15.14.3. TESTING OF A LENS SURFACE

One of the important uses of Newton's rings is in the testing of the optical components manufactured for use in telescopes and other instruments. The grinding of a lens surface is tested by keeping it on a master. A master is an optical flat which is a cylindrical disc made of fused quartz. The two faces of the optical flat are perfectly parallel to each other. The departure from the flatness of each face is less than a light wavelength. If a lens is ground perfectly, a circular fringe pattern is observed. Otherwise variations are observed (Fig. 15.39*b*)which give an indication of how the lens must be ground and polished to remove the imperfections. High quality lenses are ground with a precision of less than a light wavelength.

Fig. 15.39

15.14.4. THICKNESS OF A THIN FILM COATING

Dielectric and metallic thin films are often coated on optical components, solar cells etc. One of the methods of determination of thickness of such thin films is based on multiple beam interference. A partially coated substrate is used for the determination. The surfaces of the substrate and the thin film on it are coated with a transparent metallic film of uniform thickness. A glass plate is also coated on one of its surfaces with the transparent metallic film. When the substrate and the glass plate are placed in contact and examined under monochromatic light, the reflected light shows a fringe system, as shown in Fig.15.40. A shift occurs in the fringes as we pass from the region occupied by thin film to the region where thin film is absent. The amount of displacement of one set of the fringes with respect to the second set of fringes is given by

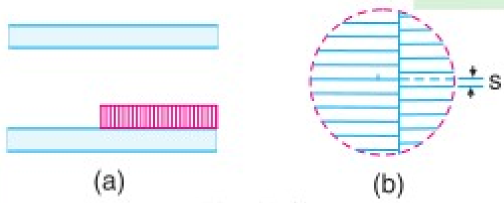


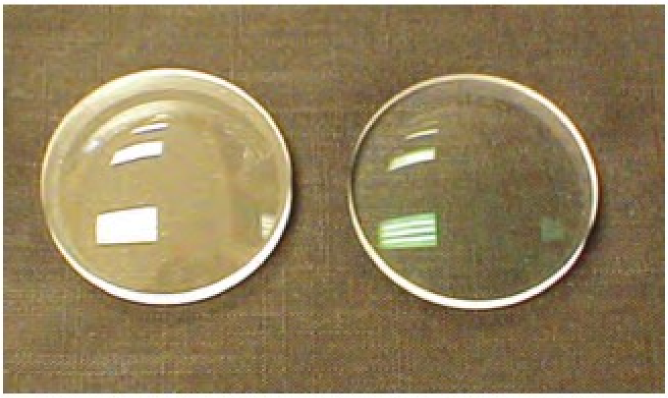
Fig. 15.40

$$s = 2t \qquad \text{or} \qquad t = s/2$$

where t is the thickness of the thin film. By measuring 's', t can be calculated.

15.15. ANTIREFLECTION COATINGS

One of the most important applications of thin film interference is in producing antireflection coatings. Optical instruments such as cameras and telescopes use multi-component glass lenses. It is noted that part of the light incident on a glass surface is reflected and that much amount is subtracted from the transmitted light. When the number of reflections are large, the quality of the image produced by the device will be poor. Alexander Smakula discovered in 1935 that the reflections from a surface can be reduced by coating the surface with a thin transparent dielectric film.



Left lens has no antireflection coatings but right lens has antireflection coatings.

A transparent thin film coated on a surface with a view to suppress the surface reflections is called an antireflection (AR) coating or a nonreflecting film.

A thin film can act as an AR coating if it meets the following two conditions:

(*i*) **Phase condition:** The waves reflected from the top and bottom surfaces of the thin film are in *opposite phase* such that their overlapping leads to destructive interference, and

(*ii*) **Amplitude condition:** The waves have *equal* amplitudes.

The above conditions enable us determine respectively (a) the required thickness of the film and (b) the refractive index of the material to be used for forming the film.

(*i*) **Phase condition and minimum thickness of the film:** Let the thickness of the film be t and the refractive index of the film-material be μ_f .The phase condition requires that the waves (ray 1 and ray 2) reflected from the top and bottom surfaces of thin film be 180° out of phase. It requires that the optical path difference between the two rays must equal one half-wave or an odd number of half-waves. Referring to Fig.15.41, the optical path difference between ray 1 and ray 2 is

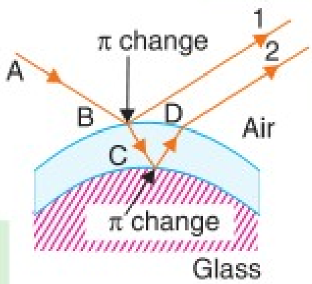


Fig. 15.41

$$\Delta = 2\mu_f t \cos r - \lambda/2 - \lambda/2$$

The first $\lambda/2$ corresponds to the π change at the top surface of the film (air-to-film boundary) and the second $\lambda/2$ to the π change that occurs at the film-to glass boundary. If we assume normal incidence of light, $\cos r = 1$ and the above equation reduces to

$$\Delta = 2\mu_f t - \lambda = 2\mu_f t$$

We wrote the above equality remembering that an *addition of a full wave or subtraction of a full wave from a train of waves does not affect the original phase relation.* The ray 1and ray 2 interfere destructively if the optical path difference satisfies the condition that

$$\Delta = (2m+1)\lambda/2$$

Thus, it requires that $2\mu_f t = (2m+1)\lambda/2$

For the film to be transparent, its thickness should be a minimum, which happens when m = 0.

$$2\mu_f t_{\min} = \lambda/2$$

$$t_{\min} = \frac{\lambda}{4\mu_f} \tag{15.85}$$

It means that the optical thickness of the AR coating should be of one-quarter wavelength. Such quarter-wavelength coatings suppress the reflections and cause the light to pass into the transmitted component.

(*ii*) **Amplitude condition :** The amplitude condition requires that the amplitudes of reflected rays, ray 1 and ray 2 are equal. That is,

$$E_1 = E_2 \tag{15.86}$$

It requires that

$$\left[\frac{\mu_f - \mu_a}{\mu_f + \mu_a}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2 \tag{15.87}$$

where μ_a,μ_f , and μ_g are the refractive indices of air, thin film and glass substrate respectively. As $\mu_a = 1$, the above expression may be rewritten as

$$\left[\frac{\mu_f - 1}{\mu_f + 1}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2$$

Expanding the above equation, we get

$$\frac{\mu_f^2 - 2\mu_f + 1}{\mu_f^2 + 2\mu_f + 1} = \frac{\mu_g^2 - 2\mu_g\mu_f + \mu_f^2}{\mu_g^2 + 2\mu_g\mu_f + \mu_f^2}$$

$$4\mu_f^3\mu_g + 4\mu_f\mu_g = 4\mu_f^3 + 4\mu_f\mu_g^2$$

Dividing by 4 μ_f and rearranging the terms

$$\mu_f^2 - \mu_g\mu_f^2 + \mu_g^2 - \mu_g = 0$$

$$\mu_f^2 = \mu_g\left(1 + \mu_f^2 - \mu_g\right)$$

$\therefore$ $$\mu_f^2 \equiv \mu_g$$

$\therefore$ $$\mu_f = \sqrt{\mu_g} \tag{15.88}$$

It implies that the refractive index of thin film should be less than that of the substrate and possibly nearer to its square root.

In case of glass, if we take $\mu_g = 1.5,\ \mu_f = \sqrt{\mu_g} = 1.22$.

The materials which have refractive index nearer to this value are magnesium fluoride, MgF_2 (μ = 1.38) and cryolite, $3NaF.AlF_3$ (μ =1.36). Apart from the refractive index, the material should possess some more additional properties. The film should adhere well, should be durable, scratch proof and insoluble in ordinary solvents. MgF_2 and cryolite satisfy these requirements. However, among the two, magnesium fluoride is cheaper and is hence widely used as AR coating.

It may be noted that the condition (15.85) is satisfied only at one particular wavelength. The wavelength normally chosen is 5500 Å for which the eye is most sensitive. This wavelength is located in the yellow-green portion of the spectrum. Consequently, the reflection of red and violet light will be larger when white light is incident on the component. Hence, the component shows *purple hue* in reflected light.

15.15.1. MULTILAYER AR COATINGS

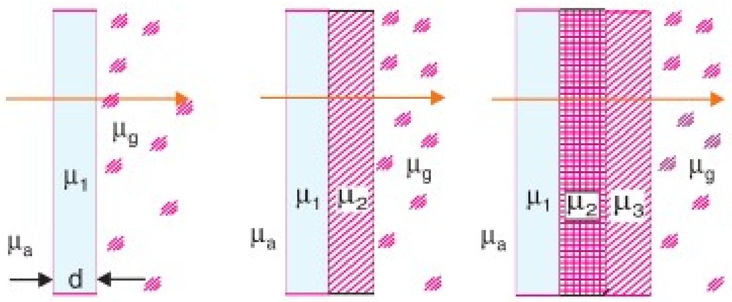
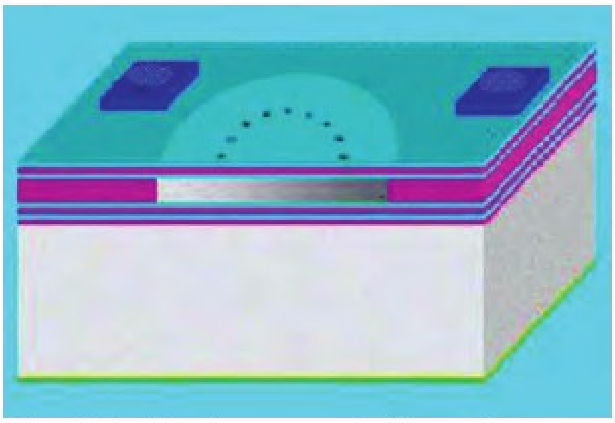


Fig. 15.42

A single layer AR coating is effective only at one particular wavelength. A much wider coverage across the spectrum is possible with multiple coatings, called *multilayers*. In practice three layer coatings are widely used and are highly effective over most of the visible spectrum. The central layer is half-wave(λ / 2) thick and is of high refractive index materials such as zirconium dioxide (ZrO_2, μ =2.1). The outside layer is of magnesium fluoride having λ / 4 thickness and the layer adjacent to the substrate is again a λ /4 thick coating of cesium fluoride (CeF_3, μ =1.63) or aluminium oxide (Al_2O_3, μ=1.76). Some of the antireflection coatings use up to 100 layers of alternating high and low refractive index materials.

15.16. DIELECTRIC MIRRORS

Another important application of the thin film interference phenomenon is in *increasing* the reflectivity of a substrate. If the refractive index of the thin film coated on glass is higher than that of



Two dielectric mirrors separated by an air gap.

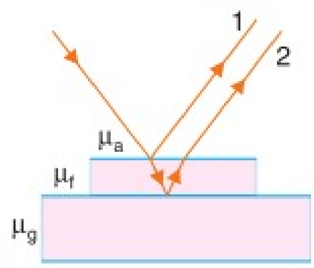


Fig. 15.43

glass ($\mu_f > \mu_g$), then the reflectivity of glass surface increases. Referring to Fig. 15.43, the optical path between rays 1 and 2 is given by

$$\Delta = 2\mu_f t - \lambda / 2$$

where normal incidence of light is assumed. Ray 1 and 2 should constructively interfere if the reflection is to be more from the surface. Therefore, the condition that $\Delta = m\lambda$ has to be satisfied. Thus, the condition for more reflection is

$$2\mu_f t - \lambda/2 = m\lambda$$

or $$2\mu_f t = (2m+1)\lambda/2$$

$m = 0$ gives the minimum thickness of the coating.

$$\therefore \qquad t_{min} = \frac{\lambda}{4\mu_f} \qquad (15.89)$$

Thus, the optical thickness of high reflectivity film is again $\lambda/4$, provided $\mu_f > \mu_g$. Thus, on a glass plate, a $\lambda/4$ thick film of a dielectric material whose refractive index is more than that of glass is deposited as a result of which the surface reflectivity is enhanced. The materials generally used are titanium oxide ($\mu = 2.8$) or zinc sulphide ($\mu = 2.3$).

15.17. INTERFERENCE FILTERS

An interference filter is an optical system that will transmit a very narrow range of wavelengths and thus provides a monochromatic beam of light.

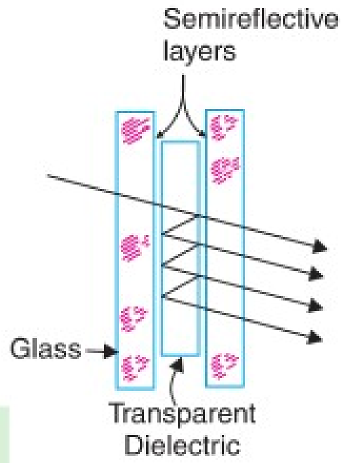


Fig. 15.44

Interference filters are fabricated earlier as follows. A thin metallic film, usually of aluminium or silver, is deposited on a glass substrate by vacuum deposition technique. Then a thin layer of cryolite is deposited over this. The structure is again covered by another metallic film. Another plate is placed over it to protect the thin film structure. The filter is shown in Fig. 15.44. By varying the thickness of the dielectric film, any particular wavelength can be filtered out. However, the filtered light will have a narrow spectrum centered on the chosen wavelength. By increasing the reflectivity of the surfaces, the transmitted spectrum can be made narrower. But it is not possible to increase the thickness of metallic films indefinitely, as they start absorbing the light.

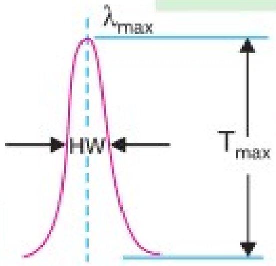


Fig. 15.45

In modern versions metallic films are not used; instead dielectric films are used. In an all dielectric interference filter, layers of dielectric materials of appropriate refractive indices are deposited. To obtain an interference filter, a $\lambda/4$ thick film of titanium oxide is deposited and then over it a film of dielectric material with lower refractive index, such as magnesium fluoride is deposited. On this, again a $\lambda/4$ thick film of titanium oxide is deposited. In this way alternately high and low refractive index materials are deposited to obtain an interference filter. With multiple coatings, it is possible to fabricate filters, which are capable of transmitting a very narrow spectrum of a width as small as 11Å or even less, about a chosen wavelength in the visible region (Fig. 15.45). Modern filters use up to 100 layers.

WORKED OUT PROBLEMS

Example 15.1: A soap film 5×10^{-5} cm thick is viewed at an angle of 35° to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light ($\mu = 1.33$).

Solution: Let i be the angle of incidence and r be the angle of refraction. Then

$$\mu = \frac{\sin i}{\sin r} \quad \therefore \quad 1.33 = \frac{\sin 35^\circ}{\sin r} \qquad \therefore r = 25.5^\circ \text{ and } \cos r = 0.90$$

The condition for destructive interference is $2\mu\, t\, \cos\, r = m\lambda$.

Using different values for m in the above relation, we get following values for wavelengths.

When m = 1, $\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-5}$ cm $\times 0.90 = 12.0 \times 10^{-5}$ cm =**120μm.**

When m = 2, $\lambda_2 = (2 \times 1.33 \times 5 \times 10^{-5}$ cm $\times 0.90) \div 2 = 6.0 \times 10^{-5}$ cm = **6000 Å.**

When m = 3, $\lambda_3 = (2 \times 1.33 \times 5 \times 10^{-5}$ cm $\times 0.90) \div 3 = 4.0 \times 10^{-5}$ cm = **4000 Å.**

When m = 4, $\lambda_4 = (2 \times 1.33 \times 5 \times 10^{-5}$ cm $\times 0.90) \div 4 = 3.0 \times 10^{-5}$ cm = **3000 Å.**

Out of the above wavelengths, λ_2 = 6000 Å, and λ_3 = 4000 Å lie in the visible region. Therefore, these two wavelengths are absent in the reflected light.

Example 15.2: A glass wedge of angle 0.01 radian is illuminated by monochromatic light of wavelength 6000 Å falling normally on it. At what distance from the edge of the wedge will the 10^{th} fringe be observed by reflected light?

Solution: Given that θ = 0.01 rad, m = 10, $\lambda = 6000 \times 10^{-8}$ cm.

The condition for dark fringe is $2\,t = m\,\lambda$

The angle of the wedge $\theta = \frac{t}{x}$ or $t = \theta x$

$\therefore \quad 2\,\theta\, x = m\,\lambda$

or $$x = \frac{m\lambda}{2\theta} = \frac{10 \times 6000 \times 10^{-8}\,\text{cm}}{2 \times 0.01} = \textbf{3 mm}$$

Example 15.3: A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge with the wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the number of dark fringes per cm of the wedge length.

Solution: Given wedge angle $\theta = 20'' = \frac{20 \times \pi}{60 \times 60 \times 180}$ radians, $\lambda = 5.82 \times 10^{-7}$ m, μ = 1.5.

$$\text{Fringe width } \beta = \frac{\lambda}{2\mu\theta} = \frac{5.82 \times 10^{-7}\,\text{m} \times 60 \times 60 \times 180}{2 \times 1.5 \times 20 \times \pi} = 2 \text{ mm.}$$

Number of fringes per cm $= \frac{1}{0.2\,\text{cm}}$ = **5 per cm.**

Example 15.4: A thin equiconvex lens of focal length 4 m and refractive index 1.50 rests on and in contact with an optical flat, and using light of wavelength 5460Å, Newton's rings are viewed normally by reflection. What is the diameter of the 5^{th} bright ring?

Solution: Given that m =5, λ = 5460Å = 5460×10^{-10} m, f = 4m, μ =1.5

We know that $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$. Here R_1 = R and R_2 = – R

$$\therefore \quad \frac{1}{f} = (\mu - 1)\left[\frac{2}{R}\right]$$

$$\therefore \quad \frac{1}{4\text{m}} = 0.5\left[\frac{2}{R}\right] \text{ or } R = 4 \text{ m.}$$

The diameter of the m^{th} bright ring is given by

$$D_m = \sqrt{2(2m-1)\lambda R} = \sqrt{2(2 \times 5 - 1) \times 5460 \times 10^{-10}\,\text{m} \times 4\text{m}} = \textbf{6.2 mm}$$

Example 15.5: Newton's rings are observed in reflected light of $\lambda = 5.9 \times 10^{-5}$ cm. The diameter of the 10^{th} dark ring is 0.5 cm. Find the radius of curvature of the lens and the thickness of the air film.

Solution: Given that $\lambda = 5.9 \times 10^{-5}$ cm, m = 10.

The radius of m^{th} dark ring is given by

$$\therefore \quad R = \frac{r^2}{m\lambda} = \frac{(0.5\,\text{cm})^2}{10 \times 5.9 \times 10^{-5}\,\text{cm}} = 106 \text{ cm} = \mathbf{1.06m}$$

The thickness of air film is given by

$$\therefore \quad t = \frac{m\lambda}{2} = \frac{10 \times 5.9 \times 10^{-5}\,\text{cm}}{2} = \mathbf{2.95\mu m}.$$

Example 15.6: In a Newton's rings experiment, the diameter of 10^{th} dark ring due to wavelength 6000 Å in air is 0.5 cm. Find the radius of curvature of the lens.

Solution: Radius of curvature, $R = \frac{(D/2)^2}{m\lambda} = \frac{(0.5 \times 10^{-2}/2)^2\,\text{m}^2}{10 \times 6000 \times 10^{-10}\,\text{m}} = \mathbf{1.04m}$

Example 15.7: In a Newton's rings experiment the diameter of the 15^{th} ring was found to be 0.59 cm and that of the 5^{th} ring was 0.336 cm. If the radius of the plano-convex lens is 100 cm, calculate the wavelength of light used.

Solution: $\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR} = \frac{D_{15}^2 - D_5^2}{4 \times 10 \times R} = \frac{(5.9 - 3.36)^2 \times 10^{-6}\,\text{m}^2}{4 \times 10 \times 1\,\text{m}} = \mathbf{5880\ Å}.$

Example 15.8: In a Newton's rings experiment the diameter of 10^{th} ring changes from 1.40 to 1.27 cm when a drop of liquid is introduced between the lens and the glass plate. Calculate the refractive index of the liquid.

Solution: $\mu = \frac{(D_m^2)_{air}}{(D_m^2)_{liq.}} = \frac{(1.40\,\text{cm})^2}{(1.27\,\text{cm})^2} = \mathbf{1.215}$

Example 15.9: In a Michelson interferometer 200 fringes cross the field of view when the movable mirror is moved through 0.0589 mm. Calculate the wavelength of light used.

Solution: $\lambda = \frac{2d}{m} = \frac{2 \times 0.0589 \times 10^{-3}\,\text{m}}{200} = \mathbf{5890\ Å}$

Example 15.10: In an experiment for determining the refractive index of gas using Michelson interferometer a shift of 140 fringes is observed, when all the gas is removed from the tube. If the wavelength of light used is 5460 Å and the length of the tube is 20 cm, calculate the refractive index of the gas.

Solution: Given that $n = 140$, $\lambda = 5460$ Å $= 5460 \times 10^{-10}$m, $l = 20$ cm = 0.2 m.

$$\mu = 1 + \left(\frac{n\lambda}{2l}\right) = 1 + \left[\frac{140 \times 5460 \times 10^{-10}\,\text{m}}{2 \times 0.2\,\text{m}}\right] = \mathbf{1.00019.}$$

Example 15.11: A glass microscope lens ($\mu = 1.50$) is coated with magnesium fluoride ($\mu = 1.38$) film to increase the transmission of normally incident yellow light ($\lambda = 5800$ Å). What minimum film thickness should be deposited on the lens?

Solution: Given that $\mu_g = 1.50$, $\mu_f = 1.38$, $\lambda = 5800$ Å.

Minimum film thickness $t_{min} = \frac{\lambda}{4\mu_f} = \frac{5800}{4 \times 1.38}$ Å $= \mathbf{1050\ Å}$

QUESTIONS

1. How will you find the wavelength of monochromatic light by using Michelson's Interferometer ? **(Nagpur, 2005)**
2. Draw a well labeled diagram of Michelson Interferometer. **(Nagpur, 2005)**
3. (*i*) Describe Michelson Interferometer with a neat diagram.
   (*ii*) How it is used to find difference between two close wavelengths?
4. (*i*) Explain the formation of circular fringes in Michelson Interferometer.
   (*ii*) Describe how you will use Michelson Interferometer to determine the difference between two wavelengths very close to each other.
5. With the help of a neat diagram, explain the construction of Fabry-Perot Interferometer.
6. Explain the construction of Michelson's interferometer. How it is used to determine the difference in wave lengths between two closely spaced spectral lines. **(Bangalore, 2005)**
7. Give the theory of Newton's rings. **(Kovempu, 2005)**
8. Describe the working of a Michelson interferometer. State the condition for obtaining white light fringes. Show with necessary theory how this interferometer can be used to measure wavelength of light. **(Agra, 2005)**
9. How is the wavelength of sodium light determined by Newton's rings method? Derive the formula used. Why are the rings circular? **(Meerut, 2005)**
10. Describe the construction and working of Michelson's interferometer. How will you use it to determine the difference between wavelength of two D-lines of sodium? **(Meerut, 2005)**
11. What will happen if wedge shaped film is placed in white light? **(Meerut, 2005)**
12. Soap bubble or a thin film of oil spread over the surface of water appears coloured in sunlight. Why? **(Lucknow, 2004)**
13. How would you obtain Newton's rings with bright center? **(Lucknow, 2004)**
14. Explain the construction and working of a Fabry-Perot interferometer. **(Lucknow 2004)**
15. Light containing two wavelengths λ_1 and λ_e falls normally on a plano convex lens of radius of curvature R, resting on a plane glass plate. If the nth dark ring due to λ_1 coincides with the $(_n+1)$ th dark ring due to λ_e, show that the radius of the $_n$th dark ring of λ_1 is :
$$\sqrt{\frac{\lambda_1 \lambda_e R}{\lambda_1 - \lambda_e}}$$ **(Lucknow, 2004)**
16. Describe a Michelson interferometer. How can it be used, for measuring the wavelength of monochromatic light? **(Garhwal, 2005)**
17. What are Newton's rings? Describe an experiment to determine the radius of curvature of a plae convex lens. **(Gulbarga, 2005)**
18. Describe Michelson interferometer and explain the formation of fringes in it. **(Punjab, 2005)**
19. What will happen if a transparent thin sheet is introduced in the path of one of the interfering beams? **(Punjab, 2005)**
20. What is the effect of phase change at each reflection in a Fabry – Perot interferometer? **(Punjab, 2005)**
21. Describe with theory, the Newton's rings experiment to determine the wavelength of monochromatic source of radiation. **(A.P.University, 2010)**
22. What happens to the rings pattern when a liquid is introduced between the plano-convex lens and plate in Newton's rings experiment? **(RTMNU, 2010)**
23. Explain the complementary nature of fringes due to reflected and transmitted light in thin films. **(RTMNU, 2010)**
24. Describe the principle and working of Fabry-Parot interferometer. **(GNDU, Amritsar, 2010)**

25. Describe the construction and working Fabry – Perot interferometer. How is it used to determine wavelength of light? How is it superior to Michelson's interferometer? **(RTMNU, Nagpur, 2010)**
26. Obtain the conditions for the observing the interference fringes. **(GNDU, Amritsar, 2010)**
27. Discuss applications of Michelson's interferometer. **(GNDU, Amritsar, 2010)**
28. Show that the separation between two successive bright Newton's rings is given by:

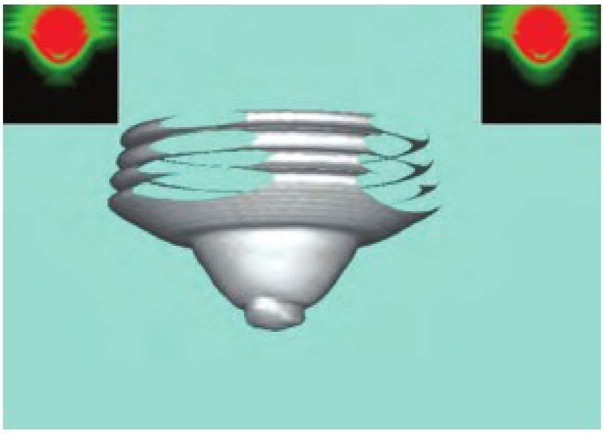
$$S = \frac{\sqrt{2\lambda R}}{\sqrt{2n+1} + \sqrt{2n-1}}$$

(RTMNU, 2010)

PROBLEMS FOR PRACTICE

1. In Michelson Interferometer 100 fringes cross the field of view when movable mirror is displaced through 2.948×10^{-3} cm. Calculate the wave length of monochromatic light used. **(Nagpur, 2005)**
2. When the movable mirror of Michelson's interferometer is shifted by 0.0589 mm, a shift of 200 fringes is obtained. Find the wavelength of light. **(Nagpur, 2005)**
3. By how much distance the movable mirror or a Michelson Interferometer be moved to obtain consecutive positions of maximum distinctness for sodium D-lines λ_1 = 5890 Å and λ_2 = 5896 Å. **(Nagpur, 2004)**
4. Calculate the distance between two successive position of a movable mirror of a Michelson Interferometer giving maximum visibility fringes for sodium D-lines of wavelength 5890 Å and 5896 Å.
5. Calculate the minimum thickness of a half wave plate of calcite for wavelength 5890 Å. Given : For Calcite $\mu_o = 1.658$, $\mu_o = 1.486$. **(Nagpur, 2004)**
6. Newton's rings are formed in a reflected light of wave length 590 nm. The diameter of the 10^{th} dark ring is 0.5×10^{-2} m. Find the radius of curvature of the lens. **(Bangalore, 2005)**
7. A shift of 100 circular fringes is observed when the movable mirror of the Michelson interferometer is shifted by 0.0295 mm. Calculate the wavelength of light. **(Kovempu, 2005)**
8. A soap film of RI 1.33 and thickness 1.5×10^{-4} cm is illuminated by white incident at an angle of 60°. The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of 5×10^{-5} cm. Calculate the order of interference of the dark band. **(Gulbarga, 2005)**
9. A shift of 200 fringes is observed when movable mirror of F-P interferometer is shifted by 0.0295 mm. Calculate the wavelength used. **(Punjab, 2005)**
10. In Michelson interferometer when the mirror M_1 is moved maximum visibility is observed for the position 0.03 cm. Calculate the difference between the wavelengths, if mean wavelength is 5893Å.
11. In a Fabry-Perot interferometer, the separation between the plates is 4×10^{-4} cm. Light of wavelength 5000Å falls normally on the plates. Find the order of the plates. Find the order of the maximum at the centre. **(RTMNU, 2010)**
12. The Michelson's interferometer experiment is performed with a source which consists of two wavelengths 4882Å. Through what distance does the mirror have to be moved between two positions of the disappearance of the fringes? **(GNDU, Amritsar, 2010)**
13. In a Newton's rings experiment, the diameter of 3^{rd} and 23^{rd} dark tings are 0.2cm and 0.6cm respectively. If the radius of curvature of plane convex lens is 92 cm, find the wavelength of light. **(A.P.University, 2010)**

16 CHAPTER



Coherence

16.1. INTRODUCTION

We have so far assumed naively that light sources emit *perfect harmonic waves.* In an ideal harmonic wave there exists a definite relationship between the phase of the wave at a given time and at a certain time later; and also at a given point and at a certain distance away. In reality, light sources do not emit perfectly harmonic waves. Even a very best practical *monochromatic* source emits a finite range of wavelengths and the light waves are quasi-monochromatic. If ***it were not so***, the light waves would have been ideally coherent and interference would be observed at all times.

