

**Assignment**  
**Sub: Group Theory**  
**FYUGP 4<sup>th</sup> Semester, 2025**

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1. Prove that  $U_n$  ( $n \geq 2$ ) is a group under the operation of multiplication
2. Let  $G$  be the group of all nonzero complex numbers  $a + bi$  ( $a, b$  real, not both 0) under the multiplication. Prove that  $H = \{a + bi \in G \mid a^2 + b^2 = 1\}$  is a subgroup of  $G$ .
3. If  $H$  is a subgroup of  $G$ , let  $N(H) = \{g \in G \mid gHg^{-1} = H\}$ . Prove that  $N(H)$  is a subgroup of  $G$  and  $H$  is normal in  $G$ .
4. In a group  $G$ ,  $a^5 = e$ ,  $aba^{-1} = b^2$  for  $a, b \in G$ . Then show that  $\text{ord}(b) = 31$ .
5. Show that a subgroup of index 2 in a group  $G$  is a normal subgroup of  $G$ .
6. Prove that  $D_n$  is non-Abelian for  $n \geq 3$ .