In practice, light is emitted from a light source when excited atoms pass from the upper excited state to a lower energy state. The atom gives up the excess energy in the form of a photon. The process of transition from upper state to a lower state lasts for a brief time of about 10^{-8}s. It means that an atom starts emitting a light wave as it leaves the excited state and ceases emission as soon as it reaches the lower energy state. Therefore, an emission event produces a light burst. Each light burst occurs over a period of 10^{-8}s only, during which period a train of finite length having a certain limited number of wave oscillations is generated. Such a light burst is known as a **wave train** or a **wave packet.** After some time the atom again receives energy and jumps into excited state and subsequently emits another burst of light. These emission events occur quite randomly. Each atom in the source acts independently and different atoms emit

At a Glance

wave trains at different instants and their combination in millions and millions constitutes the light from a light source. In order to appreciate some of the peculiarities of natural light, the following fact is to be well understood. **The light emitted by an ordinary light source is not an infinitely long, simple harmonic wave but is composed of a jumble of finite wave trains**. We therefore call a real monochromatic source as a **quasi-monochromatic** source. The wave trains issuing out of a quasi-monochromatic source are as shown in Fig. 16.1.

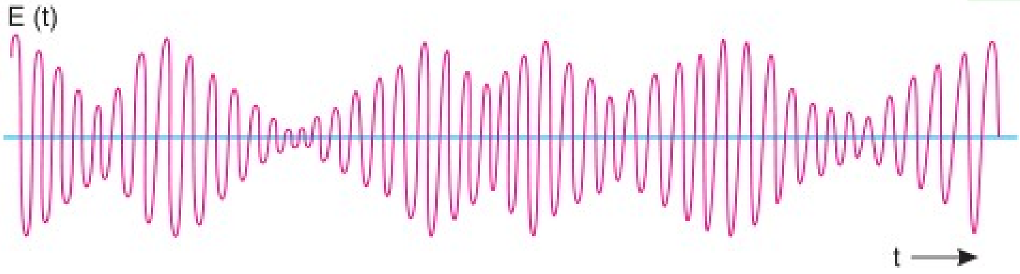


Fig. 16.1

16.2. WAVE TRAIN

Fig. 16.2 shows a wave train generated by an atom.

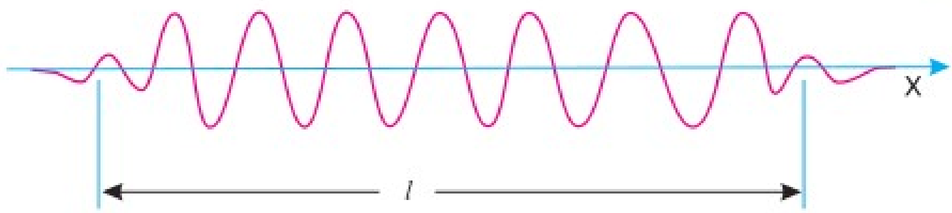


Fig. 16.2

If such a wave train lasts for a time interval Δt, then the length of the wave train in a vacuum is

$$l = c\Delta t \tag{16.1}$$

where c is the velocity of light in a vacuum.

For example, if $\Delta t = 10^{-8}$s, and c = 3×10^8m/s, then l = (3×10^8m/s)(10^{-8}s) = 3 m.

The number of oscillations present in the wave train is

$$N = \frac{l}{\lambda} \tag{16.2}$$

where λ is the wavelength. If we assume λ = 5000Å = 5×10^{-7}m, then

$$N = \frac{3\text{ m}}{5\times10^{-7}\text{ m}} = \mathbf{6\times10^6}$$

Thus, a wave train contains about a million wave oscillations in it.

Adding together the wave packets generated by all atoms in the light source, one finds a succession of wave trains, as shown in Fig. 16.3. In passing from one wave train to the next, there is an abrupt change in phase and also in plane of polarization. It is not possible to relate the phase at a point in wave train Q to a point in wave train P.

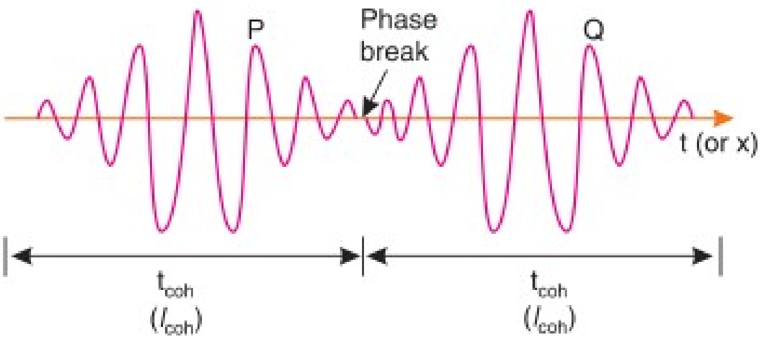


Fig. 16.3

Consequently there is no correlation between the phase of different wave trains. Each wave train has a sustained phase for only about 10^{-8} s, after which a new wave train is emitted with a totally random phase which also lasts only for about 10^{-8} s. The phase of the wave train from one atom will remain constant with respect to the phase of the wave train from another atom for utmost 10^{-8} s. It means that the wave trains can be coherent for a maximum of 10^{-8} s only. If two light waves overlap, sustained interference is not observed since the phase relationship between the waves changes rapidly, nearly at the rate of 10^8 times per second.

16.3. COHERENCE LENGTH AND COHERENCE TIME

The wave train, shown in Fig. 16.2, appears fairly sinusoidal for some number of oscillations between abrupt changes of frequency and phase. The length of the wave train over which it may be assumed to have a fairly sinusoidal character and predictable phase is known as **coherence length**. We denote it by $l_{coh.}$ We may consider coherence length as approximately equal to the length of the wave train, $c\Delta t$, over which its phase is predictable. The time interval during which the phase of the wave train can be predicted reliably is called **coherence time**. It is the time, Δt, during which the phase of the wave train does not become randomized but undergoes change in a regular systematic way. Coherence time is denoted by t_{coh}. We can therefore write

$$l_{coh} = c\,\Delta\,t \tag{16.3}$$

and

$$t_{coh} = \Delta\,t \tag{16.4}$$

$\therefore$

$$l_{coh} = c\;t_{coh} \tag{16.5}$$

A wave train consists of a group of waves, which have a continuous spread of wavelengths over a finite range $\Delta\lambda_o$ centered on a wavelength λ_o. According to Fourier analysis the frequency bandwidth $\Delta\nu$ is given by

$$\Delta\nu = \frac{1}{\Delta t}$$

where Δt is the average lifetime of the excited state of the atom. However, Δt is the time during which a wave train is radiated by the atom and corresponds to the coherence time, t_{coh}, of the wave train.

$\therefore$

$$\Delta\nu = \frac{1}{\Delta t} = \frac{1}{t_{coh}} \tag{16.6}$$

Using the relation (16.5) into equ. (16.6), we get

$$\Delta\nu = \frac{c}{l_{coh}} \tag{16.7}$$

16.4. BANDWIDTH

A wave packet is not a harmonic wave. Therefore, it cannot be represented mathematically by simple sine functions. The mathematical representation of a wave packet is done in terms of Fourier integrals. If light emitted from a source is analyzed with the help of a spectrograph, it is known to be made up of discrete spectral lines. Wave packets emitted by atoms form these spectral lines. Therefore, a spectral line and a wave packet are equivalent descriptions. The wavelength of a wave packet or a spectral line is not precisely defined. There is a continuous spread of wavelengths over a finite range, $\Delta\lambda$, centered on a wavelength λ_o. The maximum intensity of the wave packet occurs at λ_o and the intensity falls off rapidly on either side of λ_o, as shown in Fig. 16.4. The spread of wavelengths is called the *bandwidth*.

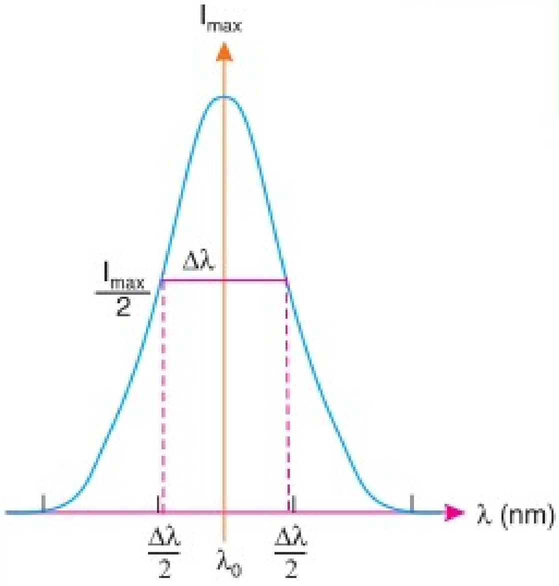


Fig. 16.4

The bandwidth is the wavelength interval from $\lambda_o - \Delta\lambda/2$ to $\lambda_o + \Delta\lambda/2$ which contains the major portion of the energy of the wave packet. In practice a source, which is said to produce line spectrum, produces a number of sharp wavelength distributions.

16.5. RELATION BETWEEN COHERENCE LENGTH AND BANDWIDTH

The frequency and wavelength of a light wave are related through the equation

$$\nu = \frac{c}{\lambda} \tag{16.8}$$

where λ_o is the vacuum wavelength.

Differentiating equ.(16.8) on both sides, we get

$$\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda \tag{16.9}$$

Using the relation (16.7) into equ.(16.9), we obtain

$$\therefore \quad \frac{c}{l_{coh}} = -\frac{c}{\lambda^2}\Delta\lambda \tag{16.10}$$

Rearranging the terms, we get

$$l_{coh} = \frac{\lambda^2}{\Delta\lambda} \tag{16.11}$$

The minus sign has no significance and hence is ignored. Equ.(16.11) means that the coherence length (the length of the wave packet) and the bandwidth of the wave packet are related to each other. The longer the wave packet, the narrower will be the bandwidth (see Fig. 16.5). In the limiting case, when the wave is infinitely long, we obtain monochromatic radiation of frequency ν_o (wavelength λ_o).

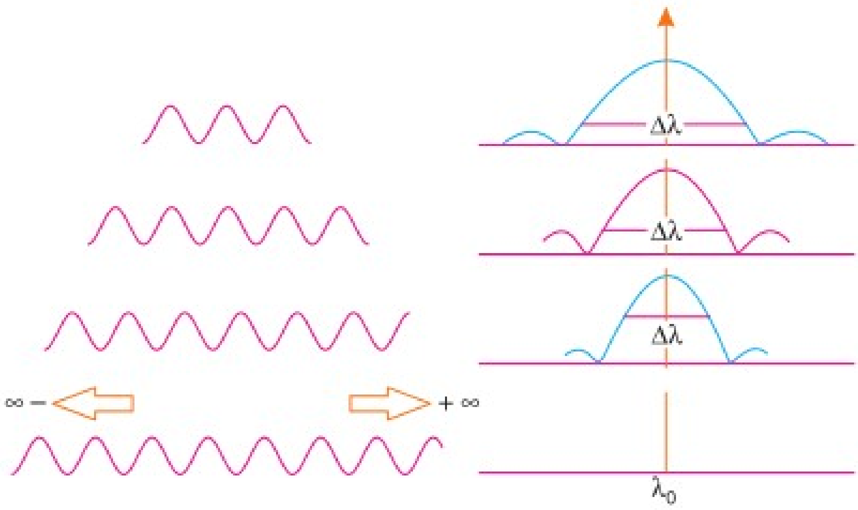


Fig. 16.5

Form the equ.(16.2), the coherence length may be defined as the product of the number of wave oscillations N contained in the wave train and of the wavelength, λ. Thus,

$$l_{coh} = N\lambda \tag{16.12}$$

Equating (16.11) and (16.12), we get

$$N = \frac{\lambda}{\Delta\lambda}$$

$$\therefore \qquad \frac{\Delta\lambda}{\lambda} = \frac{1}{N} \tag{16.13}$$

Equ.(16.13) shows that the larger the number of wave oscillations in a wave packet, the smaller is the bandwidth. In the limiting case, when N is infinitely large, that is when the wave packet is infinitely long, the wave will be monochromatic having a precisely defined wavelength. The dependence of bandwidth on the length of the wave packet is schematically shown in Fig. 16.5.

16.6. COHERENCE

Coherence is an important property of light. It refers to the connection between the phase of light waves at one point and time, and the phase of the light waves at another point and time. Coherence effects are mainly divided into two categories: *temporal and spatial*. The temporal coherence is related directly to the finite bandwidth of the source, whereas the spatial coherence is related to the finite size of the source.

16.6.1. TEMPORAL COHERENCE

Temporal coherence is also known as longitudinal coherence. Let a point source of quasi-monochromatic light S (Fig.16.6) emit light in all directions. Let us consider light travelling along the line SP_1P_2. The phase relationship between the points P_1 and P_2 depends on the distance P_1P_2 and the coherence length of the light beam. The electric fields at P_1 and P_2 will be correlated in phase when a single wave train extends over greater length than the distance P_1P_2; that is if the distance P_1P_2 is less than the coherence length l_{coh}. Then, the waves are correlated in their rising and falling and they will preserve a constant phase difference.

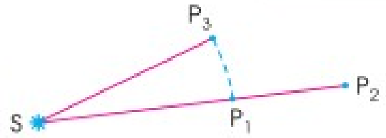
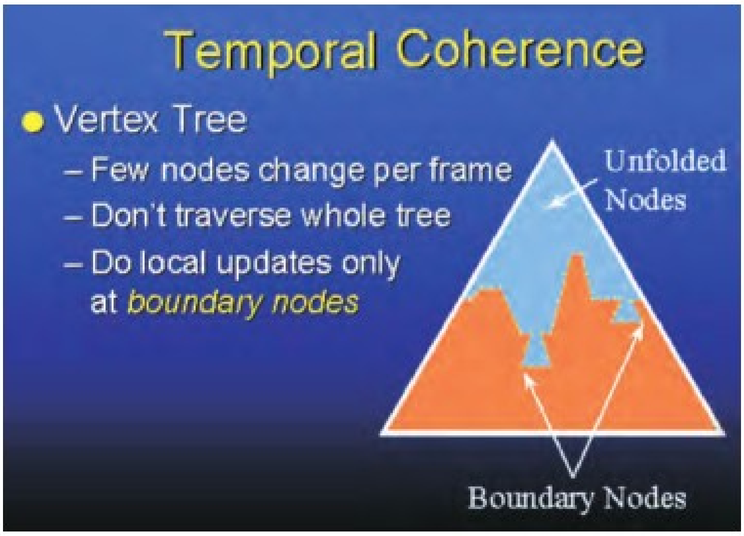


Fig. 16.6

The points P_1 and P_2 would not have any phase relationship if the longitudinal distance P_1P_2 is greater than



l_{coh}, since in such a case many wave trains would span the distance. It means different independent wave trains would be at P_1 and P_2 at any instant and therefore the phase at the two points would be independent of each other. The degree to which a correlation exists is known as the amount of *longitudinal coherence.*

16.6.1.1. Monochromaticity

Form equ.(16.11) and Fig.16.5 we conclude that temporal coherence is indicative of **monochromaticity** of the source. An ideally monochromatic source is an absolutely coherent source. The degree of monochromaticity of a source is given by

$$\xi = \frac{\Delta \nu}{\nu_o} \qquad (16.14)$$

When the ratio $\Delta\nu / \nu_o = 0$, the light wave is ideally monochromatic.

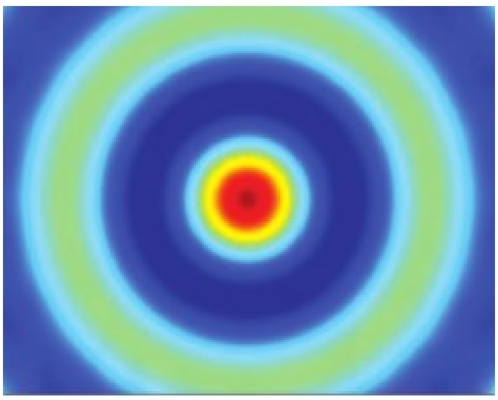
16.6.1.2. Purity of spectral line

The width of a spectral line is given by $\Delta\lambda$ (see Fig. 16.4). It is seen from equ.(16.11) that it is related to the temporal coherence. Thus,

$$\Delta\lambda = \frac{\lambda^2}{l_{coh}} \qquad (16.15)$$

16.6.2. SPATIAL COHERENCE

Spatial coherence refers to the continuity and uniformity of a wave in a direction perpendicular to the direction of propagation. If the phase difference for any two fixed points in a plane normal to the wave propagation does not vary with time, then the wave is said to exhibit *spatial coherence*. It is also known as *lateral coherence*. Again looking at the point source S (Fig.16.6), $SP_1 = SP_3$ and therefore, the fields at points P_1 and P_3 would have the same phase. Thus, an ideal point source exhibits spatial coherence, as the waves produced by it are likely to have the same phase at points in space, which are equidistant from the source. On the other hand, an extended source is bound to exhibit lesser



Spatial Coherence.

lateral spatial coherence. Two points on the source separated by a lateral distance greater than one wavelength will behave quite independently. Therefore, correlation is absent between the phases of the waves emitted by them. The degree of contrast of the interference fringes produced by a source is a measure of the degree of the spatial coherence of its waves. The higher the contrast, the better is the spatial coherence.

16.7. DETERMINATION OF COHERENCE LENGTH

The coherence length can be measured by means of Michelson interferometer. In a Michelson interferometer, a light beam from the source S is incident on a semi-silvered glass plate G (see Fig.16.7) and gets divided into two components; one component is reflected, 1,and the other,2, is transmitted. These two beams, 1 and 2, are reflected back at mirrors M_1 and M_2 respectively and are received by the telescope where interference fringes are produced. It is obvious that the beams produce stationary interference only if they are coherent.

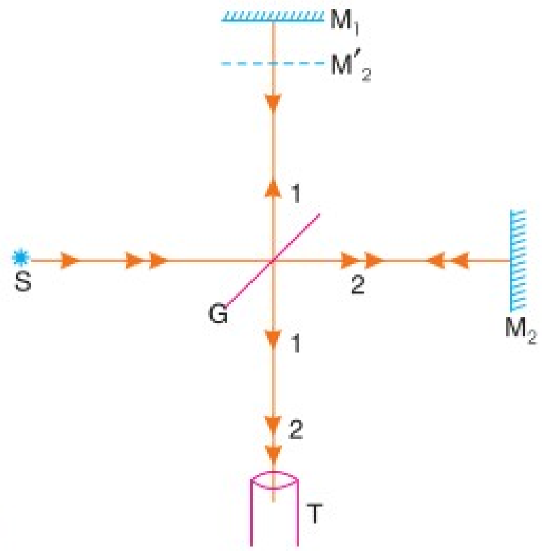


Fig. 16.7

Let M_2' be the image of M_2 formed by G. If the reflecting surfaces M_1 and M_2' (the image of M_2) are separated by a distance d, then $2d$ will be the path difference between the interfering waves. The condition of fixed phase relationship between the two waves, 1 and 2, will be satisfied if

$$2d << l_{coh}$$

In such a case distinct interference fringes will be seen. If, however,

$$2d >> l_{coh}$$

then the phases of the two waves are not correlated and interference fringes will not be seen. To determine the coherence length of waves emitted by a light source, the distance d between the mirrors M_1 and M_2' (the image of M_2) is varied by moving one of the mirrors. As the distance varies, the contrast of the fringes decreases and ultimately they disappear. The path difference $2d$ at the particular stage where the fringes disappear gives us the coherence length.

The light from a sodium lamp has coherence length of the order of 1 mm, that of green mercury line is about 1 cm, neon red line 3 cm, red cadmium line 30 cm, orange krypton line 80 cm and that of a commercial He-Ne laser is about 15m. The coherence length of light from some of the lasers goes up to a few km.

16.8. CONDITION FOR SPATIAL COHERENCE

The degree of spatial coherence of a beam of light can be deduced from the contrast of the fringes produced by it. The broader the source of the light, the lesser is the degree of coherence. In Young's double slit experiment, if the slits S_1 and S_2 are directly illuminated by a source, interference fringes are not observed. Instead the screen is uniformly illuminated. The absence of fringes indicates that the light issuing from the slits do not possess spatial coherence. If a narrow slit is introduced before the double slit, light passing through the narrow slit S illuminates S_1 and S_2. The waves emerging from them, having been derived through wave front division, are coherent and stationary interference pattern will be observed on the screen. If the width of the slit S is gradually increased the contrast of fringe pattern decreases and fringes disappear. When the slit S is wider, S_1 and S_2 receive waves from different parts of S which do not maintain coherence. When S is narrow, it ensures that the wave trains incident on slits S_1 and S_2 originate from a small region of the source and hence they have spatial coherence.

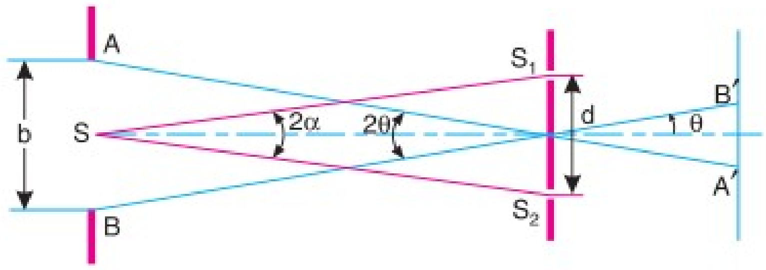


Fig. 16.8

Formation of distinct fringe pattern depends on two parameters in the double slit experiment. One is the size of the slit S and second is the separation, d, between the two slits S_1 and S_2. From Fig.16.9, it is seen that the path difference between the two waves passing at the edges A and B of the slit S is

$$\Delta = b \sin \alpha$$

Distinct fringes will be obtained when $\Delta << \frac{1}{2}\lambda$

Therefore, it requires that $$b \sin \alpha << \frac{1}{2}\lambda \tag{16.16}$$

This equation determines the size of the source, which is spatially coherent to produce fringes of satisfactory contrast. Equ.(16.16) shows that the smaller is the size of the source, the better is the spatial coherence. Spatial coherence of waves decreases with increasing size of the light source.

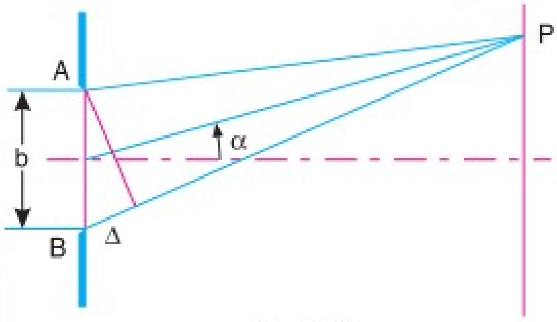


Fig. 16.9

From Fig. 16.8 it is seen that the edges A and B of the slit S subtend an angle 2θ at the centre of the double slit and the double slit subtends an angle 2α at the centre of S. Waves from A produce interference pattern which has its centre at A′. Similarly, waves from B produce fringe pattern with its centre at B′. The patterns cancel each other if the maximum of one falls on the minimum of the other. The condition for no fringes is that

$$2\theta = \frac{\lambda}{2d} \tag{16.17}$$

When the slit S is made wider, the edges A and B move away from each other. As a result the patterns at A′ and B′ also move away. When the maxima of one pattern fall on the maxima of other, the fringes appear. Therefore, the condition for fringes is that

$$2\theta = \frac{\lambda}{d}$$

or $$d = \frac{\lambda}{2\theta} \tag{16.18}$$

In other words, there is restriction imposed on the separation of the slits S_1 and S_2. d must be very small compared to $\lambda/2\theta$ in order to maintain spatial coherence and obtain distinct fringe pattern on the screen.

Light produced by lasers is highly coherent. Therefore, when a laser is used as the source, interference fringes can be obtained without the aid of slit S.

WORKED OUT PROBLEMS

Example 16.1: A sodium atom radiates for 4×10^{-12} s. What is the coherence length of light from a sodium lamp?

Solution: It is given that coherence time $t_{coh} = 4\times 10^{-12}$ s.

$$\text{Coherence length } l_{coh} = c\, t_{coh} = (3\times10^8 \text{ m/s})(4\times 10^{-12} \text{ s})$$
$$= 12\times10^{-4} \text{ m} = \mathbf{1.2\ mm.}$$

Example 16.2: Calculate the coherence length for CO_2 laser whose line width is 1×10^{-5} nm at IR emission wavelength of 10.6 μm.

Solution: Coherence length, $l_{coh} = \dfrac{\lambda^2}{\Delta\lambda} = \dfrac{\left(10.6\times10^{-6}\right)^2 \text{ m}^2}{1\times10^{-5}\times10^{-9}\text{ m}} = \mathbf{11.2\ km}$.

Example 16.3: Compute the coherence length of yellow light with 5893Å in 10^{-12} second pulse duration. Find also the bandwidth.

Solution: Coherence length is given by $l_{coh} = ct_{coh}$.

$\therefore$ $$l_{coh} = (3\times10^8 \text{ m/s})(10^{-12}\text{s}) = \mathbf{0.3\ mm}$$

$$\text{Bandwidth is given by } \Delta\lambda = \frac{\lambda^2}{l_{coh}} = \frac{(5893\times10^{-10}\text{ m})^2}{3\times10^{-4}\text{ m}} = \mathbf{11.6Å}$$

QUESTIONS

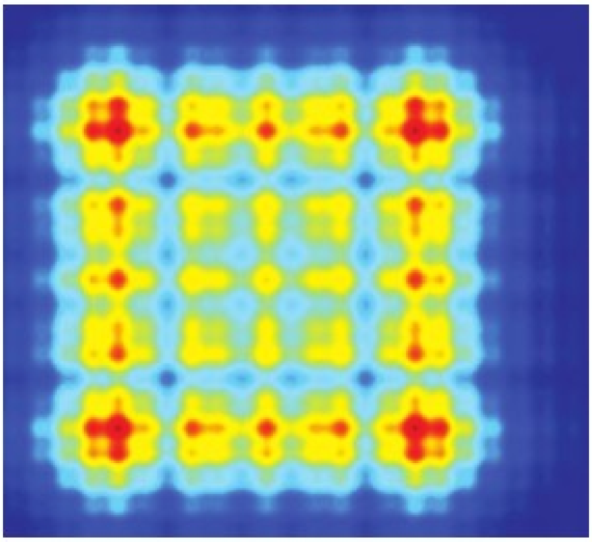
1. Explain the terms coherence length and coherence time for a light wave. Derive an expression for the coherence length of a wave train that has a frequency bandwidth Δν.
2. Explain in brief:
   (*i*) Coherence length (*ii*) spatial coherence (*iii*) temporal coherence.
   Obtain an expression for coherence length.
3. Explain in brief the terms: (*i*) Temporal Coherence (*ii*) Spatial Coherence. **(Punjab 2005)**
4. Distinguish between spatial and temporal coherence. What are coherence length and coherence time? Why is it impossible to observe interference between light waves emitted by independent sources?
5. The light of wavelength 6000Å has wave trains 13.2×10^{-6} m. Calculate the coherent time. **(GNDU, Amritsar, 2010)**
6. Determine what emission frequency width would be required to have a temporal coherence length of 10m at a source wavelength of 488 nm? **(GNDU, Amritsar, 2010)**

PROBLEMS FOR PRACTICE

1. Calculate the frequency bandwidth for white light (frequency range 4×10^{14}Hz to 7.5×10^{14}Hz). Also find (i) coherence time and (ii) coherence length of white light.
   [**Ans:** $\Delta\nu = 3.5\times10^{14}$Hz; $t_{coh} = 2.8\times10^{-15}$s; $l_{coh} = 8.6\times10^{-7}$m]
2. A quasi-monochromatic source emits radiations of mean wavelength λ = 5461Å and has a bandwidth $\Delta\nu = 10^9$Hz. Calculate coherence time, coherence length and frequency stability. [**Ans:** $t_{coh} = 10^{-9}$s; $l_{coh} = 0.3$ m; $\Delta\nu/\nu = 1.8\times10^{-6}$]
3. An optical filter has a line width of 1.5 nm and mean wavelength 550 nm. With white light incident on the filter, calculate (i) coherence length and (ii) the number of wavelengths in the wave train. [**Ans:** $l_{coh} = 2\times10^{-4}$m; N = 366]

17

CHAPTER



Fresnel Diffraction

17.1. INTRODUCTION

When waves encounter obstacles (or openings), they bend round the edges of the obstacles, if the dimensions of the obstacles are comparable to the wavelength of the waves. The bending of waves around the edges of an obstacle is called *diffraction.*

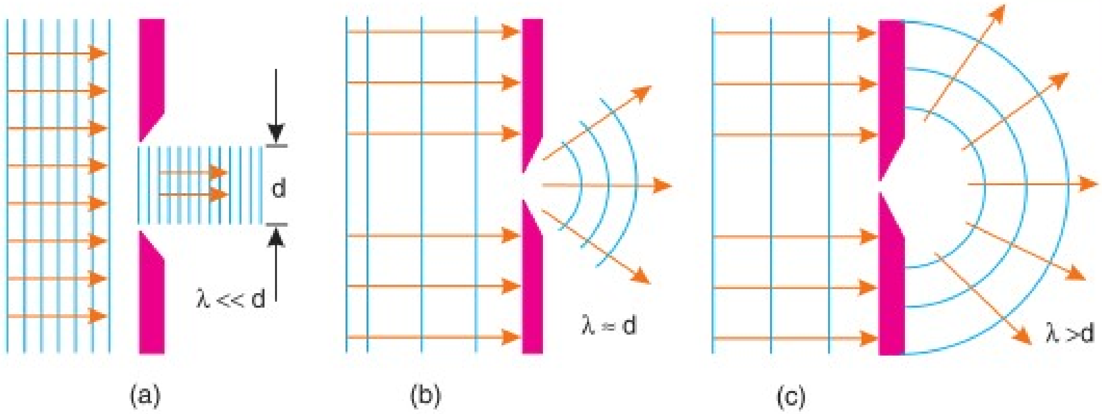
Fig. 17.1 illustrates the passage of waves through an opening. When the opening is large compared to the wavelength, the waves do not bend round the edges. When the opening is small, the bending round the edges is noticeable. When the opening is very small, the waves spread over the

Light diffraction by a razor blade.

At a Glance

- Introduction
- Huygens-Fresnel Theory
- Fresnel's Assumptions
- Reactilinear Propagation of Light
- Zone Plate
- Distinction Betwen Interference and Diffraction
- Fresnel and Fraunhoffer Types of Diffraction
- Diffraction at a Circular Aperture
- Diffraction at an Opaque Circular Disc
- Diffraction Pattern Due to A Straight Edge
- Diffraction Pattern Due to a Narrow Slit
- Diffraction Due to a Narrow Wire
- Cornu's Spiral
- Cornu's Spiral (Alternative Method)
- Diffraction at a Straight Edge

entire surface behind the opening. The opening acts an independent source of waves, which propagate in all directions. The diffraction effect is observable quite close to the opening when the size of the opening is very small. When the opening is large, diffraction effect is observed at greater distances from the opening. In general diffraction of waves becomes noticeable only when the size of the obstacle is comparable to a wavelength.



Diffraction–(*a*) A plane wave does not bend at the slit if the opening $d \gg \lambda$. (*b*) Bending is perceptible when $\lambda \approx d$ (*c*). When $\lambda > d$, the bending takes place to such an extent that light can be perceived in a direction normal to the ray propagation suggesting that the opening acts as a point source.

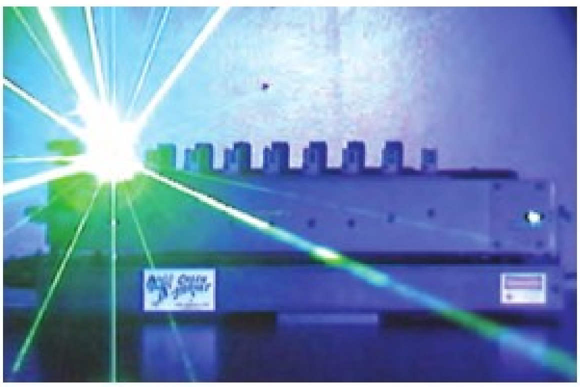
Fig. 17.1

It is a matter of common experience that the path of light entering a dark room through a hole in the window illuminated by sunlight is straight. Similarly, if an opaque obstacle is placed in the path of light, a sharp shadow is cast on the screen, indicating thereby that light travels in straight lines. Rectilinear propagation of light can be easily explained on the basis of Newton's corpuscular theory. But it has been observed that when a beam of light passes through a small opening (a small circular hole or a narrow slit) it spreads to some extent into the region of the geometrical shadow also. If light energy is propagated in the form of waves, then similar to sound waves, one would expect bending of a beam of light round the edges of an opaque obstacle or illumination of the geometrical shadow.

However, diffraction phenomenon is not readily apparent in case of light waves. It becomes significant when the aperture size is of the order of one wavelength wide. Diffraction and interference are basically equivalent.

17.2. HUYGENS-FRESNEL THEORY

According to Huygen's wave theory of light, each progressive wave produces secondary waves, the envelope of which forms the secondary wave front. In Fig. 17.2 (a), S is a source of monochromatic light and MN is a small aperture. XY is the screen placed in the path of light. AB is the illuminated portion of the screen and above A and below B is the region of the geometrical shadow. Considering MN as the primary wavefront, according to Huygen's construction, if secondary wave fronts are drawn, one would expect encroachment of light



Diffraction effect.

in the geometrical shadow. Thus, the shadows formed by small obstacles are not sharp. This bending of light round the edges of an obstacle or the encroachment of light within the geometrical shadow is known as *diffraction*. Similarly, if an opaque obstacle MN is placed in the path of light [Fig.17.2 (b)], there should be illumination in the geometrical shadow region AB also. But the illumination in the geometrical shadow of an obstacle is not commonly observed because the light sources are not point sources and secondly the obstacles used are of very large size compared to the wavelength of light. If a shadow of an obstacle is cast by an extended source, say a frosted electric bulb, light from every point on the surface of the bulb forms its own diffraction pattern (bright and dark diffraction bands) and these overlap such that no single pattern can be identified. *The term diffraction is referred to such problems in which one considers the resultant effect produced by a limited portion of a wavefront.*

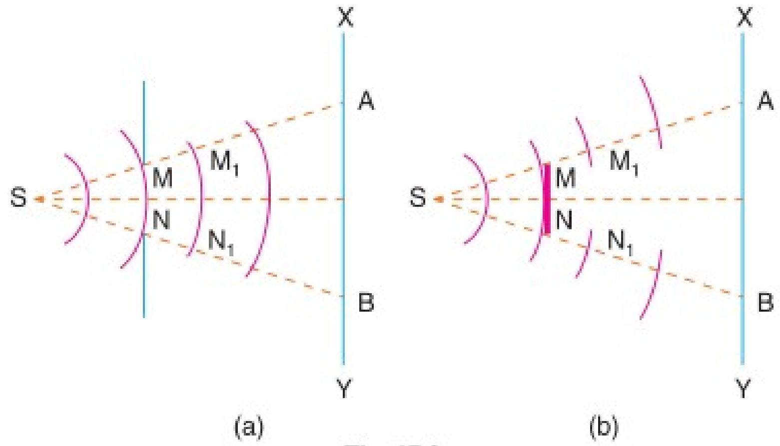


Fig. 17.2

Diffraction phenomena are part of our common experience. The luminous border that surrounds the profile of a mountain just before the sun rises behind it, the light streaks that one sees while looking at a strong source of light with half shut eyes and the colored spectra (arranged in the form of a cross) that one sees while viewing a distant source of light through a fine piece of cloth are all examples of diffraction effects.

Augustine Jean Fresnel in 1815, combined in a striking manner Huygens' wavelets with the principle of interference and could satisfactorily explain the bending of light round obstacles and also the rectilinear propagation of light.

17.3. FRESNEL'S ASSUMPTIONS

According to Fresnel, the resultant effect at an external point due to a wavefront will depend on the factors discussed below:

In Fig. 17.3, S is a point source of monochromatic light and MN is a small aperture. XY is the screen and SO is perpendicular to XY. MCN is the incident spherical wavefront due to the point source S. To obtain the resultant effect at a point P on the screen, Fresnel assumed the following:

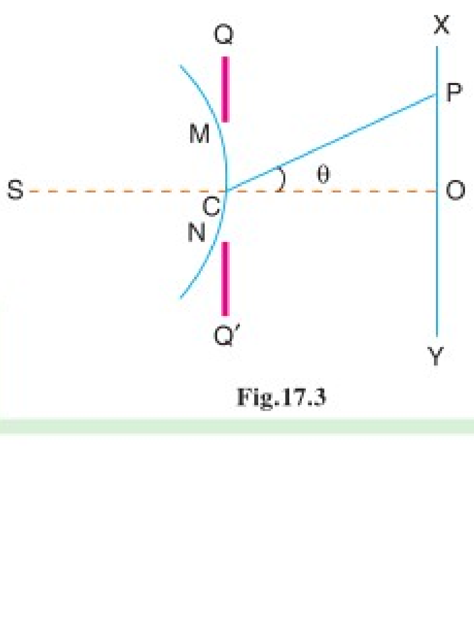


Fig.17.3

(1) A wave front can be divided into a large number of strips or **zones** called Fresnel's zones of

small area and the resultant effect at any point will depend on the combined effect of all the secondary waves emanating from the various zones;

(2) The effect at a point due to any particular zone will depend on the distance of the point from the zone;

(3) The effect at P will also depend on the obliquity of the point with reference to the zone under consideration, e.g. due to the part of the wavefront at C, the effect will be a maximum at O and decreases with increasing obliquity. It is a maximum in a direction radially outwards from C and it decreases in the opposite direction. The effect at a point due to the obliquity factor is proportional to (1+ cos θ) where ∠PCO = θ. Considering an elementary wavefront at C, the effect is maximum at O because θ = 0 and cos θ =1. Similarly, in a direction tangential to the primary wavefront at C (along CQ) the resultant effect is one half of that along CO because θ = 90° and cos 90° = 0. In the direction CS, the resultant effect is zero since θ = 180° and cos 180° = –1 and 1 + cos 180° = 1–1 = 0. This property of the secondary waves eliminates one of the difficulties experienced with the simpler form of Huygens principle viz., that if the secondary waves spread out in all directions from each point on the primary wavefront, they should give a wave traveling forward as well as backward. Now, as the amplitude at the rear of the wave is zero there will evidently be no back wave.

Augustin Jean Fresnel
(1788-1827)

17.4. RECTILINEAR PROPAGATION OF LIGHT

ABCD is a plane wavefront perpendicular to the plane of the paper [Fig. 17.4 (a)] and P is an external point at a distance b perpendicular to ABCD. To find the resultant intensity at P due to the wavefront ABCD, Fresnel's method consists in dividing the wavefront into a number of *half period elements* or *zones* called Fresnel's zones and to find the effect of all the zones at the point P.

If spheres are constructed with P as centre and radii equal to b+λ/2, b+2λ/2, b+3λ/2 etc., they will cut out circular areas of radii OM_1, OM_2, OM_3, etc., on the wave front. These circular zones are called **half period zones** or **half period elements**. Each zone differs from its neighbour by a phase difference of π or path difference of λ/2. Thus the secondary waves starting from the point O and M_1 and reaching P will have a phase difference of π or a path difference λ /2. A Fresnel half period zone with respect to an actual point P is a thin annular zone of the primary wavefront in which the secondary waves from any two corresponding points of neighboring zones differ in path by λ /2 .

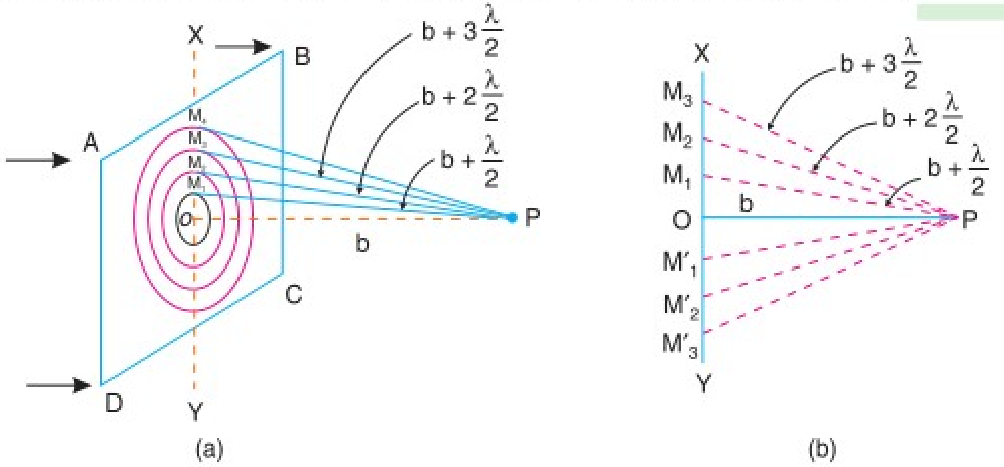


Fig. 17.4

In Fig. 17.4 (b) O is the pole of the wavefront XY with reference to the external point P. OP is perpendicular to XY. In Fig.17.4 (c) 1, 2, 3 etc are the half period zones constructed on the primary wavefront XY. OM_1 is the radius of the first zone. OM_2 is the radius of the second zone and so on. P is the point at which the resultant intensity has to be calculated.

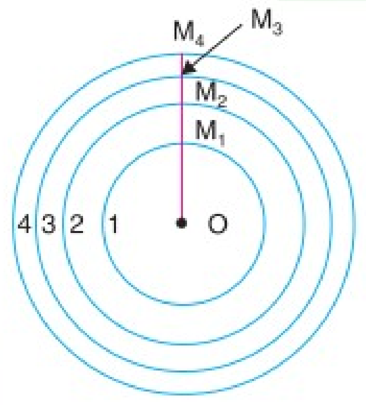


Fig. 17.4 (*c*)

$$OP = b,\ OM_1 = r_1,\ OM_2 = r_2,\ OM_3 = r_3, \text{ etc.}$$

And

$$M_1P = b + \frac{\lambda}{2}, \quad M_2P = b + \frac{2\lambda}{2}, \quad M_3P = b + \frac{3\lambda}{2} \text{ etc.}$$

The area of the first half period zone is

$$\pi\, OM_1^2 = \pi\, [M_1P^2 - OP^2] = \pi \left[\left(b + \frac{\lambda}{2}\right)^2 - b^2\right]$$

$$= \pi \left[b\lambda + \frac{\lambda^2}{4}\right] = \pi\, b\lambda \qquad (17.1)$$

As λ is small, λ^2 term is neglected.

The radius of the first half period zone is, $r_1 = OM_1 = \sqrt{b\lambda}$

The radius of the second half period zone is, $OM_2 = [M_2P^2 - OP^2]^{1/2}$

$$= \left[(b+\lambda)^2 - b^2\right]^{1/2}$$

$$= \sqrt{2b\lambda}$$

The area of the second half period zone, $= \pi\left[OM_2^2 - OM_1^2\right]$

$$= \pi\, [2b\lambda - b\lambda]$$

$$= \pi\, b\lambda \qquad (17.2)$$

Thus, the area of each half period zone is equal to $\pi\, b\, \lambda$. Also the radii of the 1st, 2nd, 3rd, etc half period zones are $\sqrt{1b\lambda}$, $\sqrt{2b\lambda}$, $\sqrt{3b\lambda}$ etc. Therefore, the radii are proportional to the square roots of the natural numbers. However, it should be remembered that the areas of the zones are not constant but are dependent on - (i) λ , the wavelength of light and (ii) b, the distance of the point from the wavefront. The area of the zone increases with increase in the wavelength of light and with increase in the distance of the point P from the wavefront.

As discussed in §17.3 the effect at a point will depend on (i) the distance of P from the wavefront, (ii) the area of the zone and (iii) the obliquity factor. Here, the area of each zone is the same. The secondary waves reaching the point P are continuously out of phase and in phase with reference to the central or the first half period zone. Let m_1, m_2, m_3, etc represent the amplitudes of vibration of the ether particles at P due to secondary waves from the 1st , 2nd, 3rd, etc. half period zones (see Fig. 17.5). As we

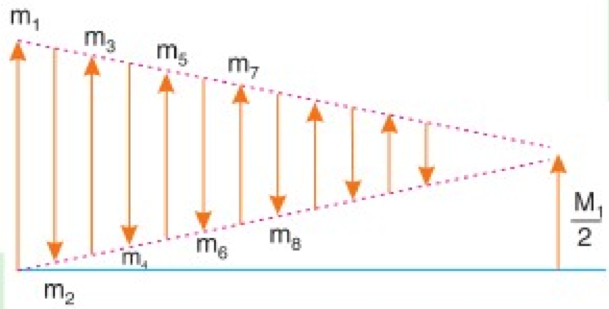


Fig. 17.5

consider the zones outwards from O, the obliquity increases and hence the quantities m_1, m_2, m_3 etc are of continuously decreasing order. Thus, m_1 is slightly greater than m_2; m_2 is slightly greater than m_3 and so on. Due to the phase difference of π between any two consecutive zones, if the displacements of the ether particles due to odd numbered zones is in the positive direction, then due to the even numbered zones the displacement will be in the negative direction at the same instant. As the amplitudes are of gradually decreasing magnitude, the amplitude of vibration at P due to any zone can be approximately taken as the mean of the amplitudes due to the zones preceding and succeeding it.

e.g. $$m_2 = \frac{m_1 + m_3}{2}$$

The resultant amplitude at P at any instant is given by,

$$A = m_1 - m_2 + m_3 - m_4 ...... + m_n \text{ if } n \text{ is odd.}$$

(If n is even, the last quantity is $- m_n$).

$$\therefore \qquad A = \frac{m_1}{2} + \left[\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right] + \left[\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right] + ...$$

But $$m_2 = \frac{m_2}{2} + \frac{m_3}{2} \quad and \quad m_4 = \frac{m_3}{2} + \frac{m_5}{2}$$

$$A = \frac{m_1}{2} + \frac{m_n}{2} ....if \ n \text{ is odd.}$$

$$A = \frac{m_1}{2} + \frac{m_{n-1}}{2} - m_n ......if \ n \text{ is even.}$$

If the whole wave front ABCD is unobstructed, the number of half period zones that can be constructed with reference to the point P is infinite i.e. $n \to \infty$. As the amplitudes are of gradually diminishing order, m_n and m_{n-1} tend to be zero.

Therefore, the resultant amplitude at P due to the whole wavefront $= A = \frac{m_1}{2}$. (17.3)

The intensity at a point is proportional to the square of the amplitude.

$$\therefore \qquad I \propto \frac{m_1^2}{4} \qquad (17.4)$$

Thus, the intensity at P is only one–fourth of that due to the first half period zone alone. Here, only half the area of the first half period zone is effective in producing the illumination at the point P. A small obstacle of the size of half the area of the first half period zone placed at O will screen the effect of the whole wavefront and the intensity at P due to the rest of the wavefront will be zero. While considering the rectilinear propagation of light, the size of the obstacle used is far greater than the area of the first half period zone and hence the bending effect of light round corners (diffraction effects) cannot be noticed. In the case of sound waves, the wavelengths are far greater than the wavelength of light, and hence the area of the first half period zone for a plane wavefront of sound is very large. If the effect of sound at a point beyond an obstacle is to be shadowed, an obstacle of very large size has to be used to get no sound effect. If the size of the obstacles placed in the path of light is comparable to the wavelength of light, then it is possible to observe illumination in the region of the geometrical shadow also. Thus, rectilinear propagation of light is only approximately true.

17.5. ZONE PLATE

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone. It can be designed so as to cut off light due to the even numbered zones or that due to the odd numbered zones. The correctness of Fresnel's method in dividing a wavefront into half period zones can be verified with its help.

To construct a zone plate, concentric circles are drawn on white paper such that the radii are proportional to the square roots of the natural numbers (as shown in §17.4, the radii are proportional to the square roots of the natural numbers). The odd numbered zones (i.e. 1st, 3rd, 5th, etc) are covered with black ink and a reduced photograph is taken. The drawing appears as shown in Fig. 17.6 (b). The negative of the photograph will be as shown in Fig. 17.6 (a). In the developed negative, the odd zones are transparent to incident light and the even zones will cut off light.

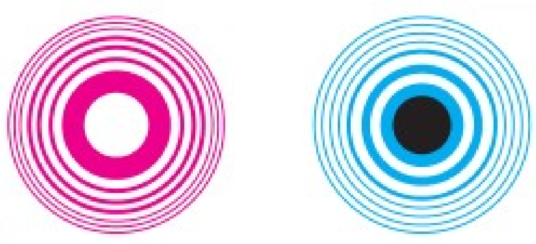


Fig. 17.6

If such a plate is held perpendicular to an incident beam of light and a screen is moved on the other side to get the image, it will be observed that maximum brightness is possible at some position of the screen say b cm from the zone plate (Fig. 17.7) XO is the upper half of the incident plane wavefront. P is the point at which the light intensity is to be considered. The distance of the point P from the wavefront is b. $OM_1(=r_1), OM_2(=r_2)$ etc are the radii of the zones.

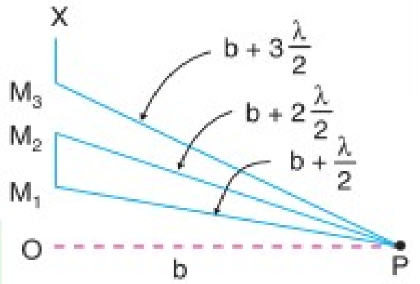


Fig. 17.7

$r_1 = \sqrt{b\lambda}$ and $r_2 = \sqrt{2b\lambda}$ where λ is the wavelength of light.

$$r_n = \sqrt{nb\lambda} \quad \text{or} \quad b = \frac{r_n^2}{n\lambda} \tag{17.5}$$

If the source is at a large distance from the zone plate, a bright spot will be obtained at P. As the distance of the source is large, the incident wavefront can be taken as a plane one with respect to the small area of the zone plate. The even numbered zones cut off the light and hence resultant amplitude at $P = A = m_1 + m_2 + m_3 + ...$ etc. In this case the focal length of the zone plate f_n is given by

$$f_n = b = \frac{r_n^2}{n\lambda} \tag{17.6}$$

Thus, a zone plate has different foci for different wavelengths. The radius of the nth zone increases with increasing value of λ. *It is very interesting to note that as the even numbered zones are opaque, the intensity at P is much greater than that when the whole wavefront is exposed to the point P.*

In the first case the resultant amplitude is given by

$$A = m_1 + m_3 + m_5 + ...m_n.... \quad (n \text{ is odd})$$

When the whole wavefront is unobstructed, the amplitude is given by

$$A = m_1 - m_2 + m_3 - m_4 ...... + m_n$$

$$= \frac{m_1}{2} \text{ (if } n \text{ is very large and } n \text{ is odd).}$$

If a parallel beam of white light is incident on the zone plate, different colours come to focus at different points along the line OP. Thus, the function of a *zone plate is similar to that of a convex* *(converging) lens* and a formula connecting the distance of the object and image points can be obtained for a zone plate also.

17.5.1. ACTION OF A ZONE PLATE FOR AN INCIDENT SPHERICAL WAVE FRONT

Let XY represent the section of the zone plate perpendicular to the plane of the paper. S is a point source of light, P is the position of the screen for a bright image, 'a' is the distance of the source from the zone plate and b is the distance of the screen from the plate. OM_1, OM_2, OM_3, (r_1, r_2, r_3) etc are the radii of the 1st, 2nd, 3rd etc. half period zones. The position of the screen is such that from one zone to the next there is an increasing path difference of $\frac{\lambda}{2}$.

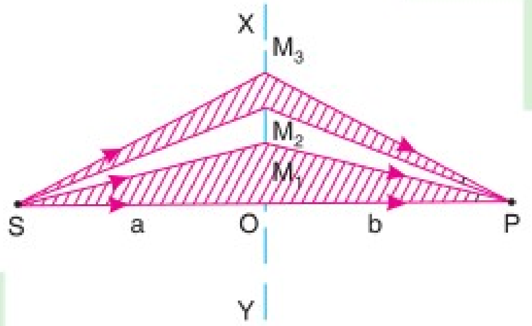


Fig. 17.8

Thus,

$$SO + OP = a + b$$

$$SM_1 + M_1P = a + b + \frac{\lambda}{2} \qquad (17.7)$$

$$SM_2 + M_2P = a + b + \frac{2\lambda}{2} \quad \text{and so on.}$$

From the $\Delta\, SM_1O$

$$SM_1 = \left(SO^2 + OM_1^2\right)^{1/2}$$

$$= \left(a^2 + r_1^2\right)^{1/2}$$

Similarly from the ΔOM_1P

$$M_1P = \left(OP^2 + OM_1^2\right)^{1/2}$$

$$= \left(b^2 + r_1^2\right)^{1/2}$$

Substituting the values of SM_1 and M_1P in equation (17.7), we get

$$(a^2 + r_1^2)^{1/2} + \left(b^2 + r_1^2\right)^{1/2} = a + b + \frac{\lambda}{2}$$

$$a\left(1 + \frac{r_1^2}{a^2}\right)^{1/2} + b\left(1 + \frac{r_1^2}{b^2}\right)^{1/2} = a + b + \frac{\lambda}{2}$$

$$a + \frac{r_1^2}{2a} + b + \frac{r_1^2}{2b} = a + b + \frac{\lambda}{2}$$

$$\frac{r_1^2}{2}\left[\frac{1}{a} + \frac{1}{b}\right] = \frac{\lambda}{2}$$

$$r_1^2\left(\frac{1}{a} + \frac{1}{b}\right) = \lambda$$

Similarly for r_n i.e. the radius of the nth zone, the relation can be written as

$$r_n^2\left[\frac{1}{a} + \frac{1}{b}\right] = n\lambda \qquad (17.8)$$

Applying the sign convention,

$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r^2} = \frac{1}{f_n} \tag{17.9}$$

or
$$f_n = \frac{r_n^2}{n\lambda}$$

Equation (17.9) is similar to the equation $\left(\frac{1}{\upsilon} - \frac{1}{u} = \frac{1}{f}\right)$ in the case of lenses with a and b as the object and image distances and f_n the focal length. Thus, a zone plate acts as a converging lens. A zone plate has a number of foci which depend on the number of zones used as well as the wavelength of light employed.

17.5.2. DIFFERENCE BETWEEN A ZONE PLATE AND A CONVEX LENS

For a given wavelength of light, a convex lens has only one focal length given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where f is the focal length of the lens, μ is the refractive index of the material of the lens and R_1 and R_2 are the radii of curvature. In a convex lens, the violet rays of light come to focus nearer the lens than the red rays of light because for a given material the refractive index for violet rays of light is more than for red rays of light.

In the case of a zone plate, there are a number of foci between the point O and P (Fig. 17.8). Each focus corresponds to the position where, with reference to P an odd number of half period elements can be constructed on each zone. As the screen is moved nearer the zone plate, the area of the half period elements decreases and more half period elements can be present on each zone. If P_m is the position on the image when $(2_m - 1)$ half period elements can be present on each zone, f_m the focal length of the zone plate is given by

$$f_m = \frac{r_n^2}{(2m - 1)n\lambda} \tag{17.10}$$

Putting m = 1, 2, 3.., etc., the different positions of the screen for a bright image can be obtained. In equation (17.10), r_n is the radius of the n^{th} zone of the wavefront, λ is the wavelength of light and (2m –1) is the number of odd half period elements present on each zone. For example, if the position of the screen is such that with reference to the point P, three half period elements can be constructed on each zone, then the focal length of the zone plate f_3 is given by

$$f_3 = \frac{r_n^2}{5n\lambda}$$

With the decrease in the focal length of the zone plate, the brightness of the image decreases. Let the first zone contain only one period element. Then, the amplitude at P due to this zone is m_1. If the first zone contains three half period elements for a particular position of the screen, then the amplitude at P due to the first zone

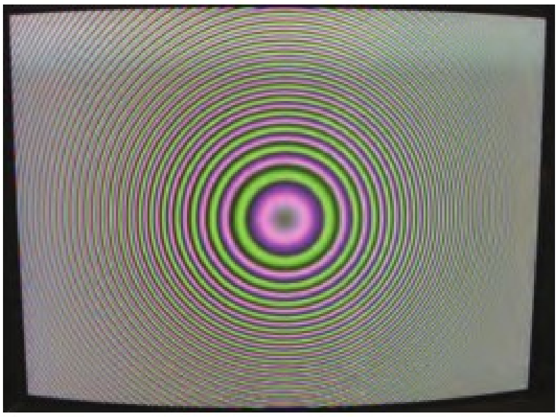
$$= m_1 - m_2 + m_3 = \frac{m_1}{2} + \left[\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right] + \frac{m_3}{2} = \frac{m_1}{2} + \frac{m_2}{2}$$

But, $\frac{m_1 + m_3}{2}$ is less than m_1, because $m_1 > m_3$. Further, in a zone plate (for the same number of odd half period elements contained in each zone) the focal length for violet light is more than for red light, which is reverse in the case of a convex lens.

Comparison between a zone plate and a convex lens:

1. Both the zone plate and convex lens form a real image of the object and the equations connecting the conjugate distances are similar.

2. The focal lengths of both depend on the wavelength, λ and hence suffer from chromatic aberration. The chromatic aberration in a zone plate is much more severe than in a convex lens.




Comparison between a zone plate and a convex lens.

3. A zone plate acts simultaneously as a convex lens and as a concave lens. In addition to a real image, a virtual image is also formed simultaneously.
   A convex lens forms only a real image.
4. In case of zone plate the image is formed by the diffraction phenomenon.
   In case of a convex lens the image is formed due to refraction of light.
5. The zone plate has got multiple foci on either side of the plate. Hence, the intensity of the image formed will be much less.
   Convex lens has only one focus. As all the light is focused at one point, the intensity of the image will be more.
6. In a zone plate, waves reaching the image point through any two alternate zones differ in path by λ and in phase by 2π.
   In case of a convex lens all the rays reaching the image point have zero path or phase difference.
7. A zone plate can be used over a wide range of wavelengths from microwaves to x-rays.
   Glass lens cannot be used beyond the visible region.

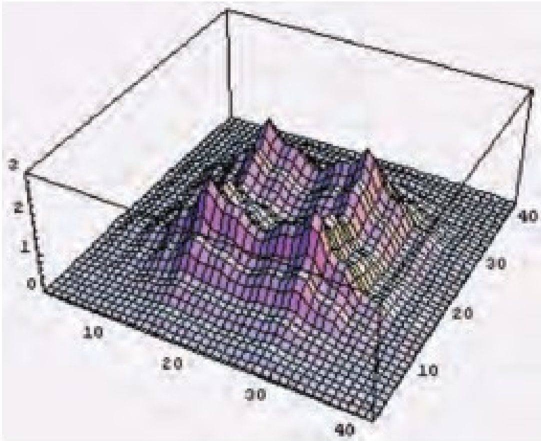
17.6. DISTINCTION BETWEEN INTERFERENCE AND DIFFRACTION

The main differences between interference and diffraction are as follows:

INTERFERENCE	DIFFRACTION
1. Interference is the result of interaction of light coming from different wave fronts originating from the source.	1. Diffraction is the result of interaction of light coming from different parts of the same wavefront.
2. Interference fringes may or may not be of the same width.	2. Diffraction fringes are not of the same width.
3. Regions of minimum intensity are perfectly dark.	3. Regions of minimum intensity are not perfectly dark.
4. All bright bands are of same intensity.	4. The different maxima are of varying intensities with maximum intensity for central maximum.

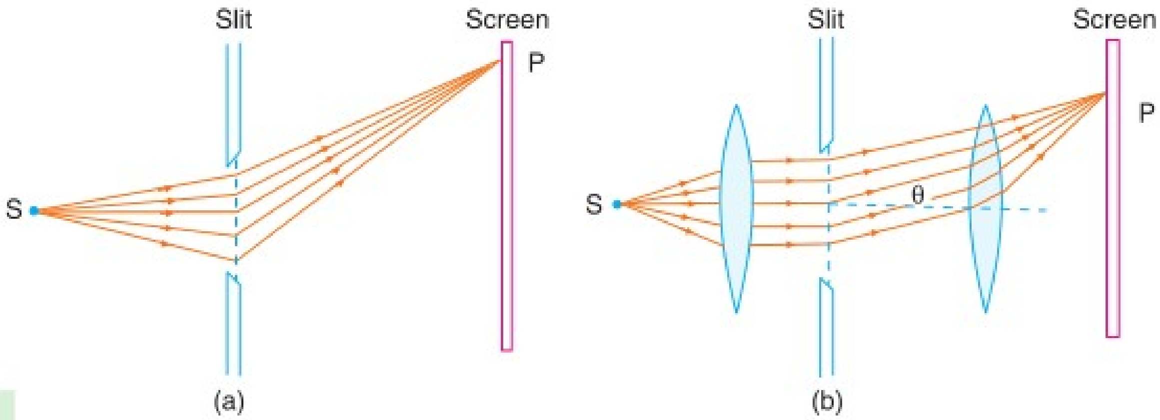
17.7. FRESNEL AND FRAUNHOFFER TYPES OF DIFFRACTION

The diffraction phenomena are broadly classified into two types: Fresnel diffraction and Fraunhoffer diffraction.



Surface of Fresnel diffraction.

1. Fresnel diffraction: In this type of diffraction, the source of light and the screen are effectively at finite distances from the obstacle (Fig. 17.9a). Observation of Fresnel diffraction phenomenon does not require any lenses. The incident wave front is not planar. As a result, the phase of secondary wavelets is not the same at all points in the plane of the obstacle. The resultant amplitude at any point of the screen is obtained by the mutual interference of secondary wavelets from different elements of unblocked portions of wave front. It is experimentally simple but the analysis proves to be very complex.



Conditions for Fresnel diffraction and Fraunhoffer diffraction.

Fig. 17.9

2. Fraunhoffer diffraction: In this type of diffraction, the source of light and the screen are effectively at infinite distances from the obstacle. Fraunhoffer diffraction pattern can be easily observed in practice. The conditions required for Fraunhoffer diffraction are achieved using two convex lenses, one to make the light from the source parallel and the other to focus the light after diffraction on to the screen (Fig. 17.9b). The diffraction is thus produced by the interference between parallel rays. The incident wave front as such is plane and the secondary wavelets, which originate from the unblocked portions of the wave front, are in the same phase at every point in the plane of the obstacle. This problem is simple to handle mathematically because the rays are parallel. The incoming light is rendered parallel with a lens and diffracted beam is focused on the screen with another lens.

Fresnel class of diffraction phenomenon is treated in this chapter.

17.8. DIFFRACTION AT A CIRCULAR APERTURE

Let AB be a small aperture (say a pin hole) and S is a point source of monochromatic light. XY is a screen perpendicular to the plane of the paper and P is a point on the screen. SP is perpendicular to the screen O is the center of the aperture and r is the radius of the aperture.

Let the distance of a source from the aperture be a (SO = a) and the distance of the screen from the aperture be b (OP=b). QOQ_1 is the incident spherical wavefront and with reference to the point P, O is the pole of the wavefront (Fig. 17.10). To consider the intensity at P, half period zones can be constructed with P as center and radii $b+\frac{\lambda}{2}, b+\frac{2\lambda}{2}$ etc. on the exposed wavefront AOB. Depending on the distance of P from the aperture (i.e. the distance b) the number of half period zones that can be constructed may be odd or even. If the distance a is such that only one half period zone can be constructed, then the intensity at P will be proportional to m_1^2 (where m_1 is the amplitude due to the first zone at P). On the other hand, if the whole of the wavefront is exposed to the point P, the resultant amplitude is $\frac{m_1}{2}$ or the intensity at P will be proportional to $\frac{m_1^2}{4}$. The position of the screen can be altered so as to construct 2, 3 or more half period zones for the same area of the aperture. If only two zones are exposed, the resultant amplitude at $P = m_1 - m_2$ (minimum) and if 3 zones are exposed, the amplitude = $m_1 - m_2 + m_3$ (maximum) and so on. Thus by continuously altering the value of b, the point P becomes alternately bright and dark depending on whether odd or even number of zones are exposed by the aperture.

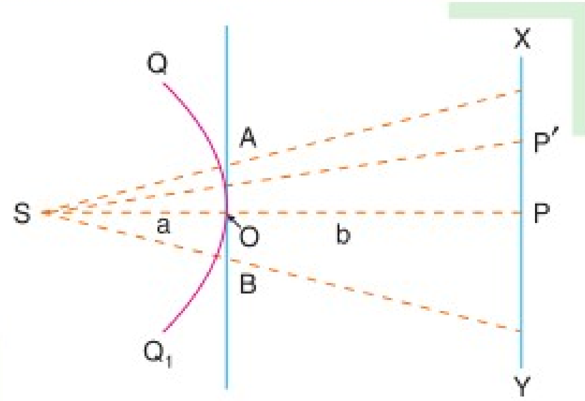


Fig. 17.10

Now let us consider a point P′ on the screen XY (Fig.17.11). Let S to P′ be joined. The line SP′ meets the wavefront at O′. O′ is the pole of the wavefront with reference to the point P′. Now let us construct half period zones with the point O′ as the pole of the wavefront. The upper half of the wavefront is cut off by the obstacle. If the first two zones are cut off by the obstacle between the points O′ and A and if only the 3rd, 4th and 5th zones are exposed by the aperture AOB, then intensity at P′ will be maximum. Thus if odd number of half period zones are exposed, point P′ will be of maximum intensity and if even number of zones are exposed, point P′ will be of minimum intensity. As the distance of P′ from P increases the intensity of maxima and minima gradually decreases. It is because with the point P′ far removed from P, the most effective central half period zones are cut off by the obstacle between the points O′ and A. With the outer zones the obliquity increases with reference to the point P′ and hence the intensity of maxima and minima also will be less. If the point P′ happens to be of maximum intensity, then all the points lying on a circle of radius PP′ on the screen also will be of maximum intensity. Thus with a circular aperture, the diffraction pattern will be concentric bright and dark rings with the centre P bright or dark depending on the distance b. The width of the rings continuously decreases.

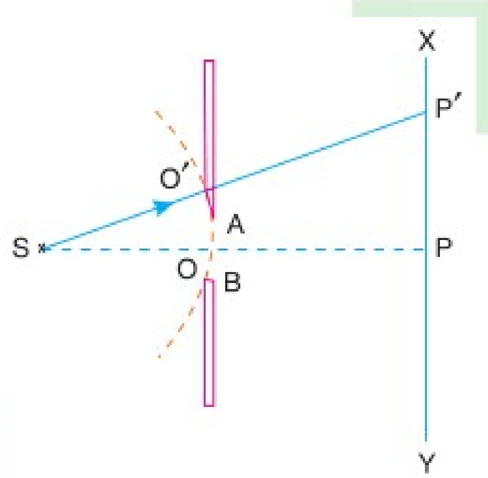


Fig. 17.11

17.8.1. MATHEMATICAL TREATMENT OF DIFFRACTION AT A CIRCULAR APERTURE

In Fig. 17.12 S is a point source of a monochromatic light, AB is the circular aperture and P is point on the screen. O is the center of the circular aperture. The line SOP is perpendicular to the circular aperture AB and the screen at P. The screen is perpendicular to the plane of the paper.

Let δ be the path difference for the wave reaching P along the paths SAP and SOP.

$$SO = a\ ;\ OP = b;\ OA = r$$

$$\delta = SA + AP - SOP$$

$$= \left(a^2+r^2\right)^{1/2} + \left(b^2+r^2\right)^{1/2} - (a+b) \quad = a\left(1+\frac{r^2}{a^2}\right)^{1/2} + b\left(1+\frac{r^2}{b^2}\right)^{1/2} - (a+b)$$

$$= a\left(1+\frac{r^2}{2a^2}\right) + b\left(1+\frac{r^2}{2b^2}\right) - (a+b) \quad = \frac{r^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right) \tag{17.11a}$$

$$\therefore \qquad \frac{1}{a}+\frac{1}{b} = \frac{2\delta}{r^2}$$

If the position of the screen is such that n full number of half period zones can be constructed on the aperture, then the path difference, $\delta = \frac{n\lambda}{2}$ or $2\delta = n\lambda$,

Substituting the value of 2δ in (17.11a), we get ,

$$\frac{1}{a}+\frac{1}{b} = \frac{n\lambda}{r^2} \tag{17.11b}$$

The point P will be of maximum or minimum intensity depending on whether n is odd or even. If the source is at infinite distance (for an incident plane wave front), then $a = \infty$ and

$$\frac{1}{b} = \frac{1}{f} = \frac{n\lambda}{r^2} \tag{17.12}$$

If n is odd, P will be a bright point. The idea of focus at P does not mean that it is always a bright point.

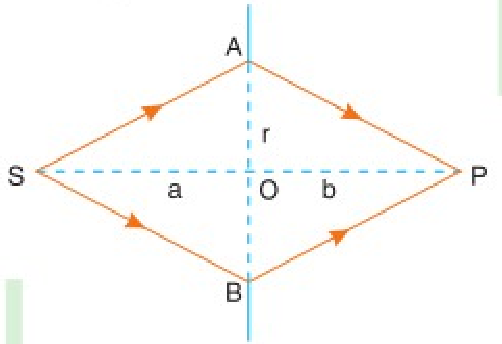


Fig. 17.12

17.8.2. INTENSITY AT A POINT AWAY FROM THE CENTRE

In Fig. 17.13 AB is a circular aperture and P and P′ are two points on the screen. PP′ = x and OP = b. OP is perpendicular to the screen.

Let r be the radius of the aperture. The path difference between the secondary waves from A and B and reaching P′ can be given by,

$$\delta = BP' - AP'$$

$$= \sqrt{b^2+(x+r)^2} - \sqrt{b^2+(x-r)^2} \quad = b\left(1+\frac{(x+r)^2}{2b^2}\right) - b\left(1+\frac{(x-r)^2}{2b^2}\right)$$

$$= \left(b+\frac{(x+r)^2}{2b}\right) - \left(b-\frac{(x-r)^2}{2b}\right) \quad = \frac{1}{2b}\left[(x+r)^2-(x-r)^2\right]$$

$$\therefore \qquad \delta = \frac{1}{2b}(4xr) = \frac{2rx}{b} \tag{17.13}$$

The point P′ will be dark if the path difference,

$\delta = 2n\frac{\lambda}{2}$ ($2n$ means even number of zones).

$$2n\frac{\lambda}{2} = \frac{2rx_n}{b}$$

or

$$x_n = \frac{nb\lambda}{2r} \tag{17.14}$$

where x_n gives the radius of n^{th} dark ring.

Similarly, if $\delta = \frac{(2n+1)\lambda}{2}$,

then $\frac{(2n+1)\lambda}{2} = \frac{2rx_n}{b}$

or $$x_n = \frac{(2n+1)\, b\,\lambda}{4r} \quad (17.15)$$

where x_n gives the radius of the n^{th} bright ring.

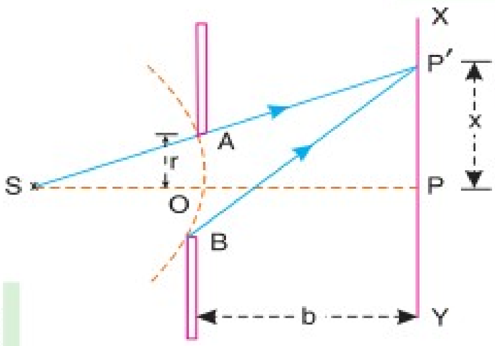


Fig. 17.13

The objective of a telescope consists of an achromatic convex lens and a circular aperture is fixed in front of the lens. Let the diameter of the aperture be D (= 2r). While viewing distant objects, the incident wave front is plane and the diffraction pattern consists of a bright centre surrounded by dark and bright rings of gradually decreasing intensity. The radii of the dark rings are given by

$$x_n = \frac{nb\lambda}{2r} = \frac{nb\lambda}{D} \quad (17.16)$$

The radius of the first dark ring is, $x_1 = \frac{b\lambda}{D}$

For an incident plane wavefront, $b = f$ the focal length of the objective.

$\therefore$ $$x_1 = \frac{f\lambda}{D}$$

The value of x_1 measures the distance of the first secondary minimum from the central bright maximum. However, according to Airy's theory, the radius of the first dark ring is given by

$$x_1 = \frac{1.22 f\lambda}{D} \quad (17.17)$$

It is interesting to note that the size of the central image depends on λ, the wavelength of light, *f*, the focal length of the lens and D, diameter of the lens aperture.

17.9. DIFFRACTION AT AN OPAQUE CIRCULAR DISC

S is a point source of monochromatic light. CD is an opaque disc and MN is the screen. P is a point on the screen such that SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. XY is the incident spherical wave front. EF is the geometrical shadow. With reference to the point P, the wave front can be divided into half period zones taking the centre of the disc (A) as the pole (Fig.17.14). If one half period zone can be constructed on the surface of the disc,

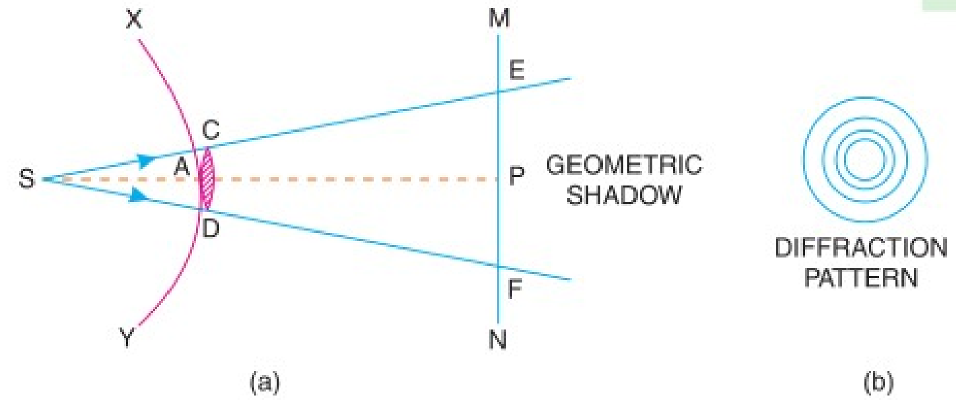


Fig. 17.14

the rest of the zones are exposed to the point P and the resultant amplitude at $P = \frac{m_2}{2}$ approximately, where m_2 is the amplitude due to the second zone $\left[m_2 - m_3 + m_4 - \ldots\ldots = \frac{m_2^2}{2} \text{ approximately}\right]$.

Similarly, if two half period zones can be constructed on the surface of the disc, the resultant amplitude at P due to the exposed zones will be $\frac{m_3}{2}$ and so on. Thus, the point P will always be bright but the intensity at P decreases with increase in the diameter of the disc. That is, with a large diameter of the disc, the most effective central zones will be cut off by the disc and the exposed outer zones are more oblique with reference to the point P. Thus (at P) the center of the geometrical shadow will be bright as if the disc were absent. The diffraction pattern consists of a central bright spot surrounded by alternate bright and dark rings, as shown in Fig. 17.14 (b).

17.10. DIFFRACTION PATTERN DUE TO A STRAIGHT EDGE

Let S be narrow slit illuminated by a source of monochromatic light of wavelength, λ. The length of the slit is perpendicular to the plane of the paper. AD is the straight edge and the length of the edge is parallel to the length of slit (Fig.17.15). XY is the incident cylindrical wavefront. P is a

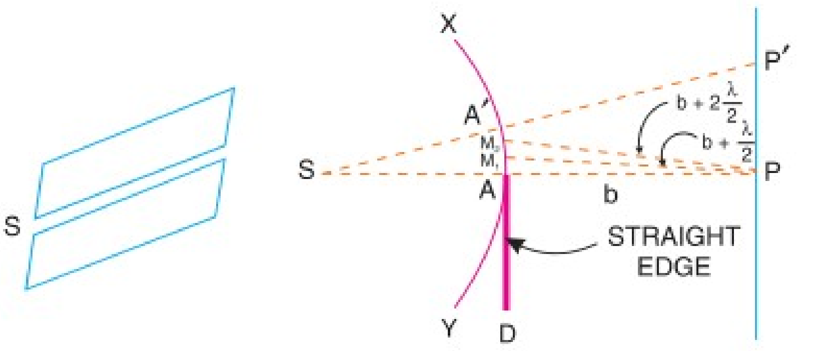


Fig. 17.15

point on the screen and SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. Below the point P is the geometrical shadow and above P is the illuminated portion. Let the distance AP be *b*. With reference to the point P, the wave front can be divided into a number of half period strips, as shown in Fig. 17.16. XY is the wave front, A is the pole of the wave front and AM_1, M_1M_2, M_2M_3 etc measure of the thickness of the 1st, 2nd, 3rd etc half period strips. With the increase in the order of the strip, the area of the strip decreases (Fig. 17.16).

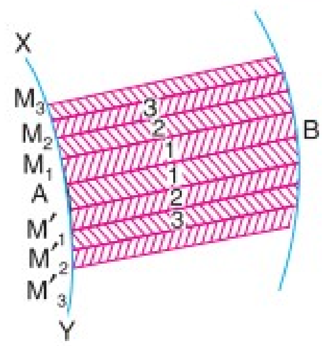


Fig. 17.16

In Fig.17.15, AP = b,

$$PM_1 = b + \frac{\lambda}{2} \quad \text{and} \quad PM_2 = b + \frac{2\lambda}{2} \text{ etc.}$$

Let P′ be a point on the screen in the illuminated portion (Fig. 17.17). To calculate the resultant effect at P′ due to the wave front XY, let us join S to P′. This line meets the wave front at B. B is the pole of the wave front with reference to the point P′ and the intensity at P′ will depend mainly on the number of half period strips enclosed between the points A and B. The effect at P′ due to the wave front above B is same at all points on the screen whereas it is different at different points due to the wave front between B and A. The point P′ will be of maximum intensity, if the number of half period strips enclosed between B and A is odd and the intensity at P′ will be minimum if the number of half period strips enclosed between B and A is even.

17.10.1. POSITIONS OF MAXIMUM AND MINIMUM INTENSITY

Let the distance between the slit and the straight edge be a and the distance between the straight edge and the screen be b (Fig. 17.17). Let PP′ be x.

The path difference, $\delta = AP' - BP'$

$$= (b^2 + x^2)^{1/2} - [SP' - SB]$$

$$= (b^2 + x^2)^{1/2} - \left(\sqrt{(a+b)^2 + x^2} - a\right)$$

$$= b\left[1 + \frac{x^2}{2b^2}\right] - (a+b)\left[1 + \frac{x^2}{2(a+b)^2}\right] + a$$

$$= \frac{x^2}{2}\left(\frac{1}{b} - \frac{1}{a+b}\right) = \frac{x^2}{2}\left(\frac{a+b-b}{b(a+b)}\right)$$

$$\therefore \quad \delta = \frac{x^2}{2} \cdot \frac{a}{b(a+b)}$$

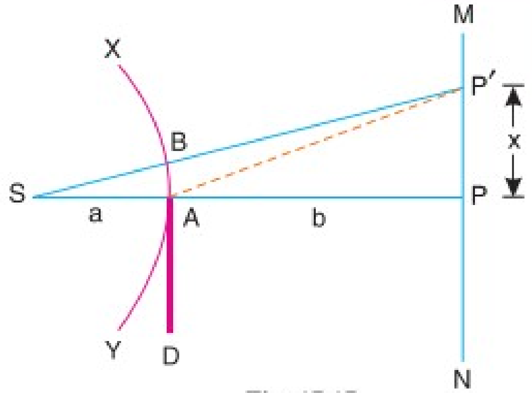


Fig. 17.17

The point P′ will be of maximum intensity if $\delta = (2n+1)\frac{\lambda}{2}$

$$\therefore \quad (2n+1)\frac{\lambda}{2} = \frac{ax_n^2}{2b(a+b)}$$

$$x_n^2 = \frac{(2n+1)(a+b)b\lambda}{a}$$

or

$$x_n = \sqrt{\frac{(2n+1)(a+b)b\lambda}{a}} \quad (17.18)$$

where x_n is the distance of the n^{th} bright band from P.

Similarly, P′ will be of minimum intensity if $\delta = 2n\frac{\lambda}{2}$.

$$\therefore \quad 2n\frac{\lambda}{2} = \frac{ax_n^2}{2b(a+b)} \quad \text{or} \quad x_n = \frac{\sqrt{2n(a+b)b\lambda}}{a}$$

where x_n is the distance of the n^{th} dark band from P. Thus, diffraction bands of varying intensity (roughly corresponding to maxima and minima) are observed above the geometrical shadow i.e., above P and the bands disappear and uniform illumination occurs if P′ is far away from P.

17.10.2. INTENSITY AT A POINT INSIDE THE GEOMETRICAL SHADOW (STRAIGHT EDGE)

If P′ is a point below P (Fig. 17.18) and B is the new pole of the wave front with reference to the point P′, then the half period strips below B are cut off by the obstacle and only the uncovered half period strips above B will be effective in producing the illumination at P′. As P′ moves farther from P, more number of half period strips above B is also cut off and the intensity gradually falls. Thus within the geometrical shadow, the intensity gradually falls off depending on the position of P′ with respect to P.

The intensity distribution on the screen due to a straight edge is shown in Fig. 17.19. S is the source, AD is the straight edge and MN is the screen. In the illuminated portion PM, alternate bright and dark bands of gradually diminishing intensity will be observed and the intensity falls off gradually in the region of the geometrical shadow. Thus according to the wave theory, the shadows cast by obstacles in the path of light are not sharp and hence rectilinear propagation of light is only approximately true. In general, there is gradual fading of intensity in the region of the geometrical shadow and with monochromatic light bright and dark bands (diffraction bands) are observed in the illuminated portion of the screen. However, with white light coloured bands will be observed and the bands of shorter wavelength are nearer the point P.

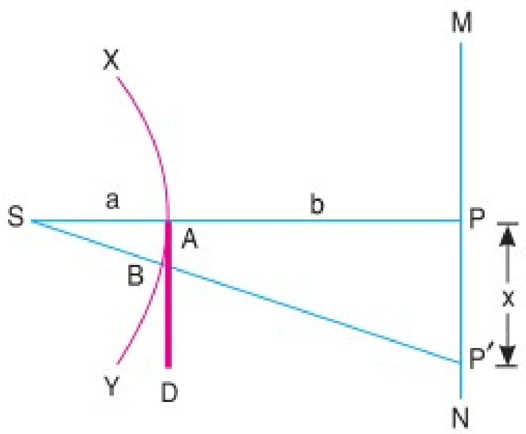


Fig. 17.18

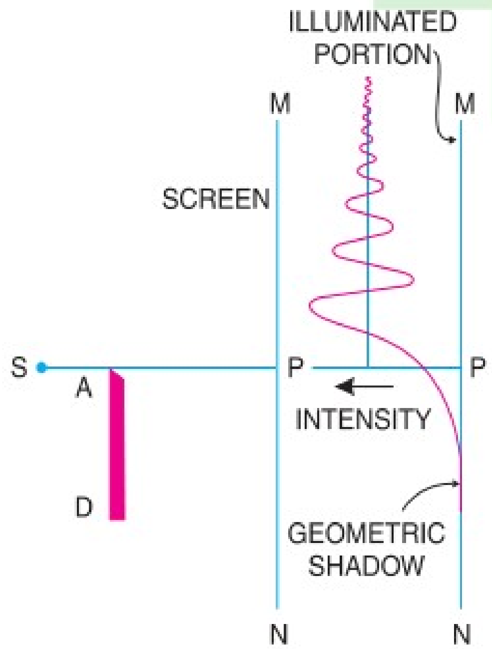


Fig. 17.19

17.11. DIFFRACTION PATTERN DUE TO A NARROW SLIT

S is a narrow slit illuminated by monochromatic light. The length of the slit is perpendicular to the plane of the paper. AB is a rectangular aperture parallel to the slit, MN is the screen and P is a point on the screen such that SOP is perpendicular to the plane of the paper; XY is the incident cylindrical wave front (Fig. 17.20). On the screen, EF is the illuminated portion and above E and below F is the region of the geometrical shadow.

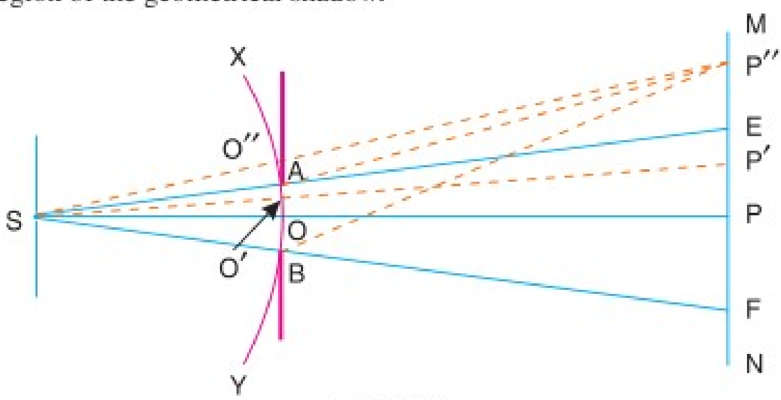


Fig. 17.20

If the slit AB is wide, then with reference to the point P, the cylindrical wave front can be divided into a large number of half period strips and the resultant amplitude at P will be $\frac{m_1}{2}$ where m_1 is the amplitude due to the first half period strip. Thus, the point P will be illuminated. Even points very near to P will be equally illuminated. If the wave front is divided with reference to points nearer P, the number of half period strips above and below the new pole in the exposed portion of the wavefront will be quite large and hence this results in uniform illumination.

Now, let us consider a point P′ nearer to the edge of the geometrical shadow (see Fig.17.20). Let us join S to P′. Here O′ is the pole of the wave front with reference to the point P′. If the wave front is divided into half period strips, the number of half period strips between O′ and B will be quite large and the illumination at P′ due to the lower portion of the wave front will be the same at all points near the edge of the geometrical shadow. But the intensity at P due to the exposed portion of the wave front between A and O′ will depend on the number of half period strips present. If the number of half period elements is odd, the point P′ will be of maximum intensity and if it is even the point will be of minimum intensity.

Let P″ be a point in the region of the geometrical shadow. Let us join S to P″. Here O″ is the pole of the wave front with reference to the point P″. If the wave front is divided into half period elements, then the upper half of the wave front between X and O″ is cut off by the obstacle and only a portion between A and B is exposed to the point P″. If the number of half period elements exposed by AB is odd, then P″ will be of maximum intensity and if it is even, it will be of minimum intensity. But as the most effective central half period strips between O″ and A are cut off, the intensity falls off rapidly in the region of the geometrical shadow and maxima and minima cannot be distinguished. The intensity distribution due to a wide aperture is shown in Fig. 17.21 (b).

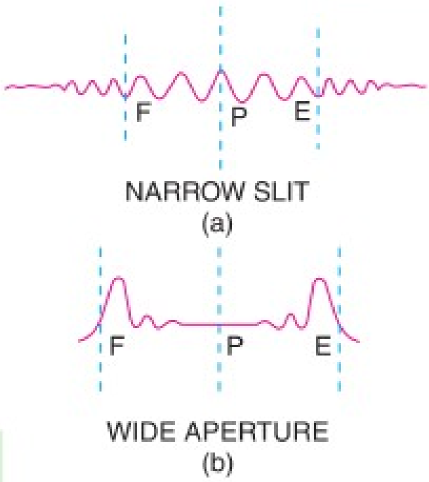


Fig. 17.21

On the other hand if the slit is narrow, the intensity at the point P will depend on the number of half period strips that can be constructed on the exposed wave front between A and B. If the number of half period strips is odd, the intensity at P will be maximum and if it is even, the intensity at P will be minimum (Fig. 17.20). Thus, the point P can be bright or dark. If we consider a point P′ in the illuminated portion EF of the screen, the intensity at P will depend on number of half period strips that can be constructed between A and O′ where O′ is the pole of the wave front with reference to the point P′. If the number of half periods strips between A and O′ is odd, P′ will be a point of maximum intensity. Thus, between E and F alternate bright and dark bands will be observed and the point P may be bright or dark.

Now consider a point P″ in the region of the geometrical shadow (Fig.17.20). O″ is the pole of the wavefront with reference to the point P″ and the intensity at P″ will depend on the number of half period strips exposed by the slit AB. The upper half of the wavefront above O″ is obstructed by the obstacle and even the most effective central half period strips between O″ and A are cut off by the obstacle. Thus, at P″ which is far away from E, the maxima and minima become indistinguishable. There is no marked transition between the diffraction bands observed in the geometrical shadow and the illuminated portion. In the intensity distribution on the screen due to a narrow slit (say less than the wavelength of light), a broad central maximum will be observed in the illuminated portion and the intensity variation cannot be distinguished. The intensity gradually falls off in the region of geometrical shadow.

17.12. DIFFRACTION DUE TO A NARROW WIRE

In Fig. 17.22, S is a narrow slit illuminated by monochromatic light, AB is the diameter of the narrow wire and MN is the screen. The length of the wire is parallel to the illuminated slit and perpendicular to the plane of the paper. The screen is also perpendicular to the plane of the paper. XY is the incident cylindrical wave front. P is a point on the screen such that SOP is perpendicular to the screen. EF is the region of the geometrical shadow and above E and below F, the screen is illuminated.

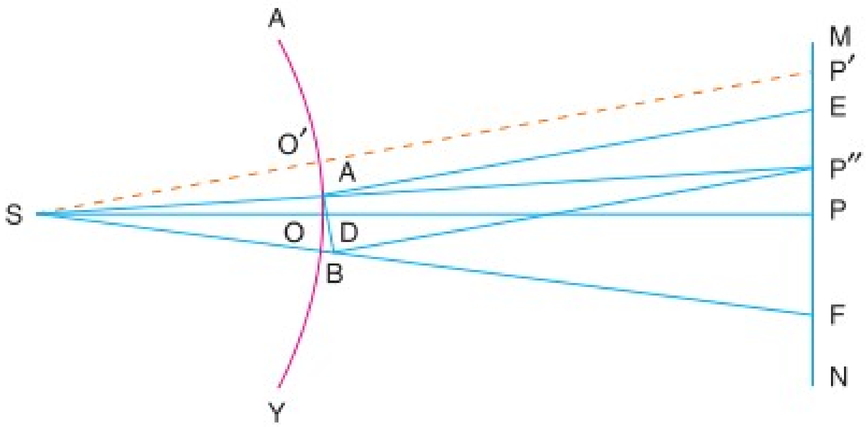


Fig. 17. 22

Now, let us consider a point P′ on the screen in the illuminated portion. Let us join S to O′, a point on the wave front. O′ is the pole of the wave front with reference to P′. The intensity P′ due to the wave front above O′ is the same at all points and the effect due to the wave front BY is negligible. The intensity at P′ will be a maximum or a minimum depending on whether the number of half period strips between O′ and A is odd or even. Thus, in the illuminated portion of the screen, diffraction bands of gradually diminishing intensity will be observed. The distinction between maxima and minima will become less if P′ is far away from the edge E of the geometrical shadow. Maxima and minima cannot be distinguished if the wire is very narrow, because in that case the portion BY of the wavefront also produces illumination at P.

Next let us consider a point P″ in the region of the geometrical shadow. Interference bands of equal width will be observed in this region due to the fact that the points A and B, of the incident wave front, are similar to two coherent sources. The point P″ will be of maximum or minimum intensity, depending on whether the path difference (BP″ – AP″) is equal to even or odd multiples of $\frac{\lambda}{2}$. The fringe width β is given by

$$\beta = \frac{D\lambda}{d}$$

where D is the distance between the wire and the screen, λ is the wave length of light and d is the distance between the two coherent sources. In this case $d = 2r$ where $2r$ is diameter of the wire ($AB = 2r$).

$$\therefore \qquad \beta = \frac{D\lambda}{2r} \qquad (17.19)$$

$$\therefore \qquad r = \frac{D\lambda}{2\beta} \qquad (17.20)$$

$$\text{or} \qquad \lambda = \frac{2r\beta}{D} \qquad (17.21)$$

Here, β the fringe width corresponds to the distance between any two consecutive maxima. Thus, from equations (17.20) and (17.21), knowing the values of *r* or λ ; λ or r can be determined. In Fig. 17.23 the bands marked "*a*" represent the interference bands in the region of the geometrical shadow and the bands marked "*b*" and "c" represent the diffraction bands in the illuminated portion. The intensity distribution due to a narrow wire is shown in Fig. 17.24 (*a*). The center of the geometrical shadow is bright.

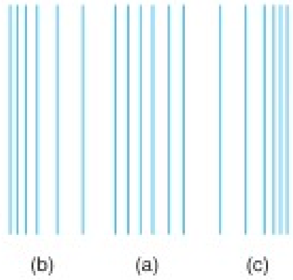


Fig. 17.23

On the other hand, if the wire is very thick, the interference bands cannot be noticed. From equation (17.19), $\beta = \frac{D\lambda}{2r}$; where β is the fringe width. As the diameter of the wire increases the fringe width decreases and if the wire is sufficiently thick, the width of the interference fringes decreases considerably and they cannot be distinguished. The intensity falls off rapidly in the geometrical shadow. The diffraction pattern in the illuminated portion will be similar to that of a thin wire Fig. 17.24(b). Coloured fringes will be observed with white light.

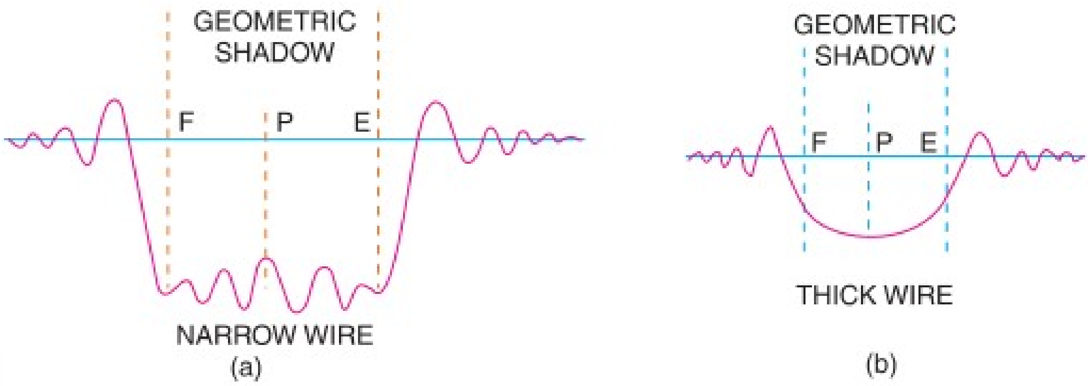
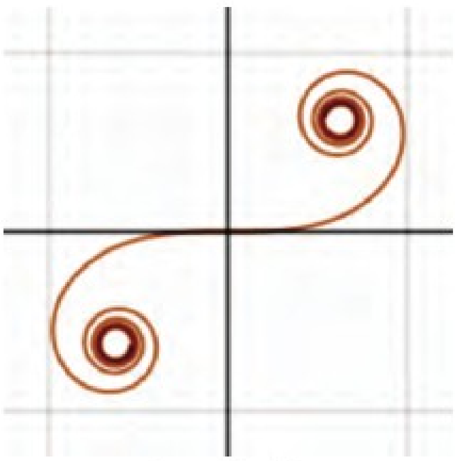


Fig. 17.24

17.13. CORNU'S SPIRAL

To find the effect at a point due to an incident wave front Fresnel's method consists in dividing the wavefront into half period strips or half period zones. The path difference between secondary



Cornu's Spiral

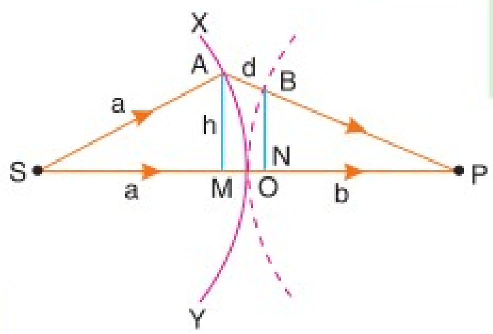


Fig. 17.25

waves from two corresponding points of neighboring zones is equal to $\frac{\lambda}{2}$.

In Fig. 17.25, S is a point source of light and XY is the incident spherical wave front. With reference to the point P, O is the pole of the wave front. Let a and b be the distances of the points S and P from the pole of the wave front. With P as centre and radius b, let us draw a sphere touching the incident wavefront at O. The path difference between the waves travelling in the directions SAP and SOP is given by

$$d = SA + AP - SOP = SA + AP - (SO + OP) = a + AB + b - (a + b) = AB$$

For large distances of a and b, AM and BN can be taken to be approximately equal and the path difference d can be written as

$$d = AB = MO + ON$$

But, from the property of a circle,

$$MO = \frac{AM^2}{2SO} = \frac{h^2}{2a} \text{ and } ON = \frac{BN^2}{2OP} = \frac{h^2}{2b} \qquad \text{(approximately)}$$

$$\therefore \qquad d = \frac{h^2}{2a} + \frac{h^2}{2b} = \frac{h^2}{2ab}(a+b) \qquad (17.22)$$

If AM happens to be the radius of the n^{th} half period zone, then this path difference is equal to $\frac{n\lambda}{2}$ according to the Fresnel's method of constructing the half period zones.

$$\therefore \qquad \frac{h^2}{2ab}(a+b) = \frac{n\lambda}{2} \qquad (17.23)$$

The resultant amplitude at an external point due to the wave front can be obtained by the following method. Let the first half period strip of the Fresnel's zones be divided into eight sub- strips and these vectors are represented from O to M_1 (Fig. 17.26). The continuous phase change is due to the continuous increase in the obliquity factor from O to M_1. The resultant amplitude at the external point due to the first half period strip is given by OM_1 (= m_1). Similarly, if the process is continued, we obtain the vibration curve M_1M_2. The portion M_1M_2 corresponds to the second half period strip.

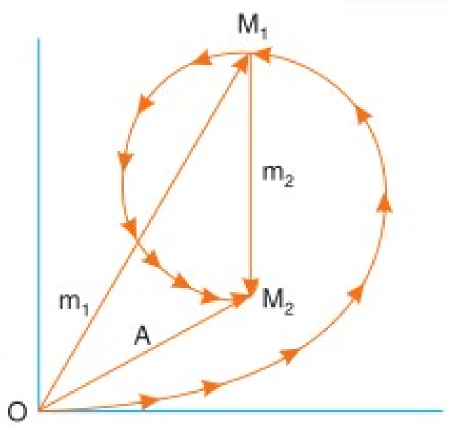


Fig. 17.26

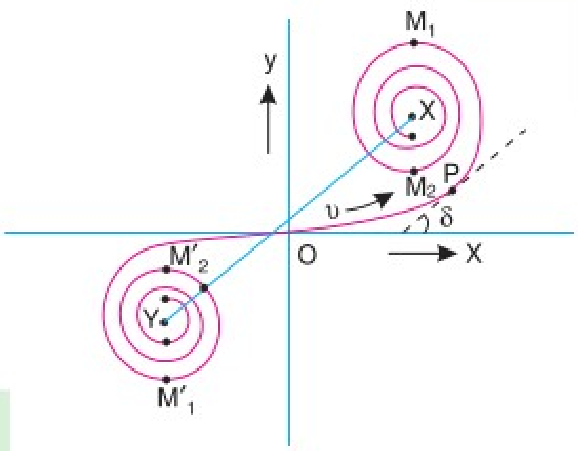


Fig. 17.27

The resultant amplitude at the point due to the first two half period strips is given by OM_2 (= A). If instead of eight sub-strips, each period zone is divided into sub-strips of infinitesimal width, a smooth curve will be obtained. The complete vibration curve for whole wave front will be a spiral as shown in Fig. 17.27. X and Y correspond to the two extremities of the wave front and M_1 and M_2

etc refer to the edge of the first, second, etc. half period strips. Similarly $M_1{}'$ and $M_2{}'$ etc refer to the edge of the first, second, etc half period strips of the lower portion of the wave front. This is called **Cornu's spiral**. The characteristic of this curve is that for any point *P* on the curve, the phase lag δ is directly proportional to the square of the distance υ. The distance is measured along the length of the curve from the point *O*. For a path difference of λ, the phase difference 2π. Hence, for a path difference of *d*, the phase difference δ is given by,

$$\delta = \frac{2\pi}{\lambda} d.$$

Substituting the value of *d* from equation (17.22) we get,

$$\delta = \frac{\pi}{2}\left[\frac{2h^2(a+b)}{ab\lambda}\right] \qquad (17.24)$$

$$\delta = \frac{\pi}{2}\upsilon^2 \qquad (17.25)$$

The value of υ is given by

$$\upsilon^2 = \frac{2h^2(a+b)}{ab\lambda}$$

or

$$\upsilon = h\sqrt{\frac{2(a+b)}{ab\lambda}} \qquad (17.26)$$

Cornu's spiral can be used for any diffraction problem irrespective of the values of *a*, *b* and λ.

17.13.1. FRESNEL'S INTEGRALS

For any point on the Cornu's spiral, the *x* and *y* co-ordinates are given by two integrals known as *Fresnel's integrals*. Let us consider the point P on the spiral (Fig 17.27). The distance of the point P along the curve from the origin is υ. The tangent to the curve at P makes an angle δ with the *x*-axis. δ corresponds to the phase change from O to P. For a small displacement $d\upsilon$ of the point along the curve, let the corresponding changes in the co-ordinates of the point be *dx* and *dy*.

Then, $dx = d\upsilon\ \cos\delta$ and $dy = d\upsilon\ \sin\delta$

Substituting the value of δ from equation (17.25) of §17.13, we get

$$dx = \cos\left(\frac{\pi\upsilon^2}{2}\right)d\upsilon \qquad (17.27)$$

and

$$dy = \sin\left(\frac{\pi\upsilon^2}{2}\right)d\upsilon \qquad (17.28)$$

The coordinates x and y of the Cornu's spiral are given by,

$$x = \int dx = \int_0^{\upsilon} \cos\left(\frac{\pi\upsilon^2}{2}\right)d\upsilon \qquad (17.29)$$

and

$$y = \int dy = \int_0^{\upsilon} \sin\left(\frac{\pi\upsilon^2}{2}\right)d\upsilon \qquad (17.30)$$

These are called **Fresnel's integrals.**

17.13.2. MAXIMA AND MINIMA IN DIFFRACTION PATTERNS (CORNU'S SPIRAL)

The various diffraction patterns as discussed in the earlier articles and the positions of maxima and minima can be easily explained with the help of Cornu's spiral.

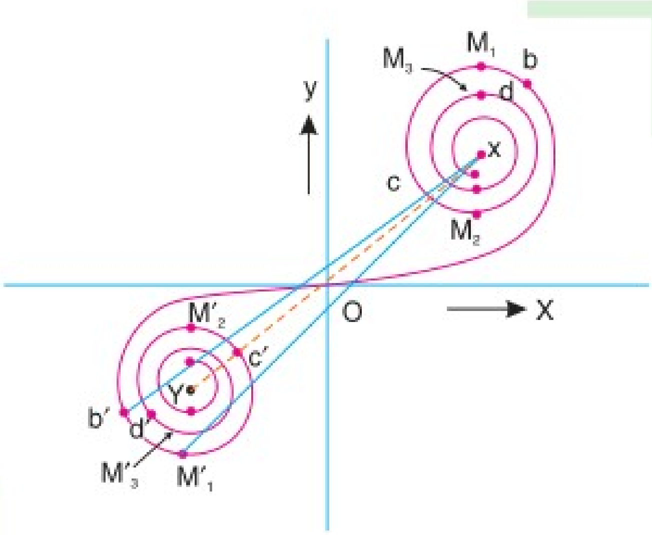


Fig. 17.28

In Fig. 17.28, O is the origin of co-ordinates, OX is the vibration curve for the upper half of the wave front and OY refers to the vibration curve for the lower half of the wave front. If the whole wave front is unobstructed, the resultant amplitude at a point is given by XY.

If a cylindrical wave front is incident on a straight edge, the amplitude at a point P on the edge of the geometrical shadow (refer to the discussion on diffraction at a straight edge) is given by XY. If points above P in the illuminated portion are considered, gradually more of the lower half of the wave front is also exposed to the screen and amplitude vector passes through maxima and minima. Xb′, Xd′ etc refer to maximum amplitudes and Xc′ refers to the minimum amplitude. Thus, in the illuminated portion alternate bright and dark bands parallel to the length of the slit are observed on the screen. If points below P and in the region of (the geometrical shadow are considered) the lower half of the wave front and a portion of the upper half of the wave front are cut off and the tail of the amplitude vector moves to the right of O. The amplitude gradually decreases and becomes zero when the tail approaches X. Thus, in the region of the geometrical shadow the intensity falls off gradually. Quantitative values of intensity for different points on the screen can be obtained by finding the amplitude A for different values of υ. The square of the amplitude measures the intensity at that point. The point M_1, M_2 etc correspond to the edges of the first, second etc half period strips in the upper half of the wave front and the points M_1', M_2' etc refer to the lower half of the wave front. The points b′, d′ etc on the spiral, corresponding to maximum intensity, occur a little before the points M_1', M_3' etc are reached.

The coordinates x and y of the Cornu's spiral are given by

$$x = \int dx = \int_0^{\upsilon} \cos\left(\frac{\pi \upsilon^2}{2}\right) d\upsilon \qquad (17.31)$$

$$y = \int dy = \int_0^{\upsilon} \sin\left(\frac{\pi \upsilon^2}{2}\right) d\upsilon \qquad (17.32)$$

The values of these integrals can be calculated for different values of υ. The graph is as shown in Fig. 17.28. The two integrals represent the horizontal and vertical components of the resultant amplitude. The intensity is proportional to the square of the resultant amplitude.

$$I_p = k\left[x^2 + y^2\right] \qquad (17.33)$$

When the whole of the wave front is exposed to the point, $\upsilon \to \infty$, the values of the integrals will be,

$$x = \int_0^{\infty} \cos\left(\frac{\pi \upsilon^2}{2}\right) d\upsilon = \frac{1}{2}$$

$$y = \int_0^{\infty} \sin\left(\frac{\pi \upsilon^2}{2}\right) d\upsilon = \frac{1}{2}$$

Thus, for the point X in Fig 17.28, the x and y co-ordinates are [1/2, 1/2]. Similarly, for the point Y on the lower half of the spiral, the coordinates are [–1/2, –1/2].

At the origin, i.e. when $\upsilon = 0$, $x = 0$ and $y = 0$, the spiral passes through the origin and it is symmetrical with the origin. At any point on the spiral, the tangent to the curve makes an angle ϕ with the x-axis and $\tan\phi = \frac{dy}{dx}$.

$$\tan = \phi \frac{\sin\left(\frac{\pi\upsilon^2}{2}\right)d\upsilon}{\cos\left(\frac{\pi\upsilon^2}{2}\right)d\upsilon} = \tan\left(\frac{\pi\upsilon^2}{2}\right)$$

or
$$\phi = \left(\frac{\pi\upsilon^2}{2}\right) \tag{17.34}$$

When $\upsilon = 0$, $\phi = 0$.

It means the curve is parallel to the x-axis at the origin.

The element of length $d\upsilon$ along the spiral is given by,

$$d\upsilon = \sqrt{(dx)^2 + (dy)^2} \tag{17.35}$$

Differentiating equation (17.34), we get

$$d\phi = \frac{2\pi\upsilon}{2}d\upsilon \quad \text{or} \quad \frac{d\upsilon}{d\phi} = \frac{1}{\pi\upsilon} \tag{17.36}$$

Hence $\frac{d\upsilon}{d\phi}$ measures the radius of curvature of the spiral at the point under consideration.

From equation (17.36), $\frac{d\upsilon}{d\phi} \propto \frac{1}{\upsilon}$

It shows that with the increase in the value of υ, the radius of curvature of the curve gradually decreases and takes the shape of a spiral. Finally for $\upsilon \to \infty$, the curve ends in a point (X or Y).

17.14. CORNU'S SPIRAL (ALTERNATIVE METHOD)

A narrow slit illuminated by light gives rise to a cylindrical wave front. Let S be a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light of wavelength λ (Fig 17.29).

For the cylindrical wave front XY, the slit is the axis of the cylindrical surface. The effect of the wave front at a point P in the plane is the same as that at all points along a line passing through the point P and parallel to the length of the slit. Let y be the displacement at all points on the wavefront XY at time t such that

$$y = a\sin\omega t = a\sin\frac{2\pi t}{T}$$

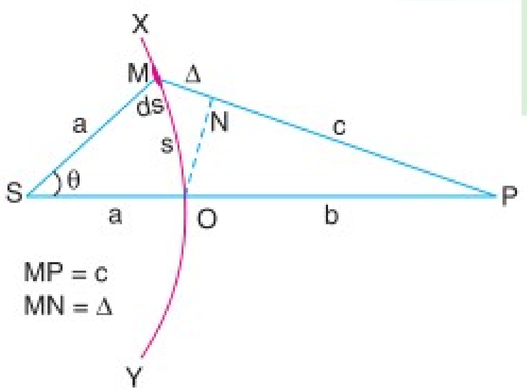


Fig. 17.29

According to Huygens principle, every point on the primary wave front is a source of secondary disturbance and the resultant intensity at P can be obtained by combining the effect of all the secondary wavelets. Let us consider a small element ds of the wave front at M. The distance of the element measured along the curve is s. For the secondary disturbances from O to P, i.e., for a distance OP = b, the phase difference is $\frac{2\pi b}{\lambda}$. The disturbance at the point P due to a small element ds is given by

$$dy = K \sin 2\pi \left[\frac{t}{T} - \frac{b}{\lambda}\right] ds \tag{17.37}$$

Similarly, the disturbance at P due to a small element at M is given by

$$dy = K \sin 2\pi \left[\frac{t}{T} - \frac{c}{\lambda}\right] ds \tag{17.38}$$

where MP = $c = b + \Delta$

$$dy = K \sin 2\pi \left[\frac{t}{T} - \frac{b}{\lambda} - \frac{\Delta}{\lambda}\right] ds \tag{17.39}$$

$$dy = K \sin \left[2\pi\left(\frac{t}{T} - \frac{b}{\lambda}\right) - \frac{2\pi\Delta}{\lambda}\right] ds$$

or $$dy = K\left[\sin 2\pi\left(\frac{t}{T} - \frac{b}{\lambda}\right)\cos\frac{2\pi\Delta}{\lambda}\right] ds - K\left[\cos 2\pi\left(\frac{t}{T} - \frac{b}{\lambda}\right)\sin\frac{2\pi\Delta}{\lambda}\right] ds \tag{17.40}$$

Integrating the above equation between the limits O and s, we get

$$y = K \sin 2\pi\left(\frac{\mathrm{t}}{\mathrm{T}} - \frac{b}{\lambda}\right)\int_0^s \cos\frac{2\pi\Delta}{\lambda}\, ds - K\cos 2\pi\left(\frac{\mathrm{t}}{\mathrm{T}} - \frac{b}{\lambda}\right)\int_0^s \sin\frac{2\pi\Delta}{\lambda}\, ds \tag{17.41}$$

Let $$K\int_0^s \cos\frac{2\pi\Delta}{\lambda} ds = R\cos\phi \quad \text{and} \quad K\int_0^s \sin\frac{2\pi\Delta}{\lambda} ds = R\sin\phi$$

Substituting these values in equation (17.41), we obtain

$$y = R \sin 2\pi\left(\frac{t}{T} - \frac{b}{\lambda}\right)\cos\phi - R\cos 2\pi\left(\frac{t}{T} - \frac{b}{\lambda}\right)\sin\phi$$

$$= R\sin\left[2\pi\left(\frac{t}{T} - \frac{b}{\lambda}\right) - \phi\right]$$

Also $R^2 = R^2\sin^2\phi + R^2\cos^2\phi$

$$= K^2\left[\left\{\int_0^s \sin\frac{2\pi\Delta}{\lambda} ds\right\}^2 + \left\{\int_0^s \cos\frac{2\pi\Delta}{\lambda} ds\right\}^2\right]$$

From the ΔSMP

$$c^2 = (a+b)^2 + a^2 - 2a(a+b)\cos\theta$$

$$= (a+b)^2 + a^2 - 2a(a+b)\left[1 - \frac{\theta^2}{2\,!}\right] = b^2 + a(a+b)\theta^2$$

But $\theta = \frac{s}{a}$ $\quad \therefore\ c^2 = b^2 + \frac{a(a+b)s^2}{a^2} = b^2 + \frac{(a+b)}{a}s^2$

$\therefore$ $$c = b\left[1+\frac{(a+b)}{a}\frac{s^2}{b^2}\right]^{1/2} = b+\frac{(a+b)}{2ab}s^2$$

$$MP - NP = \Delta = c - b$$

$$= b+\frac{(a+b)}{2ab}s^2 - b$$

or $$\Delta = \frac{(a+b)s^2}{2ab} \quad (17.42)$$

Let $\frac{2\pi\,\Delta}{\lambda} = \frac{\pi\,\upsilon^2}{2}$, where υ is a new variable depending on the values of a, b, λ and s. Substituting the value of Δ from equ. (17.42), we obtain

$$\frac{2\pi}{\lambda}\left[\frac{(a+b)s^2}{2ab}\right] = \frac{\pi\,\upsilon^2}{2}$$

or $$s^2 = \upsilon^2\left[\frac{ab\lambda}{2(a+b)}\right]$$

$\therefore$ $$s = \upsilon\sqrt{\frac{ab\lambda}{2(a+b)}} \quad (17.43)$$

or $$\upsilon = s\sqrt{\frac{2(a+b)}{ab\lambda}} \quad (17.44)$$

From equation (17.43),

$$ds = d\upsilon\sqrt{\frac{ab\lambda}{2(a+b)}}$$

Substituting the value of ds and $\frac{2\pi\,\Delta}{\lambda}$ in the following relation

$$R^2 = K^2\left[\left\{\int_0^s \cos\frac{2\pi\,\Delta}{\lambda}\,ds\right\}^2 + \left\{\int_0^s \sin\frac{2\pi\,\Delta}{\lambda}\right\}^2\right]$$

we get

$$R^2 = \frac{K^2 ab\lambda}{2(a+b)}\left[\left(\int_0^\upsilon \cos\frac{\pi\,\upsilon^2}{2}\,d\upsilon\right)^2 + \left(\int_0^\upsilon \sin\frac{\pi\,\upsilon^2}{2}\,d\upsilon\right)^2\right]$$

or $$R^2 = K_1\left[X^2 + Y^2\right]$$

where $X = \int_0^\upsilon \cos\frac{\pi\,\upsilon^2}{2}\,d\upsilon$ and $Y = \int_0^\upsilon \sin\frac{\pi\,\upsilon^2}{2}\,d\upsilon$

$$\int_0^\upsilon \cos\frac{\pi\,\upsilon^2}{2}\,d\upsilon = M\cos\left(\frac{\pi\,\upsilon^2}{2}\right) + N\sin\left(\frac{\pi\,\upsilon^2}{2}\right)$$

and $$\int_0^\upsilon \sin\left(\frac{\pi\,\upsilon^2}{2}\right)d\upsilon = M\sin\left(\frac{\pi\,\upsilon^2}{2}\right) - N\sin\left(\frac{\pi\,\upsilon^2}{2}\right)$$

Here $$M = \frac{\pi^{o}\upsilon}{1} - \frac{\pi^{2}\upsilon^{5}}{1.3.5} + \frac{\pi^{4}\upsilon^{9}}{1.3.5.7.9} \quad \text{and} \quad N = \frac{\pi\,\upsilon^{3}}{1.3} - \frac{\pi^{3}\upsilon^{7}}{1.3.5.7} + \frac{\pi^{5}\upsilon^{11}}{1.3.5.7.9.11}$$

For a given value of s (the extent of the wavefront contributing for the intensity at a point on the screen) the corresponding value of υ is evaluated from the relation,

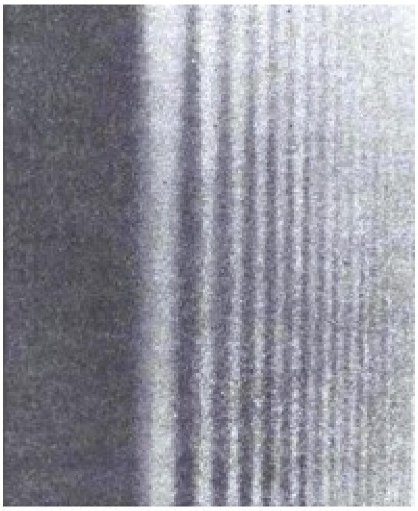
$$\upsilon = \sqrt{\frac{2(a+b)}{ab\lambda}}.$$

From the value of υ thus obtained, the values of X and Y corresponding to the X and Y coordinates of a point on the spiral are obtained from Fresnel's integrals. For the properties of Cornu's spiral refer to §17.13.2. Cornu's spiral is useful in understanding the Fresnel's diffraction patterns due to obstacles such as straight edge, thin wire, thick wire, narrow slit, wide aperture etc placed in the path of light.

Application of Cornu's spiral to Fresnel's diffraction at a straight edge is dealt in §17.15. Proceeding in the same way, the intensity distribution on the screen due to Fresnel's diffraction at a narrow wire, narrow slit, thick wire, wide aperture etc can also be obtained.

17.15. DIFFRACTION AT A STRAIGHT EDGE

S is a narrow slit illuminated by monochromatic light of wavelength λ. The length of the slit is perpendicular to the plane of the paper. *AD* is a straight edge and the length of the edge is parallel length of the slit (Fig.17.30). *XY* is the incident cylindrical wave front. Cornu's spiral helps to obtain qualitatively the intensity distribution at a point on the screen.



Diffract spatially filtered beam from 60-mW helium-neon laser around edge of razor blade mounting on rotating stage.

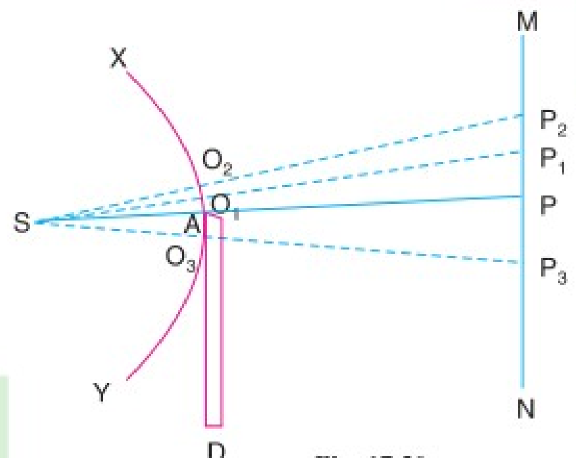


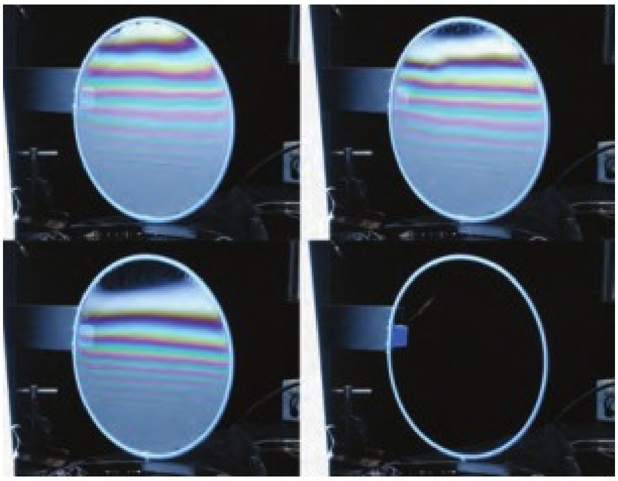
Fig. 17.30

Let us consider the points P, P_1 and P_2 in the illuminated portion and point P_3 in the geometrical shadow region of the screen MN. A, O_1, O_2 and O_3 are the poles of the wave front for the points P, P_1, P_2 and P_3 respectively.

Intensity at the point P:

The pole of the wave front is the origin of coordinates. The lower half of the wave front is cut off by the obstacle and the intensity at P is due to the upper half of the wave front between A and X. This intensity is equal to $I_o/4$, where I_o is the intensity due to the whole wave front (see Fig.17.31).

15 CHAPTER



Interference In Thin Films

15.1. THIN FILM

An optical medium is called a **thin film** when its thickness is about the order of 1 wavelength of light in visible region. Thus, a film of thickness in the range 0.5 μm to 10 μm may be considered as a thin film. A thin film may be a thin sheet of transparent material such as glass, mica, an air film enclosed between two transparent plates or a soap bubble. When light is incident on such a film, a small part of it gets reflected from the top surface and a major part is transmitted into the film. Again, a small part of the transmitted component is reflected back into the film by the bottom surface and the rest of it emerges

Soap bubble.

At a Glance

out of the film. A small portion of the light thus gets reflected partially several times in succession within the film (see Fig. 15.1).

In transparent thin films, the two bounding surfaces strongly transmit light and only weakly reflect the incident light. Therefore, only the first reflection at the top surface and the first reflection at the bottom surface will be of appreciable strength. For example, if we consider a glass plate, having a refractive index 1.52, the reflectivity of the top surface is given by

$$r = \left[\frac{1.52-1}{1.52+1}\right]^2 = 0.042$$

It means that about 4% of the incident light is reflected by the top surface of the glass plate, while 96% of it is transmitted into the plate. Out of the light reaching the bottom surface, again 3.8% is reflected and 92% is transmitted out of the plate. Then, again out of the 3.8% of the light 0.15% is reflected at the inner boundary of the top surface and about 3.65% is transmitted out into the air. After two reflections, the intensity will become insignificantly small. At each reflection, the intensity and hence the *amplitude of light wave is divided* into a reflected component and a refracted component. The reflected and refracted components travel along different paths and subsequently overlap to produce interference. Therefore, the interference in thin films is called interference by division of amplitude. Newton and Robert Hooke first observed the thin film interference. However, Thomas Young gave the correct explanation of the phenomena. A thin film may be uniform or non-uniform in its structure. However, as long as its thickness lies within the specified limits, interference of light occurs.

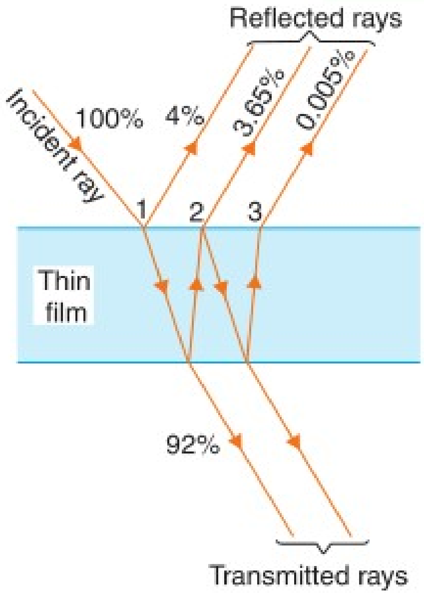


Fig. 15.1

15.2. PLANE PARALLEL FILM

A transparent thin film of uniform thickness bounded by two parallel surfaces is known as a *plane parallel thin film.*

When light is incident on a parallel thin film, a small portion of it gets reflected from the top surface and a major portion is transmitted into the film. Again, a small part of the transmitted component is reflected back into the film by the bottom surface and the rest of it is transmitted from the lower surface of the film. Thin films transmit incident light strongly and reflect only weakly. After two reflections, the intensities of reflected rays drop to a negligible strength. Therefore, we consider the first two reflected rays only (see Fig. 15.2). These two rays are derived from the same incident ray but appear to come from two sources located below the film. The sources are virtual coherent sources. The reflected waves 1 and 2 travel along parallel paths and interfere at infinity. This is a case of *two-beam* interference.

The condition for maxima and minima can be deduced once we have calculated the optical path difference between the two rays at the point of their meeting.

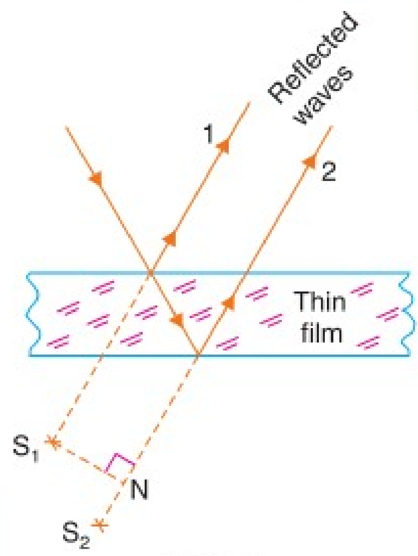


Fig. 15.2

15.2.1. INTERFERENCE DUE TO REFLECTED LIGHT

Let us consider a transparent film of uniform thickness 't' bounded by two parallel surfaces as shown in Fig. 15.3. Let the refractive index of the material be μ. The film is surrounded by air on both the sides. Let us consider plane waves from a monochromatic source falling on the thin film at an angle of incidence '*i*'. Part of a ray such as AB is reflected along BC, and part of it is transmitted

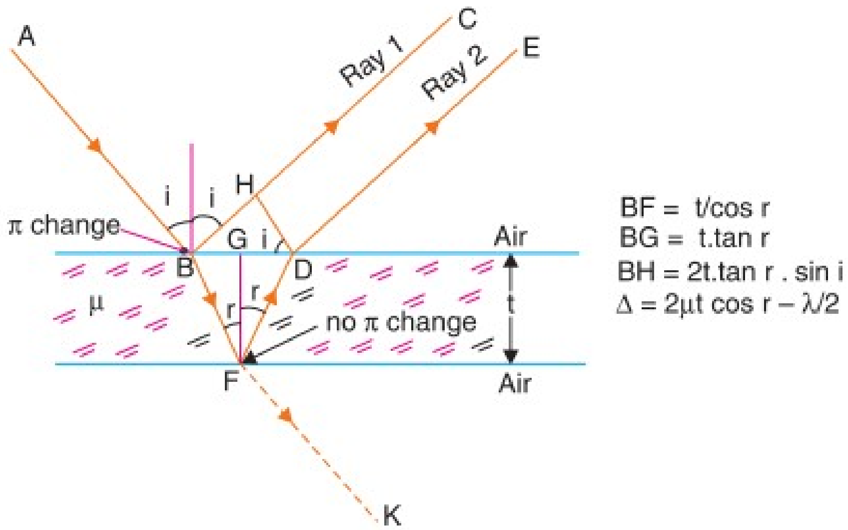


Fig. 15.3

into the film along BF. The transmitted ray BF makes an angle 'r' with the normal to the surface at the point G. The ray BF is in turn partly reflected back into the film along FD while a major part refracts into the surrounding medium along FK. Part of the reflected ray FD is transmitted at the upper surface and travels along DE. Since the film boundaries are parallel, the reflected rays BC and DE will be parallel to each other. The waves travelling along the paths BC and BFDE are derived from a single incident wave AB. Therefore they are coherent and can produce interference if they are made to overlap by a condensing lens or the eye.

(*i*) Geometrical Path Difference:

Let DH be normal to BC. From points H and D onwards, the rays HC and DE travel equal path. The ray BH travels in air while the ray BD travels in the film of refractive index μ along the path BF and FD. The geometric path difference between the two rays is

$$BF+FD-BH.$$

(*ii*) Optical Path Difference:

Optical path difference $\Delta_a = \mu L$

∴ $$\Delta_a = \mu\,(BF+FD) - 1(BH) \quad (15.1)$$

∴ In the ΔBFD, $\angle BFG = \angle GFD = \angle r$

$$BF = FD$$

$$BF = \frac{FG}{\cos r} = \frac{t}{\cos r}$$

∴ $$BF + FD = \frac{2t}{\cos r} \quad (15.2)$$

Also, $$BG = GD$$

∴ $$BD = 2BG$$

$$BG = FG \tan r = t \tan r$$

$\therefore$ $$BD = 2\,t\,\tan r$$

In the Δ^{le} BHD $\quad \angle HBD = (90 - i)$

$$\angle BHD = 90^0$$

$\therefore$ $$\angle BDH = i$$

$\therefore$ $$BH = BD \sin i = 2\,t \tan r \sin i \qquad (15.3)$$

From Snell's law,

$$\sin i = \mu \sin r$$

$\therefore$ $$BH = 2t \tan r\,(\mu \sin r) = \frac{2\mu\,t\,\sin^2 r}{\cos\,r} \qquad (15.4)$$

Using the equations (15.2) and (15.4) into eq.(15.1), we get

$$\Delta_a = \mu\left[\frac{2t}{\cos\,r}\right] - \left[\frac{2\mu\,t\,\sin^2\,r}{\cos\,r}\right]$$

$$= \frac{2\mu\,t}{\cos\,r}\left[1 - \sin^2\,r\right]$$

$$= \frac{2\mu\,t}{\cos\,r}\,\cos^2 r$$

$\therefore$ $$\Delta_a = 2\mu\,t\,\cos\,r \qquad (15.5)$$

(*iii*) Correction on account of phase change at reflection:

When a ray is reflected at the boundary of a rarer to denser medium, a path change of $\lambda/2$ occurs for the ray BC (see Fig.15.3). There is no path difference due to transmission at D. Including the change in path difference due to reflection, the true path difference

$$\Delta_t = 2\,\mu\,t \cos\,r - \frac{\lambda}{2} \qquad (15.6)$$

15.2.2. CONDITIONS FOR MAXIMA (BRIGHTNESS) AND MINIMA (DARKNESS)

Maxima occur when the optical path difference $\Delta = m\,\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves *interfere constructively*. Thus, when

$$2\mu\,t\,\cos\,r - \frac{\lambda}{2} = m\lambda \qquad (15.7)$$

the reflected rays undergo constructive interference to produce brightness or maxima at the point of their meeting.

$$2\mu\,t\,\cos\,r = m\lambda + \lambda/2$$

or $$2\mu\,t\,\cos\,r = (2m+1)\lambda\,/2$$ **Condition for Brightness** (15.8)

Minima occur when the optical path difference is $\Delta = (2m+1)\,\lambda\,/\,2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave falls on the troughs of the others and *the waves interfere destructively*. Thus, when

$$2\mu\,t\,\cos\,r - \lambda\,/2 = (2m+1)\,\lambda\,/2 \qquad (15.9)$$

the reflected rays undergo destructive interference to produce darkness. Equ.(15.9) may be rewritten as

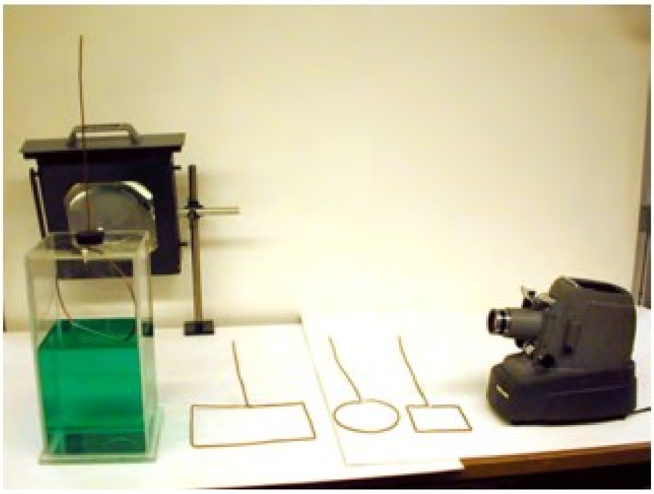
$$2\mu\ t\ \cos\ r\ = (m+1)\ \lambda$$

The phase relationship of the interfering waves does not change if one full wave is added to or subtracted from any of the interfering waves. Therefore (m + 1)λ can as well be replaced by mλ for simplicity in expression. Thus,

$$2\mu\ t\ \cos\ r = m\lambda \qquad \textbf{Condition for Darkness} \qquad (15.10)$$

15.2.3. SOME IMPORTANT POINTS

(a) It is seen that the conditions of interference depend on four parameters, namely μ, t, λ and r. In the case of constant thickness (parallel) film, (μt) is constant. When a parallel beam of light is incident on such a film, r also remains constant. Then the interference conditions solely depend on the wavelength λ.



Thin film interference -soap films.

(b) When monochromatic light falls on a parallel beam, the whole film will appear *uniformly* dark or bright. If the condition of constructive interference is satisfied, the film will show intense colour corresponding to the incident light.

(c) If a parallel beam of white light falls on a parallel film, those wavelengths for which the path difference is mλ, will be absent from the reflected light. The other colours will be reflected. Therefore, the film will appear uniformly coloured with one colour being absent.

15.2.4. NARROW LIGHT SOURCE VERSUS EXTENDED LIGHT SOURCE

In case of Fresnel's biprism and Lloyd's mirror, interference fringes were produced by two coherent sources. The initial source is narrow. The fringes obtained on a screen are viewed with an eyepiece. In case of a thin film, a narrow source limits the area of the film that can be viewed. Consider a thin film illuminated by a narrow source of light S (Fig. 15.4). The ray 1 produces

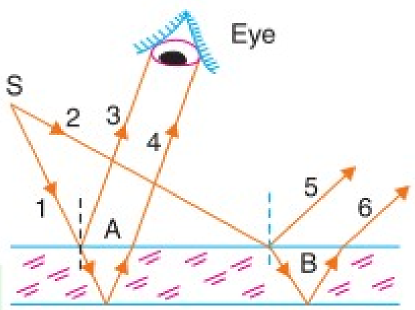


Fig. 15.4

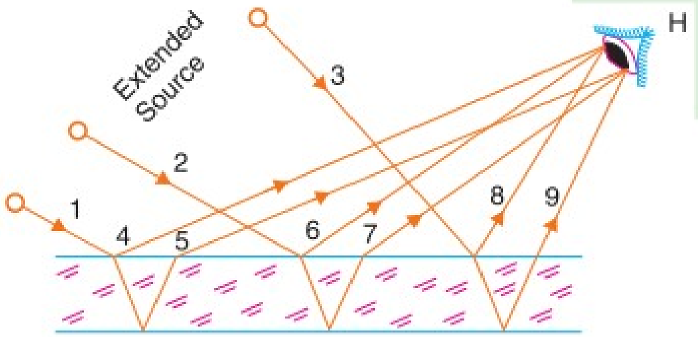


Fig. 15.5

interference fringes because rays 3 and 4 reach the eye. The ray 2 is incident on the surface of the film at a different angle and is reflected along 5 and 6. The rays 5 and 6 do not reach the eye. Similar is the case for other rays incident at different angles on the film surface. The reflected rays do not reach the eye. Thus, *only* the portion A of the film is visible to the eye.

If an extended (or broad) light source is used to illuminate the film, as in Fig.15.5, a larger area of the film surface is observed. The ray 1 after reflection from the upper and the lower surface of the film emerges as rays 4 and 5, which reach the eye. Ray 2 from some other point of the source after reflection from the upper and lower surfaces of the film emerge as rays 6 and 7 which also reach the eye. Also, ray 3 from some other point of the source after reflection from the upper and lower surfaces of the film emerge as rays 8 and 9 which also reach the eye. Therefore, the rays incident at different angles on the film are accommodated by the eye and the field of view is large. Therefore, a broad source of light is required to observe interference in thin films.

15.2.5. RESTRICTION ON THICKNESS OF THE FILM

We know that interference colours are observed only in thin films but not in thick plates such as windowpanes or glass slabs. This is due to the fact that light waves can interfere only when both

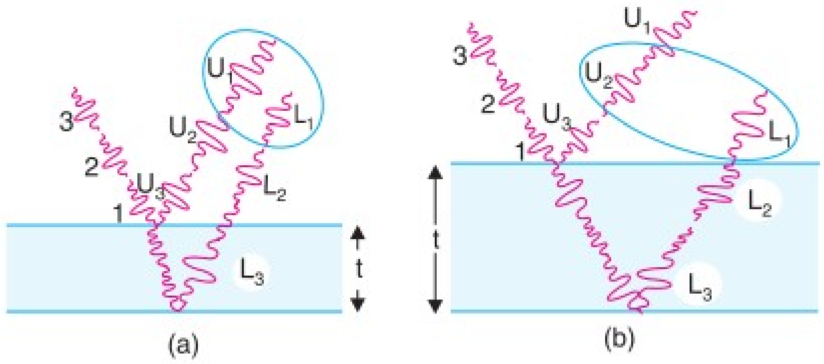
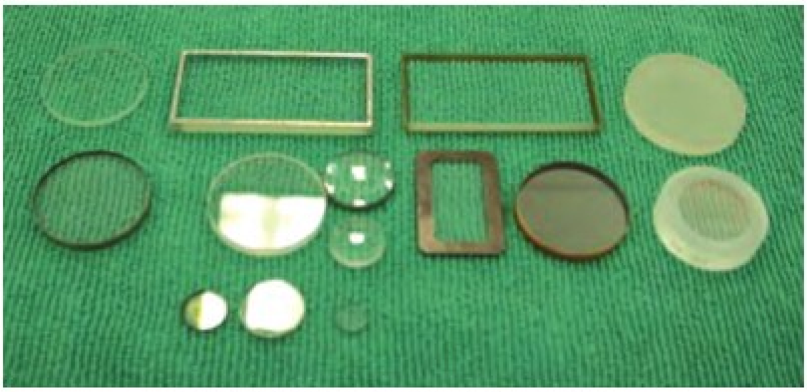


Fig. 15.6

the conditions of temporal and spatial coherence are satisfied. In Fig. 15.3 we have assumed that a monochromatic wave of infinite length is incident on the film. In reality, the incident light consists of wave trains of finite length and coherence extends over the length of each wave train only. Interference can occur only when parts of the same group of wave trains overlap. Superposition of different wave trains cannot produce interference because they will be incoherent and do not maintain any constant phase relationship with each other.



Thick films do not exhibit interference.

Fig. 15.6 shows the real situation. Wave trains 1,2,3 of finite length are incident in succession on a thin film. Portions of each wave train are reflected by the top and bottom surfaces of the film. Each wave train is divided into two reflected wave trains (U_1, L_1, U_2, L_2 and U_3, L_3). In Fig. 15.6 (a) the film is thin and the difference in the optical path lengths of U_1 and L_1 is small compared to the length of the wave train. Their superposition produces interference, as U_1 and L_1 are parts of the same wave train 1 and hence are coherent. In Fig.15.6 (b) the film is thicker and the optical path difference between U_1 and L_1 is large than the coherence length. Consequently, superposition takes place between parts of different wave trains, U_2 and L_1 and U_3 and L_2. Therefore interference does not take place.

It implies that interference occurs *only when* the optical path difference, Δ, between the superposing waves is less than the coherence length (see § 16.3).

i.e., $\Delta << l_{coh}$ (15.11)

∴ $(2\mu t \cos r - \lambda/2) << l_{coh}$ (15.12)

But $l_{coh} = \dfrac{\lambda^2}{\Delta\lambda}$ (Refer to equ. 16.11)

∴ $(2\mu t \cos r - \lambda/2) < \lambda^2/\Delta\lambda$

Rearranging the terms, we obtain

$$t < \frac{\lambda\left[\frac{\lambda}{\Delta\lambda} + \frac{1}{2}\right]}{2\mu \cos r} \quad (15.13)$$

$\lambda/\Delta\lambda >> 1/2$ and for normal incidence $\cos r = 1$.

$$\therefore \quad t < \frac{\lambda^2}{2\mu\Delta\lambda} \quad (15.14)$$

The above equation indicates that *interference in thin film will be observed if the thickness of the film is less than the coherence length of the incident light waves*. Normally, the coherence length of the light from ordinary sources is of the order of a fraction of a millimeter. Therefore, interference is seen with the films of thickness of the order of a few hundred microns only. It is because of this reason that thick films do not exhibit interference.

15.3. INTERFERENCE DUE TO TRANSMITTED LIGHT

Consider a thin transparent film of thickness t and refractive index μ. A ray SA after refraction goes along AB. At B it is partly reflected along BC and partly refracted along BR. The ray BC, after reflection at C, finally emerges along DQ. Here at B and C reflection takes place at the rarer medium. Therefore, no phase change occurs. Draw BM normal to CD and DN normal to BR. The optical path difference between DQ and BR is given by

$$\Delta = \mu(BC + CD) - BN$$

Also, $\mu = \dfrac{\sin i}{\sin r} = \dfrac{BN}{MD}$ or $BN = \mu.MD$

In Fig.15.7, $\angle BPC = r$ and CP = BC = CD

∴ BC + CD = PD

∴ Δ = μ(PD) − μ (MD) = μ(PD − MD) = μ.PM

In ΔBPM, $\cos r = \dfrac{PM}{BP}$ or PM = BP. $\cos r$

But BP = 2 t

∴ PM = 2 $t \cos r$

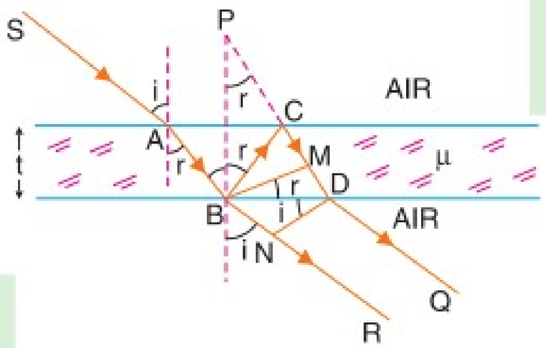


Fig. 15.7

∴ $\Delta = \mu.PM = 2\mu t \cos r$ (15.15)

Bright Fringes:

When the optical path difference $\Delta = m\lambda$, bright fringe occurs.

∴ $2\mu t \cos r = m\lambda$ (15.16)

where m = 0,1,2,3,….

Dark Fringes:

When the optical path difference $\Delta = (2m + 1) \lambda / 2$, dark fringe occurs.

$$\therefore \qquad 2\mu\ t\ \cos\ r = \frac{(2m+1)}{2}\lambda \qquad (15.17)$$

where m = 0,1,2,3,….

In case of transmitted light, the fringes are less distinct because the difference in amplitudes of BR and DQ is very large. However, when the angle of incidence is nearly 45° the fringes are more distinct.

15.4. HAIDINGER FRINGES

In thin films interference fringes are produced due to the path difference $2\mu\ t \cos r$ between the overlapping rays. For a given film the path difference may arise due to (*i*) the angle of refraction r inside the film or (*ii*) the change in thickness. We can express the change in path difference by differentiating the expression $2\ \mu\ t \cos r$.

$$\text{Change in path difference, } \delta(\Delta) = 2\mu\ t \cdot \delta(\cos\ r) + 2\mu\ \cos\ r\ (\delta\ t) \qquad (15.18)$$

When the film is of **uniform** (constant) thickness, the change in path difference is only due to the change in r. If the thickness of the film is large, the path difference will change appreciably

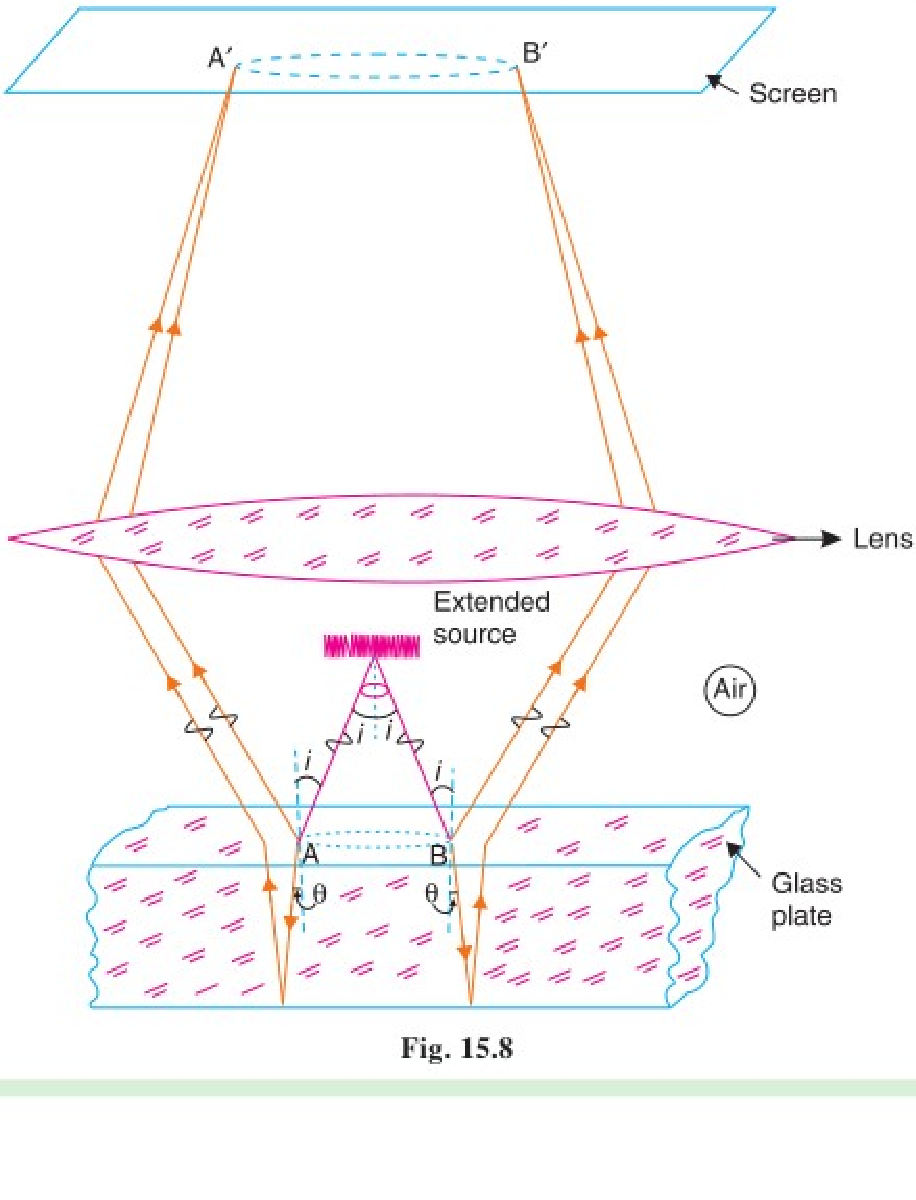


Fig. 15.8

even when *r* changes in a small way. Fringes are produced in this case due to the superposition of rays, which are equally inclined to the normal. These fringes are called **fringes of equal inclination**. The fringes of equal inclination are known as **Haidinger fringes.** In this case all the pairs of interfering rays of equal inclination pass through the plate as a parallel beam and hence meet at infinity. The other pairs of differrent inclination meet at different points at infinity. Therefore, they can be located with a telescope focussed to infinity. The fringes are therefore said to be *localized at infinity*.

To produce Haidinger fringes, the source must be an extended source, the film thickness must be appreciably large and the observing instrument is to be focussed for parallel rays.

Fig. 15.8 shows the formation of Haidinger fringes. Let us consider that a thin plate is illuminated by an extended monochromatic light source. A lens is arranged parallel to the plate and a screen is kept in the focal plane of the lens. Light from the extended source is incident on the plate in diverse directions. The waves propagating parallel to the plane of the page and falling on the plate at an angle *i* at points A and B get reflected from the top and bottom surfaces of the plate. The reflected pairs of waves will meet at points A′ and B′ respectively on the screen due to the focussing action of the lens. Depending on their path difference, the reflected waves produce either brightness or darkness on the screen. In fact the waves incident at the top surface of the plate at an angle *i* travel along the generators of a cone as shown in Fig. 15.8. Each pair of parallel reflected waves interfere at diametrically opposite points. Thus, a circular fringe is produced. Similarly, the waves incident at a different angle will produce a collection of identical points arranged along a circle of another radius. As a result, a system of alternating bright and dark circular fringes with a common centre will be observed on the screen.

Each fringe is characterized by a particular value of *m*. Bright fringes are produced when the condition $2 \mu t \cos r = m\lambda$ is satisfied; and dark fringes are produced where the condition

$2 \mu t \cos r = (2m + 1) \lambda / 2$ is satisfied. The parallel pairs of reflected rays meet only at infinity; therefore a lens is used to focus them. Accordingly, these fringes of equal inclination are said to be localized at infinity.

15.5. VARIABLE THICKNESS (WEDGE-SHAPED) FILM

Let us now study the interference of light in a film of varying thickness. *A thin film having zero thickness at one end and progressively increasing to a particular thickness at the other end is called a wedge.* A thin wedge of air film can be formed by two glass slides resting on each other at one edge and separated by a thin spacer at the opposite edge.

The arrangement for observing the interference pattern in a wedge shaped film is shown in Fig. 15.9(a). The wedge angle is usually very small and of the order of a fraction of a degree. When a parallel beam of *monochromatic* light illuminates the wedge from above, the rays reflected from its two bounding surfaces will not be parallel. They appear to diverge from a point near the film. The path difference between the rays reflected from the upper and lower surfaces of the air film varies along its length due to variation in film thickness. Therefore, alternate bright and dark fringes are observed on its top surface (see Fig. 15.9b). The fringes are localized at the top surface of the film.

When the light is incident on the wedge from above, it gets partly reflected from the glass-to-air boundary at the top of the air film. Part of the light is transmitted through the air film and gets reflected partly at the air-to-glass boundary, as shown in Fig. 15.10. The two rays BC and DE, thus reflected from the top and bottom of the air film, are coherent as they are derived from the same ray AB through *division of amplitude*. The rays are close enough if the thickness of the film is of the order of a wavelength of light. For small film thickness the rays interfere producing darkness or brightness depending on the phase difference. The thickness of the glass plates is large compared

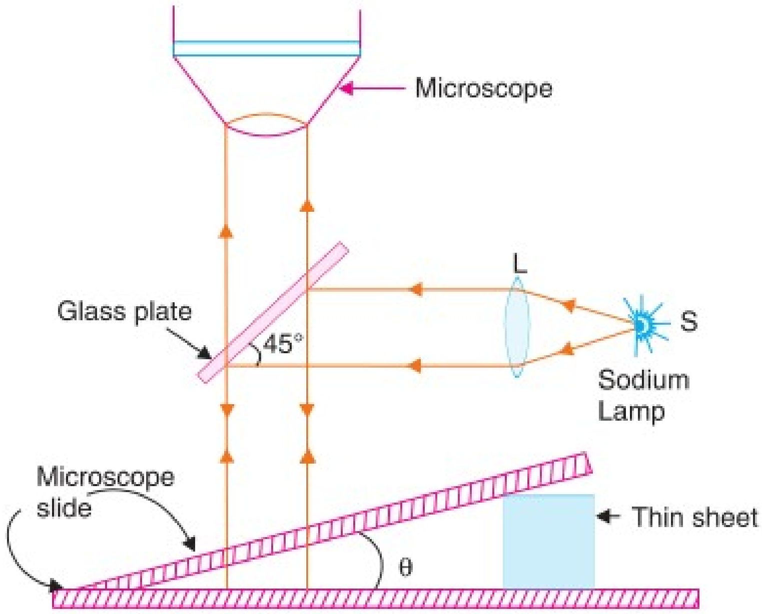


Fig. 15.9 (a)

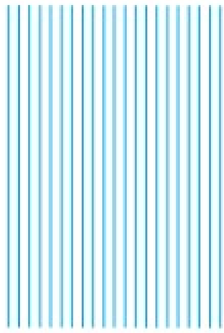


Fig. 15.9 (b)

with the wavelength of the incident light. Hence, the observed interference effects are entirely due to the wedge-shaped air film.

The optical difference between the two rays BC and DE is given by

$$\Delta = 2\mu\, t \cos r - \lambda/2$$

where $\lambda/2$ takes account the gain of half–wave due to the abrupt jump of π radians in the phase of the wave reflected from the bottom boundary of air – to – glass.

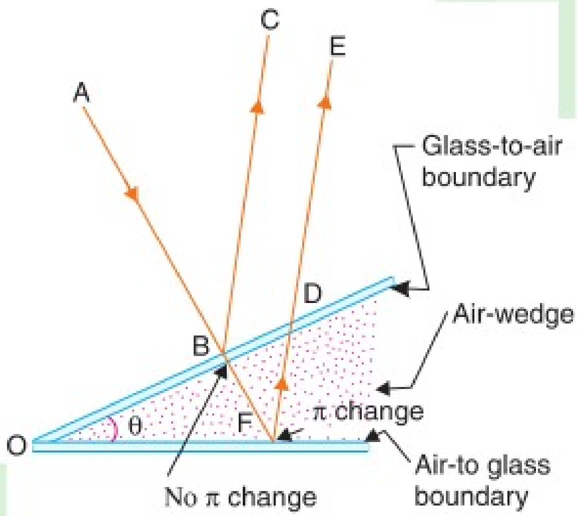


Fig. 15.10

Maxima occur when the optical path difference $\Delta = m\,\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves *interfere constructively*. This needs that

$$\Delta = 2\mu\, t \cos r - \lambda/2$$

Minima occur when the optical path difference is $\Delta = (2\,m + 1)\,\lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave falls on the troughs of the others and *the waves interfere destructively*. It needs that

$$2\mu\, t \cos r = m\lambda.$$

Referring to Fig.15.11, let us say a dark fringe occurs at A where the relation

$$2\mu\, t \cos r = m\lambda$$

is satisfied. If normal incidence is assumed, $\cos r = 1$ and if the thickness of air film at A is denoted by t_1, then at A

$$2\mu\, t_1 = m\lambda \tag{15.19}$$

The next dark fringe will occur, say, at C where the thickness CL = t_2. Then at C

$$2\mu\, t_2 = (m+1)\,\lambda \tag{15.20}$$

Subtracting eq. (15.19) from eq. (15.20), we get

$$2\mu\,(t_2 - t_1) = \lambda \tag{15.21}$$

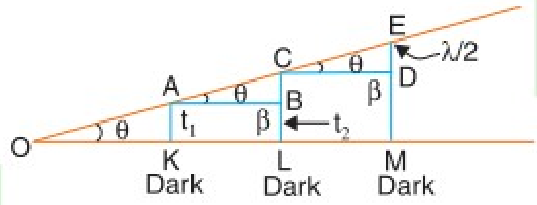


Fig. 15.11

But $\qquad (t_2 - t_1) = BC$

$\therefore \qquad 2\mu(BC) = \lambda$

or $$BC = \frac{\lambda}{2\mu} \tag{15.22}$$

From the Δ^{le}ABC, ∠CAB = θ and BC = AB tan θ

$$\therefore \qquad (AB)\ \tan\theta = \frac{\lambda}{2\mu} \tag{15.23}$$

AB is the distance between successive dark fringes and it also equals the separation of the successive bright fringes. It is, therefore, called the **fringe width,β**. That is AB = β. We may write eq. (15.23) as

$$\beta = \frac{\lambda}{2\mu\,\tan\theta} \tag{15.24}$$

For small values of θ, tan θ ≈ θ.

$$\therefore \qquad \beta = \frac{\lambda}{2\mu\theta} \tag{15.25}$$

As the quantities on the right side of the above equation are all constant, β is constant for a given wedge angle. According to equ.(15.25), an increase in the angle θ makes the fringes move closer. At an angle θ ≈ 1°, the interference pattern vanishes. On the other hand, if θ is gradually decreased, the fringe separation increases, and ultimately the fringes disappear as the faces of the film become parallel.

The interference pattern has the following salient features.

(*i*) Fringe at the apex is dark.

(*ii*) Fringes are straight and parallel.

(*iii*) Fringes are equidistant.

(*iv*) Fringes are localized.

(*v*) Fringes are of equal thickness.

(*i*) Fringe at the apex is dark: At the apex, the two glass slides are in contact with each other. Therefore, the thickness of the air film at the contact edge is negligible (t ≅ 0). The optical path difference there becomes

$$\Delta = 2\mu t - \lambda/2 = 0 - \lambda/2 = -\lambda/2 \tag{15.26}$$

It implies that a path difference of λ/2 or a phase difference of π occurs between the reflected waves at the edge. The two waves interference destructively. Therefore, the fringe at the apex is always dark (See Fig. 15.12).

(*ii*) Straight and parallel fringes: Each fringe in the pattern is produced by the interference of rays reflected from sections of the wedge having the same thickness. The locus of points having the same thickness lie along lines parallel to the contact edge. Therefore, the fringes are straight. Since the fringes are equidistant [see (iii)], they will be parallel (See Fig. 15.12).

***(iii)* Equidistant fringes:** The fringe width β is given by

$$\beta \approx \lambda/2\theta \qquad (15.27)$$

where λ is the wavelength of the incident light and θ is the angle of the wedge. As the quantities λ and θ are constants, β is constant for a given wedge angle. Therefore, the fringes are equidistant (see Fig.15.12).

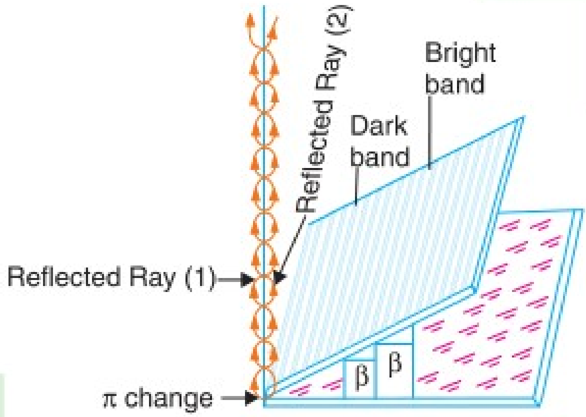


Fig. 15.12

***(iv)* Localized fringes:** The fringes form very close to the top surface of the wedge and can be seen with a microscope.

***(v)* Fringes of equal thickness:** In thin films of thickness of the order of a few λ, the rays from various parts of the film have almost the same inclination and hence the path difference between the overlapping waves changes mainly due to change of thickness. The fringes produced in such cases are mainly due to the variation in thickness of the film. Each fringe will be the locus of points of the same thickness. Such fringes are called **fringes of equal thickness**.

15.5.1. DETERMINATION OF THE WEDGE ANGLE

The wedge angle θ can be experimentally determined with the help of a travelling microscope. Using the microscope the positions of dark fringes at two distant points Q and R are noted (Fig. 15.13). Let the distance OQ be x_1 and OR be x_2. Let the thickness of the wedge be t_1 at Q and t_2 at R.

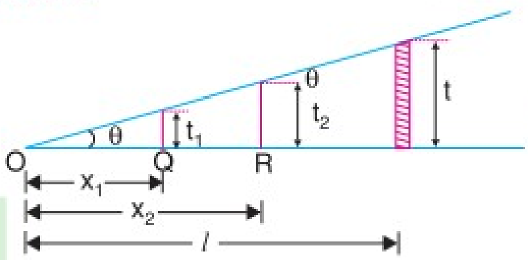


Fig. 15.13

The dark fringe at Q is given by

$$2\mu t_1 = m\lambda \qquad (15.28)$$

But as θ is very small, we can write

$$t_1 = x_1 \tan\theta \cong x_1\theta$$

$$\therefore \quad 2\mu\, x_1\, \theta = m\lambda \qquad (15.29)$$

We can write similarly for the dark fringe at R as

$$2\mu\, x_2\, \theta = (m+N)\lambda \qquad (15.30)$$

where N is the number of dark fringes lying between the positions Q and R. Subtracting equ.(15.29) from equ.(15.30), we get

$$2\mu(x_2 - x_1)\theta = N\lambda$$

$$\therefore \quad \theta = \frac{N\lambda}{2\mu(x_2 - x_1)} \qquad (15.31)$$

In case of air μ = 1 and the above relation reduces to

$$\theta = \frac{N\lambda}{2(x_2 - x_1)} \qquad (15.32)$$

15.5.2. DETERMINATION OF THE THICKNESS OF THE SPACER

The thickness of the spacer used to form the wedge shaped air film between the glass slides can be determined from the above measurements. If 't' is the thickness of the spacer (foil or wire) used, we can write from Fig.15.13 that

$$t = l \tan \theta \cong l\,\theta \qquad (15.33)$$

where l is the length of the air wedge. Using the equ.(15.32) into equ.(15.33), we obtain

$$\therefore \qquad t = \frac{lN\lambda}{2(x_2 - x_1)} \qquad (15.34)$$

15.5.3. FIZEAU FRINGES

If a parallel beam of light is incident perpendicularly or nearly perpendicular on a variable thickness film, then dark and bright fringes are seen in reflected light. These fringes are fringes of equal thickness, because each fringe corresponds to lines of equal optical thickness. These fringes or localized fringes and are observed at the top of the film. These localized fringes of equal thickness are known as *Fizeau fringes*. Contours following lines of equal optical thickness are seen if the area is large. The fringes may be obtained in case of thick films also if the source is small.

15.5.4. COLOURS IN THIN FILMS

Colours in thin films of oil.

The colours exhibited in reflection by thin films of oil, mica, soap bubbles and coatings of oxides on heated metals etc are due to interference of light from an extended source such as sky. Thomas Young explained the origin of colours in thin films. It may be understood as follows. The films are usually observed by reflected light. The eye looking at the thin film receives light waves reflected from the top and bottom surfaces of the film. The reflected rays are very close to each other and are in a position to interfere. The optical path difference between the interfering rays is $\Delta = 2\mu t \cos r - \lambda/2$. It is seen that the path difference depends upon the thickness t of the film, the wavelength λ and the angle r, which is related to the angle of incidence of light on the film. White light consists of a range of wavelengths and for specific values of t and r, waves of only certain wavelengths (colours) constructively interfere. Therefore, only those colours are present in the reflected light. The other wavelengths interfere destructively and hence are absent from the reflected light. Hence, the film at a particular point appears coloured. As the thickness and the angle of incidence vary from point to point, different colours are intensified at different places. The colours seen are not isolated colours, as at each place there is a mixture of colours. The composition of colours is different at different places and contours of impressive hues are observed over the entire surface of the film.

15.6. NEWTON'S RINGS

Newton's rings are an example of fringes of equal thickness. Newton's rings are formed when a plano-convex lens P of a large radius of curvature placed on a sheet of plane glass AB is illuminated from the top with monochromatic light (Fig. 15.14). The combination forms a thin circular air film of variable thickness in all directions around the point of contact of the lens and the glass plate. The locus of all points corresponding to specific thickness of air film falls on a circle whose centre is at O. Consequently, interference fringes are observed in the form of a series of concentric rings with their centre at O. Newton originally observed these concentric circular fringes and hence they are called **Newton's rings**.

The experimental arrangement for observing Newton's rings is shown in Fig. 15.14.

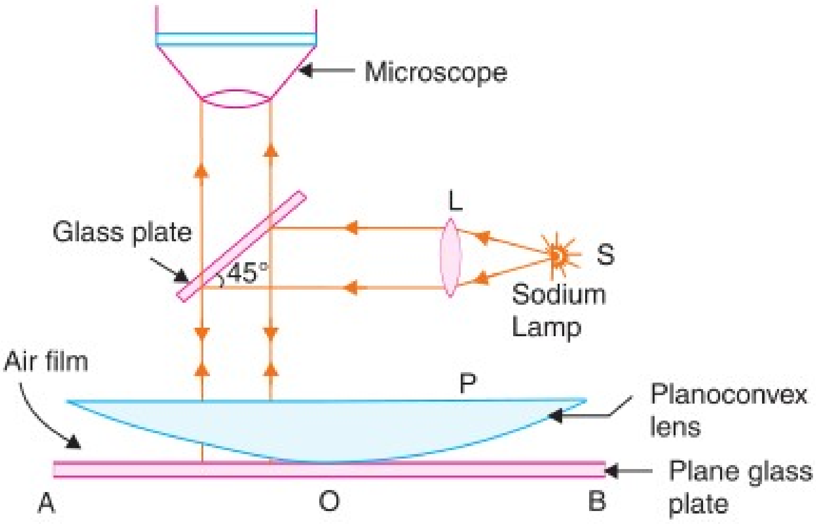


Fig. 15.14

Monochromatic light from an extended source S is rendered parallel by a lens L. It is incident on a glass plate inclined at 45° to the horizontal, and is reflected normally down onto a plano-convex lens placed on a flat glass plate. Part of the light incident on the system is reflected from the glass-to-air boundary, say from point D (Fig. 15.15). The remainder of the light is transmitted through the air film. It is again reflected from the air-to-glass boundary, say from point J. The two rays reflected from the top and bottom of the air film are derived through division of amplitude from the same incident ray CD and are therefore coherent. The rays 1 and 2 are close to each other and interfere to produce darkness or brightness. The condition of brightness or darkness depends on the path difference between the two reflected light rays, which in turn depends on the thickness of the air film at the point of incidence.

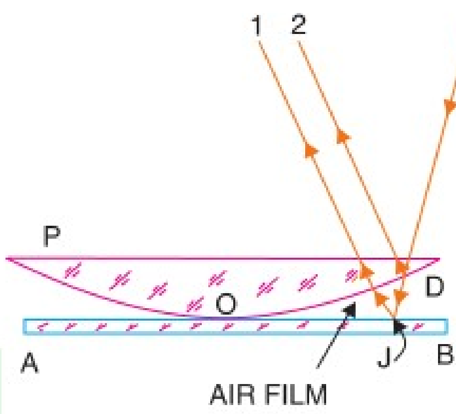


Fig. 15.15

15.6.1. CONDITION FOR BRIGHT AND DARK RINGS

The optical path difference between the rays is given by $\Delta = 2\mu t \cos r - \lambda/2$. Since $\mu = 1$ for air and $\cos r = 1$ for normal incidence of light,

$$\Delta = 2t - \lambda/2 \qquad (15.35)$$

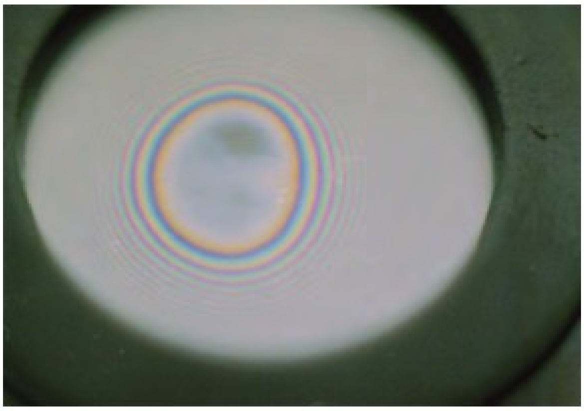
Intensity maxima occur when the optical path difference $\Delta = m\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the other and the waves *interfere constructively*. Thus, if $2t - \lambda/2 = m\lambda$

$$\boxed{2t = (2m+1)\lambda/2} \qquad (15.36)$$

bright fringe is obtained.

Intensity minima occur when the optical path difference is $\Delta = (2m + 1)\lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave fall on the troughs of the other and *the waves interfere destructively*.

Hence, if $\quad 2t - \lambda/2 = (2m + 1)\lambda/2$



Circular fringes.

$$\boxed{2t = m\lambda} \qquad (15.37)$$

dark fringe is produced.

15.6.2. CIRCULAR FRINGES

In Newton's ring arrangement, a thin air film is enclosed between a plano-convex lens and a glass plate. The thickness of the air film at the point of contact is zero and gradually increases as we move outward. The locus of points where the air film has the same thickness then fall on a circle whose centre is the point of contact. Thus, the thickness of air film is constant at points on any circle having the point of lens–glass plate contact as the centre. The fringes are therefore circular.

Fig. 15.16

15.6.3. RADII OF DARK FRINGES

Let R be the radius of curvature of the lens (Fig.15.17). Let a dark fringe be located at Q. Let the thickness of the air film at Q be PQ = t. Let the radius of the circular fringe at Q be OQ= r_m. By the Pythagorus theorem,

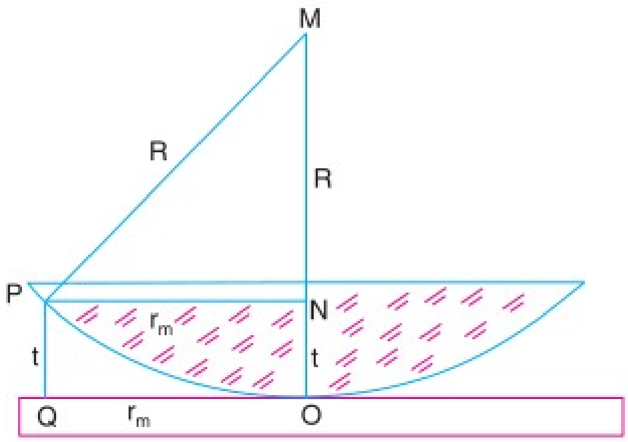


Fig. 15.17

$$PM^2 = PN^2 + MN^2$$

$\therefore \quad R^2 = r_m^2 + (R - t)^2$

or $\quad r_m^2 = 2Rt - t^2 \quad (15.38)$

As $\quad R >> t, \quad 2Rt >> t^2$.

$\therefore \quad r_m^2 \cong 2Rt \quad (15.39)$

The condition for darkness at Q is that

$$2t = m\lambda$$

$\therefore \quad r_m^2 \cong m\lambda R$

$$r_m = \sqrt{m\lambda R} \quad (15.40)$$

The radii of dark fringes can be found by inserting values 1,2,3, ...for m. Thus,

$$r_1 = \sqrt{1\lambda R} \quad \text{or} \quad r_1 \propto \sqrt{1}$$

$$r_2 = \sqrt{2\lambda R} \quad \text{or} \quad r_2 \propto \sqrt{2}$$

$$r_3 = \sqrt{3\lambda R} \quad \text{or} \quad r_3 \propto \sqrt{3} \quad \text{and so on}$$

It means that *the radii of the dark rings are proportional to under root of the natural numbers.* The above relation also implies that

$$r_m \propto \sqrt{\lambda}$$

Thus, *the radius of the m^{th} dark ring is proportional to under root of wavelength.*

Ring Diameter:

Diameter of m^{th} dark ring $D_m = 2r_m$

$$D_m = 2\sqrt{2Rt}$$

$$D_m = 2\sqrt{m\lambda R} \quad (15.41)$$

15.6.4. SPACING BETWEEN FRINGES

It is seen that the diameter of dark rings is given by

$$D_m = 2\sqrt{m\lambda R}$$

where m = 1,2,3, ...

The diameters of dark rings are proportional to the square root of the natural numbers. Therefore, the diameter of the ring does not increase in the same proportion as the order of the ring, for example, if m increases as 1,2,3,4, ...the diameters are

$$D_1 = 2\sqrt{\lambda R}$$

$$D_2 = 2\,(1.4)\sqrt{\lambda R}$$

$$D_3 = 2\,(1.7)\sqrt{\lambda R}$$

$$D_4 = 2\,(2)\sqrt{\lambda R} \quad \text{and so on.}$$

Therefore, the rings get closer and closer, as m increases. This is why the rings are not evenly spaced.

15.6.5. FRINGES OF EQUAL THICKNESS

Newton's rings are formed as result of interference between light waves reflected from the top and bottom surfaces of a thin air film enclosed between a plano-convex lens and a plane glass plate.

Newton's rings arrangement.

The occurrence of alternate bright and dark rings depends on the optical path difference arising between the reflected rays. If the light falls normally on the air film the optical path difference between the waves reflected from the two surfaces of the film is

$$\Delta = 2t - \lambda/2$$

It is seen that the path difference between the reflected rays arises due to the variation in the thickness 't' of the air film. Reflected light will be of minimum intensity for those thickness for which the path difference is m λ and maximum intensity for those thickness for which the path difference is $(2m+1)\lambda/2$. Thus, each maxima and minima is a locus of constant film thickness. Therefore, the fringes are known as fringes of equal thickness.

15.6.6. DARK CENTRAL SPOT

The central spot is dark as seen by reflection. Newton's rings are produced due to superposition of light rays reflected from the top and bottom surfaces of a thin air film enclosed between a plano-convex lens and a plane glass plate. The occurrence of brightness or darkness depends on the optical path difference arising between the reflected rays. The optical path difference is given by $\Delta = 2t - \lambda/2$.

At the point of contact 'O' of the lens and glass plate (Fig.15.18), the thickness of air film is negligibly small compared to a wavelength of light.

$$\therefore \qquad t \cong 0$$

$$\therefore \qquad \Delta \cong \lambda/2$$

The wave reflected from the lower surface of the air film suffers a phase change of π while the wave reflected from the upper surface of the film does not suffer such change.

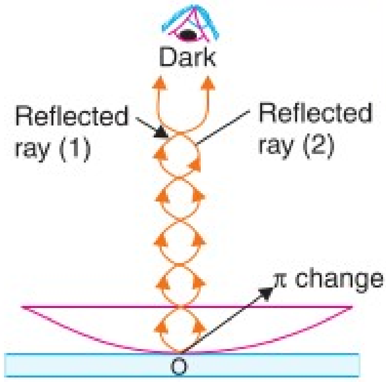


Fig. 15.18

Thus, the superposing waves are out of step by $\lambda/2$ which is equivalent to a phase difference of 180° (or π rad). Thus the two interfering waves at the centre are opposite in phase and produce a dark spot.

15.6.7. DETERMINATION OF WAVELENGTH OF LIGHT

A plano-convex lens of large radius of curvature (about 100 cm) and a flat glass plate are cleaned. The lens is kept with its convex face on the glass plate and they are held in position with the help of a metal ring arrangement. The system is held under a low power travelling microscope kept before a sodium vapour lamp. It is arranged that the yellow light coming from the sodium lamp falls on a glass plate held at 45° light beam. The light is turned through 90° and is incident normally on the lens-plate system. The microscope is adjusted till the circular rings came into focus. The centre of the cross-wire is made to come into focus on the centre of the dark spot, which is at the centre of the circular ring system. Now, turning the screw the microscope is moved on the carriage slowly towards one side, say right side. As the cross-wires move in the field of view, dark rings are counted. The movement is stopped when the 22nd dark ring is reached. Then the microscope is moved in the opposite direction and stopped at the 20th or 19th dark ring. The vertical cross-wire is made tangential to

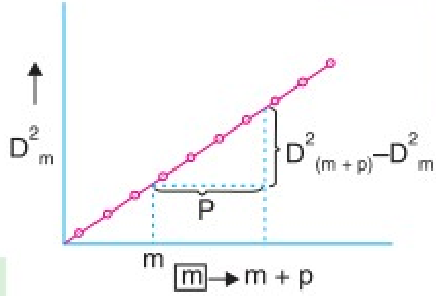


Fig. 15.19

the 19th ring and the reading is noted with the help of the scale graduated on the carriage. Thus, starting from the 19th ring, the tangential positions of the 18th, 17th, 16th,…..,5th dark rings are noted down. Now, the microscope is moved quickly to the left side of the ring system and it is stopped at the 5th dark ring. The cross-wire is again made tangential to the 5th dark ring and its position is noted. The difference between the readings on right and left sides of the 5th dark ring gives its diameter value. The procedure is repeated till 19th ring is reached and its reading is noted. From the value of the diameters the squares of the diameters are calculated. A graph is plotted between D_m^2 and the ring number '*m*'. A straight line would be obtained, as shown in Fig. 15.19.

We have

$$D_m^2 = 4m\lambda R \tag{15.42}$$

For the (m+p)th ring,

$$D_{m+p}^2 = 4(m+p)\lambda R \tag{15.43}$$

$$\therefore \quad D_{m+p}^2 - D_m^2 = 4p\lambda R$$

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR} \tag{15.44}$$

The slope of the straight line (Fig.15.18) gives the value of 4λR. Thus,

$$\lambda = \frac{Slope}{4R} \tag{15.45}$$

The radius of curvature R of the lens may be determined using a spherometer and λ is computed with the help of the above equation.

15.6.8. REFRACTIVE INDEX OF A LIQUID

The liquid, whose refractive index is to be determined, is filled in the gap between the lens and plane glass plate. Now the liquid film substitutes the air film. The condition for interference may then be written as

$$2\mu t \cos r = m\lambda \qquad \text{Darkness}$$

where μ is the refractive index of the liquid. For normal incidence the equation becomes

$$2\mu t = m\lambda$$

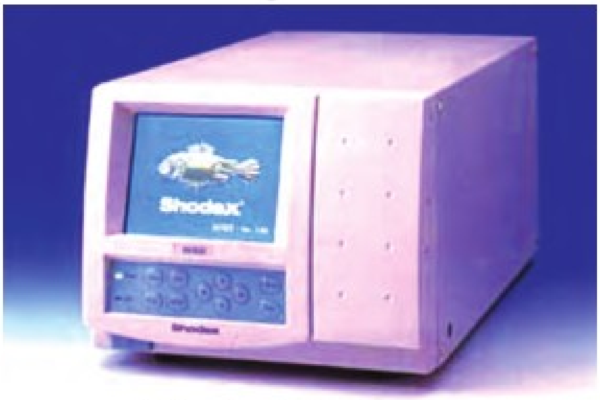
As $t = \dfrac{r^2}{2R}$,

$$\frac{2\mu r^2}{2R} = m\lambda$$

Or

$$r^2 = \frac{m\lambda R}{\mu}$$

$$\therefore \quad D^2 = \frac{4m\lambda R}{\mu}$$



Refractive index detector.

Following the above relation, the diameter of mth dark ring may be expressed as

$$\left[D_m^2\right]_L = \frac{4m\lambda R}{\mu} \tag{15.46}$$

Similarly, the diameter of the (m+p)th ring is given by

$$\left[D_{m+p}^2\right]_L = \frac{4(m+p)\lambda R}{\mu} \tag{15.47}$$

Subtracting eq. (15.46) from eq. (15.47), we get

$$\left[D_{m+p}^2\right]_L - \left[D_m^2\right]_L = \frac{4p\lambda R}{\mu} \tag{15.48}$$

But we know that

$$\left(D_{m+p}^2\right)_{air} - \left(D_m^2\right)_{air} = 4p\lambda R \tag{15.49}$$

$$\therefore \qquad \mu = \frac{\left(D_{m+p}^2\right)_{air} - \left(D_m^2\right)_{air}}{\left(D_{m+p}^2\right)_{liq} - \left(D_m^2\right)_{liq.}} \tag{15.50}$$

15.6.9. NEWTON'S RINGS IN TRANSMITTED LIGHT

Newton's rings in transmitted light may be observed with the arrangement made as in Fig. 15.20. The condition for maxima or bright rings is

$$2\mu\, t\, \cos\, r = m\lambda$$

and for dark rings $\quad 2\mu\, t\, \cos\, r = (2m+1)\lambda/2$

As $\mu = 1$ for air and r = 0 for normal observation, the above expressions may be simplified to

For bright fringes $\quad 2t = \mu\lambda$

and for dark rings $\quad 2t = (2m+1)\lambda/2$

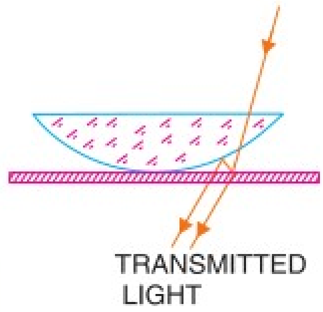


Fig. 15.20

As $t = \dfrac{r^2}{2R}$, the radius for the bright ring is given by $r_m^2 = m\lambda R$ (15.51)

and the radius for dark rings is given by $r_m^2 = (2m+1)\lambda R/2$ (15.52)

15.6.10. NEWTON'S RINGS FORMED BY TWO CURVED SURFACES

Case 1: Lower surface concave:

Let us consider two curved surfaces of radii of curvature R_1 and R_2 in contact at the point O. A thin air film is enclosed between the two surfaces. The dark and bright rings are formed and can be viewed with a travelling microscope. Suppose the radius of the mth dark ring is r. The thickness of the air film at P is

$$PQ = PT - QT$$

From geometry, $\quad PT = \dfrac{r^2}{2R_1} \quad$ and $\quad QT = \dfrac{r^2}{2R_2}$

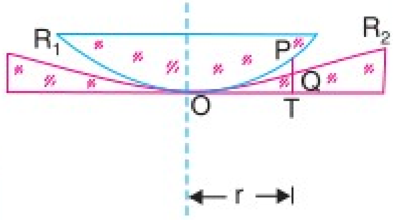


Fig. 15.21

$$\therefore \qquad PQ = \frac{r^2}{2R_1} - \frac{r^2}{2R_2} \tag{15.53}$$

But PQ = t. The condition for dark rings in reflected light is given by $2\mu\, t\, \cos\, r = m\lambda$. As $\mu = 1$ and $\cos r = 1$ for normal incidence, the above condition reduces to $2t = m\lambda$.

$$\therefore \qquad 2\left(\frac{r^2}{2R_1} - \frac{r^2}{2R_2}\right) = m\lambda$$

$$r^2\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=m\lambda \qquad \text{where } m = 0,1,2,3,\ldots.. \qquad (15.54)$$

For bright fringes the condition is $2\mu\ t\ \cos\ r\ =(2m+1)\lambda/2$

which reduces to $\quad 2t=(2m+1)\lambda/2$

or
$$r^2\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{(2m+1)\lambda}{2} \qquad \text{where } m = 0,1,2,3,\ldots.. \qquad (15.55)$$

Case 2: Lower surface convex:

Let us consider two curved surfaces of radii of curvature R_1 and R_2 in contact at the point O. A thin air film is enclosed between the two surfaces. The dark and bright rings are formed and can be viewed with a travelling microscope. Suppose the radius of the mth dark ring is r. The thickness of the air film at P is

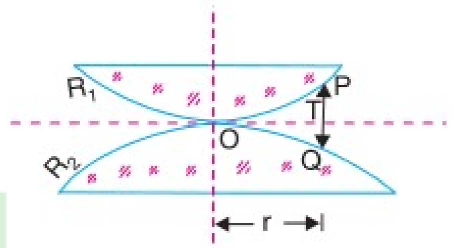


Fig. 15.22

$$PQ = PT + QT$$

From geometry $\quad PT=\dfrac{r^2}{2R_1} \quad$ and $\quad QT=\dfrac{r^2}{2R_2}$

$$\therefore \qquad PQ=\frac{r^2}{2R_1}+\frac{r^2}{2R_2}$$

But PQ = t. The condition for dark rings in reflected light is given by $2\mu\ t\cos r = m\lambda$.
As $\mu = 1$ and $\cos r = 1$ for normal incidence, the above condition reduces to $2\ t = m\lambda$

$$\therefore \qquad 2\left(\frac{r^2}{2R_1}+\frac{r^2}{2R_2}\right)=m\lambda$$

$$r^2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=m\lambda \qquad \text{where } m = 0,1,2,3,\ldots(15.56)$$

For bright fringes the condition is $2\mu\ t\ \cos\ r\ =(2m+1)\lambda/2$

which reduces to $\quad 2t=(2m+1)\lambda/2$

or
$$r^2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{(2m+1)\lambda}{2} \qquad \text{where } m = 0,1,2,3,\ldots(15.57)$$

15.7. MICHELSON'S INTERFEROMETER

An interferometer is an instrument in which the phenomenon of interference is used to make precise measurements of wavelengths or distances.


Michelson's interferometer.

15.7.1. PRINCIPLE

In Michelson interferometer, a beam of light from an extended source is divided into two parts of equal intensities by partial reflection and refraction. These beams travel in two mutually perpendicular directions and come together after reflection from plane mirrors. The beams overlap on each other and produce interference fringes.

15.7.2. CONSTRUCTION

The schematic of a simple Michelson interferometer is shown in Fig.15.23. It consists of a beam splitter G_1, a compensating plate G_2, and two plane mirrors M_1 and M_2. The beam splitter G_1 is a partially silvered plane parallel glass plate. The compensating plate G_2 is a simple plane parallel glass plate having the same thickness as G_1. The two plates G_1 and G_2 are held parallel to each other and are inclined at an angle of 45° with respect to the mirror M_2. The mirror M_1 is mounted on a carriage and can be moved exactly parallel to itself with the help of a micrometer screw. The distance through which the mirror M_1 is moved can be read with the help of a graduated drum attached to the screw. Displacements of the order of 0.1 μm (1000 Å) can be easily read. The plane mirrors M_1 and M_2 can be made perfectly perpendicular with the help of the fine screws attached to them. The interference bands are observed in the field of view of the telescope T.

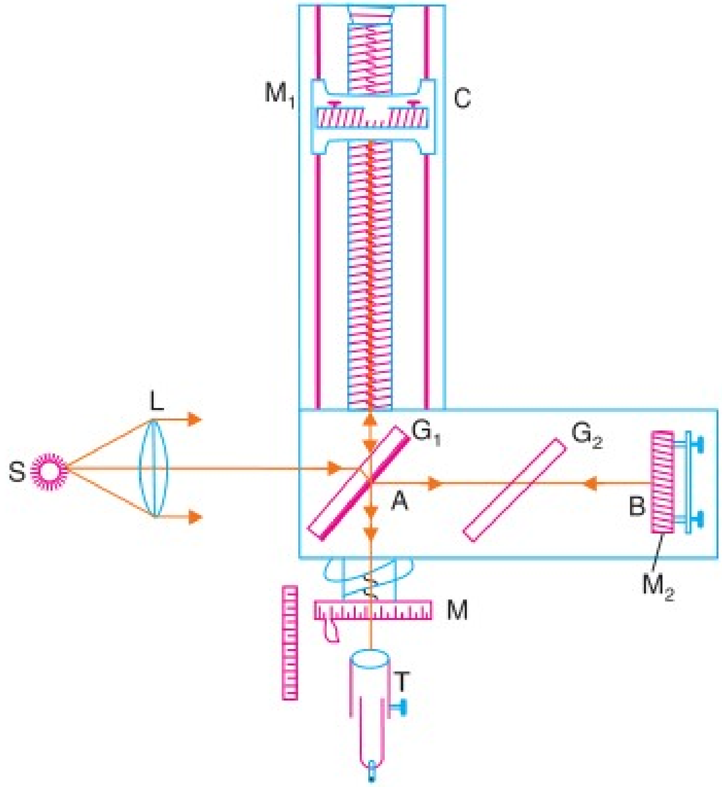


Fig. 15.23

15.7.3. WORKING

Monochromatic light from an extended source S is rendered parallel by means of a collimating lens L and is made incident on the beam splitter G_1. It is partly reflected at the back surface of G_1 along AC and partly transmitted along AB. The beam AC travels normally towards the plane mirror M_1 and is reflected back along the same path and comes out along AT. The transmitted beam travels toward the mirror M_2 and is reflected along the same path. It is reflected at the back surface of G_1 and proceeds along AT. The two beams received along AT are produced from a single source through division of amplitude and are hence *coherent*. The superposition of these beams leads to interference and produces interference fringes.

From the Fig. 15.23 it is clearly seen that a light ray starting from the source S and undergoing reflection at the mirror M_1 passes through the glass plate G_1 three times. On the other hand, in the absence of plate G_2, the ray reflected at M_2 travels through the glass plate G_1 only once. For compensating this path difference, a compensating plate G_2 of the same thickness is inserted into the path AB and is held exactly parallel to G_1.

If we look into the instrument from T, we see mirror M_1 and in addition we see a virtual image, M'_2, of mirror M_2. Depending on the positions of the mirrors, image M'_2 may be in front of, or behind, or exactly coincident with mirror M_1.

15.7.4. CIRCULAR FRINGES

Circular fringes are produced with monochromatic light when the mirrors M_1 and M_2 are exactly perpendicular to each other. The origin of the circular fringes can be understood as follows.

If we look into the instrument from T, we see mirror M_1 directly, and in addition we will see the virtual image M'_2 of mirror M_2 formed by reflection in the glass plate G_1 (Fig 15.24). It means that one of the interfering beams come from M_1 and the other beam appears to come from the virtual image M'_2. The situation is similar to an air film enclosed between mirrors M_1 and M'_2 with the difference that in case of a real film between two surfaces, multiple reflections take place, whereas in this case only two reflections take place.

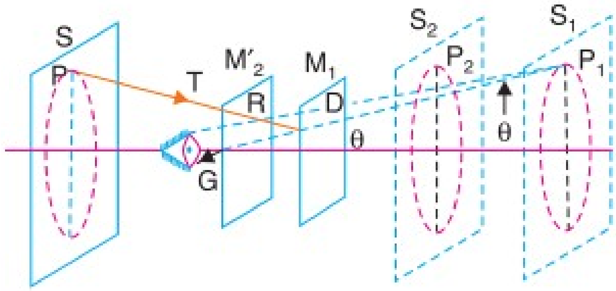


Fig. 15.24

If the two arms of the interferometer are equal in length, image M'_2 coincides with mirror M_1. If M'_2 and M_1 do not coincide, the distance between them is finite, $M'_2 M_1 = d$. Now if a light ray comes from a point S and is reflected by both M'_2 and M_1, the observer will see two virtual images- S_1 due to reflection at M'_2 and S_2 due to reflection at M_1. The virtual images are separated by a distance 2d. If the observer looks into the system at an angle θ, the path difference between the two beams will be 2d cos θ. The light that comes from M_2 and goes to T undergoes rare-to-dense reflection and therefore a π-phase change occurs. In view of this, the total path difference between the two beams is given by

$$\Delta = 2d \cos \theta + \lambda / 2.$$

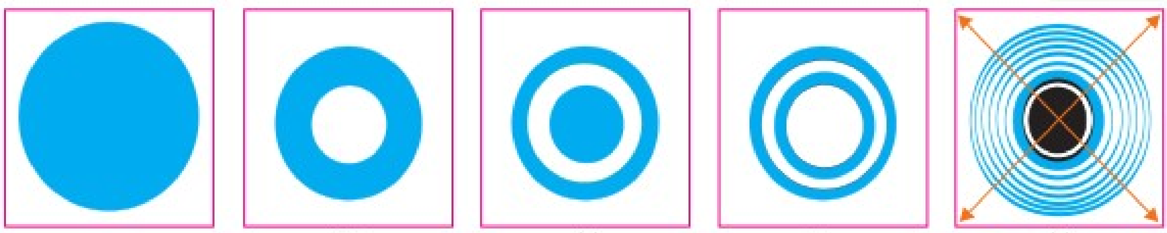
The condition for obtaining brightness

$$2d \cos \theta + \lambda / 2 = m\lambda$$

where m = 0,1,2, ...

For a given mirror separation d, a given wavelength λ and order m, angle θ is constant. This means that the fringes are of circular shape. They are called fringes of equal inclination.

In case the mirror M_1 coincides with the virtual image M'_2, d = 0. The path difference between the interfering beams will be λ / 2. Consequently, we obtain a minimum at the coincidence position and the centre of the field will be dark, as shown in Fig. 15.25 (a).



(*a*) (*b*) (*c*) (*d*) (*e*)

Fig. 15.25

If one of the mirrors is now moved through a distance λ/4, the path difference changes by λ/2 and therefore a maximum is obtained. By moving the mirror through another λ/4, a minimum is obtained; moving it by another λ/4 again a maximum is obtained and so on. Therefore, a new ring appears in the centre of the field each time the mirror is moved through λ/2. As d increases new rings appear in the centre faster than rings already present disappear in the periphery; and the field becomes more crowded with thinner rings (Fig. 15.25e). Conversely, as d is made smaller, the rings contract and disappear in the centre.

15.7.5. LOCALIZED FRINGES

When the two mirrors are tilted, they are not exactly perpendicular to each other and therefore the mirror M_1 and the virtual image M'_2 are not parallel. In this case the air path between them is wedge-shaped and the fringes appear to be straight. If one of the mirrors is moved, the fringes move across the field. The position of any particular bright fringe is taken up by the one next to it. The fringes can be counted as they pass a reference mark. If m fringes move across the field of view when M_1 moves through a distance d, then

$$d = m\lambda/2$$

or

$$\lambda = \frac{2d}{m} \quad (15.58)$$

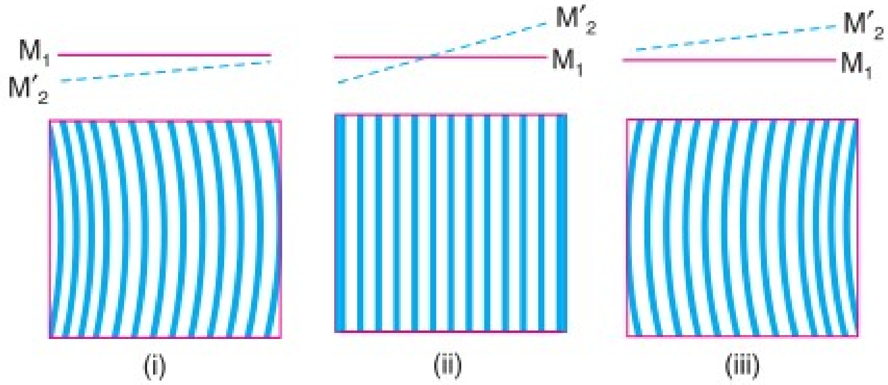
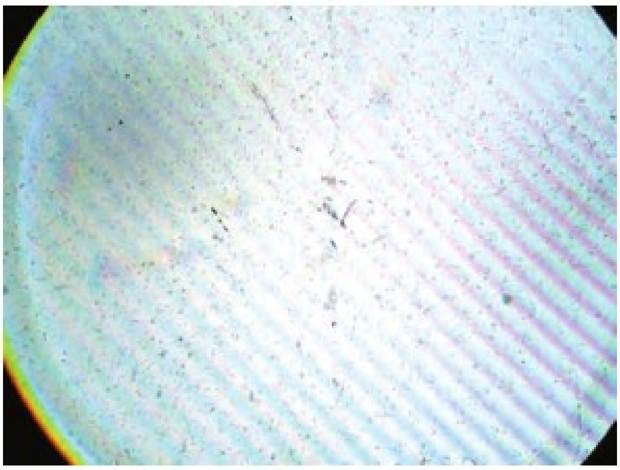


Fig. 15.26

15.7.6. WHITE LIGHT FRINGES

Instead of a monochromatic source, if a white light source is used, a few coloured fringes with a central dark fringe can be observed. In observing these fringes, the mirrors are slightly tilted as for localised fringes and position of M_1 is found where it intersects M'_2. This position is often difficult to find with white light. The position can best be located with monochromatic light when the fringes become straight. Then a very slow motion of M_1 in this region using white light will bring these fringes into view, when a central dark fringe is surrounded by 8 to 10 coloured fringes on either side are observed. These fringes are useful for the determination of zero path difference.



White light fringes.

15.7.7. VISIBILITY OF FRINGES

In case of Michelson interferometer, the intensity is given by

$$I = 4A^2 cos^2 \frac{\delta}{2}$$

Here $$\delta = \frac{2\pi}{\lambda}(2d \cos\theta)$$

where d is the distance between M_1 and M_2'. The intensity is maximum when δ is an integral multiple of 2π. The intensity is zero when δ is an odd multiple of π. When a monochromatic source of light is used, the minimum intensity of the fringes is zero. The visibility of fringes in the case of a Michelson interferometer is

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

for monochromatic light, $I_{min} = 0$ and therefore, $V = 1$

However, if the source of light is not strictly monochromatic, but contains two nearby wavelengths, the condition for maximum intensity for both the wavelengths is satisfied only for particular values of path difference ($2d \cos\theta$).

As the value of d is altered, the two wavelengths do coincide over a considerable range and here the fringe visibility is a maximum. For values of d other than maximum intensity positions for both wavelengths, the two fringe patterns will be complementary, provided the intensities for both the wavelengths are equal. If intensities are not equal, the minimum visibility will not be zero. The minimum visibility will be

$$V_{min} = \frac{A_1^2 - A_2^2}{A_1^2 + A_2^2}$$

where A_1 and A_2 are the amplitudes. Hence the source will be perfectly monochromatic if visibility is maximum and constant for different values of $2d \cos\theta$. If the visibility changes with the change of $2d \cos\theta$, the source is not strictly monochromatic.

15.8. APPLICATIONS OF MICHELSON INTERFEROMETER

Michelson interferometer can be used to determine (i) the wavelength of a given monochromatic source of light (ii) the difference between the two neighbouring wavelengths or resolution of the spectral lines, (iii) refractive index and thickness of various thin transparent materials and (iv) for measurement of the standard metre in terms of the wavelength of light.

15.8.1. MEASUREMENT OF WAVELENGTH

Michelson interferometer is used to determine the wavelength of light from a monochromatic source. The monochromatic source is kept at S. If the mirrors M_1 and M_2 are exactly perpendicular, circular fringes are obtained. If the mirror M_1 is moved forward or backward, the circular fringes appear or disappear at the centre. Now, as the mirror is moved through a known distance d and the number of fringes disappearing at the centre is counted. Suppose d_1 is the initial thickness of the air film between the mirror M_1 and the image of M_2 corresponding to the bright fringe of order m_1 and d_2 is the final thickness of the air film corresponding to a bright fringe of order m_n in the same position. Then,

$$2d_1 = m_1\lambda$$

and $$2d_2 = m_n\lambda$$

By subtraction, we get $2(d_2 - d_1) = (m_n - m_1)\lambda$

$\therefore \quad 2d = N\lambda$ where $(d_2 - d_1) = d$ and $(m_n - m_1) = N$

$$\therefore \quad \lambda = \frac{2d}{N} \tag{15.59}$$

15.8.2. DETERMINATION OF THE DIFFERENCE IN THE WAVELENGTH OF TWO WAVES

If a source of light consists of two wavelengths λ_1 and λ_2, which differ slightly, then two sets of fringes corresponding to the two wavelengths are produced in a Michelson interferometer. By adjusting the position of the mirror M_1 of the interferometer, the position is found when the fringes are very bright. In this position, the bright fringe due to λ_1 coincides with the bright fringes due to λ_2. When the mirror M_1 is moved, the two sets of fringes get out of step because their wavelengths are different. When the mirror M_1 has been moved through a certain distance, the bright fringe due to one set will coincide with the dark fringe due to the other set and no fringes will be seen in this case. Again by moving the mirror M_1, a position is reached when a bright fringe of one set falls on the bright fringe of the other and the fringes are again distinct. This is possible when the m^{th} order of the longer wavelength coincides with the $(m + 1)^{th}$ order of the shorter wavelength.

Let m_1 and m_2 be the changes in the order at the centre of the field when the mirror M_1 is displaced through a distance d between two consecutive positions of maximum distinctness of the fringes.

$$\therefore \quad 2d = m_1 \lambda_1 = m_2 \lambda_2$$

If λ_1 is greater than λ_2,

$$m_2 = m_1 + 1$$

$$\therefore \quad 2d = m_1 \lambda_1 = (m_1 + 1)\lambda_2$$

$$\therefore \quad m_1 = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$\therefore \quad 2d = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

$$\text{or} \quad \lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2d}$$

Taking λ as the mean wavelength of the two wavelengths λ_1 and λ_2, the small difference $\Delta\lambda$ is given by

$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2d} \tag{15.60}$$

15.8.3. THICKNESS OF A THIN TRANSPARENT SHEET

Let a transparent sheet of thickness t and refractive index μ be inserted in the path of one of the interfering beams of Michelson interferometer. The optical path of that beam increases because of the sheet. It becomes μt instead of t. The increase in the optical path is $(\mu t - t)$ or $(\mu - 1)t$. Since the beam traverses the medium twice, the extra path difference between the two interfering beams is $2(\mu - 1)t$. If m is the number of fringes by which the fringe system is displaced, then

$$2(\mu - 1)t = m\lambda$$

When monochromatic light is used, it is difficult to distinguish the sudden shift of fringes when the thin sheet is inserted. It is also not possible to count the number of fringes shifted. The difficulty is overcome by using white light first to locate the central dark fringe and it is made to coincide with the cross-wire of the telescope. The thin sheet is then introduced into the path of the beam. Position of mirror M_1 is adjusted till again a dark fringe of zero path difference coincides with the cross-wire of the telescope. The distance *d* through which the mirror is moved is noted. The white light is now replaced with the monochromatic light and the mirror M_1 is moved back slowly and the number of fringes contained in *d* is found. The thickness t is obtained from the relation

$$t = \frac{m\lambda}{2(\mu - 1)} \tag{15.61}$$

15.8.4. DETERMINATION OF THE REFRACTIVE INDEX OF GASES

When a tube containing a gas is introduced in the path of the beam going towards M_1, a path difference equal to $2(\mu - 1)\, l$ is introduced between the two interfering beams. Here, μ is the refractive index of the gas and l is the length of the tube. If m fringes cross the centre of the field of view, then $2(\mu - 1)\, l = m\lambda$. Knowing l, m, and λ, μ an be calculated.

In the path of the rays going towards M_1, a tube containing air at atmospheric pressure is introduced and the fringes are obtained in the centre of the field of view. In that case, refractive index of the air at various pressures can be determined. Let the length of the tube be l and let it contain air at atmospheric pressure. The tube is completely evacuated and m fringes cross the centre of the field of view. The path difference introduced between the two interfering beams is $2(\mu - 1)\, l$.

$$\therefore \quad 2(\mu - 1)\, l = m\lambda$$

$$\therefore \quad \mu = \frac{m\lambda}{2l} + 1$$

15.8.5. STANDARDISATION OF THE METRE

The experiment to measure the standard metre in terms of the wavelength of the cadmium red line was first performed by Michelson and Benoit in 1895. It is not possible to count the number of fringes which cross the field of view when one of the mirrors of the Michelson interferometer is moved through whole length of one metre. Moreover, for a path difference of more than 20 cm, it is not possible to obtain the fringes. Therefore, the mirror must not be moved through a distance of more than 10cm. In practice nine **etalons** were used, each being twice the length of the preceding etalon. The length of the shortest etalon used is 0.390625 mm and of the longest was 10 cm. The experiment is divided into two main parts.

(*i*) The number of wavelengths of the monochromatic cadmium light is counted for the shortest etalon.

(*ii*) The length of the second etalon is compared with the shorter etalon and the process is repeated until the number of wavelengths for a length of 10 cm-etalon is known. From this 10 cm-etalon, the number of wavelengths for a length of one metre in terms of the wavelength of cadmium red line is known. This acts as a standard metre because, even if the original standard metre is destroyed, the standard metre can be formed again from the knowledge of the number of wavelengths. The standard metre is represented in terms of the wavelengths of red, green and blue lines of cadmium.

Etalon

An etalon is a substandard for length. It consists of two mirrors, which are plane-parallel and silvered on their front faces. The distance between their surfaces is l (Fig. 15.27). The mirrors can be made perfectly parallel by means of screws attached to them.

Experiment:

(*i*) The Michelson interferometer is used as shown in Fig. 15.28. Light from the source S