2024

MATHEMATICS

Paper : MATMIN1014

(Foundation of Mathematics)

Full Marks : 70

Pass Marks : 28

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer :   1×10=10

শুদ্ধ উত্তৰটো বাছি উলিওৱা ঃ

(i)   Let $f(x) = x^4 + 2x^2 + 3x - 1$. Then the number of positive roots of $f(x) = 0$ is

ধৰা হ'ল $f(x) = x^4 + 2x^2 + 3x - 1$. তেতিয়াহ'লে $f(x) = 0$ সমীকৰণটোৰ ধনাত্মক মূলৰ সংখ্যা হ'ল

(A)   1

(B)   2

(C)   3

(D)   0

(ii) If z is a complex number, then $|z|^2 =$

যদি z এটা জটিল সংখ্যা হয়, তেনেহলে $|z|^2 =$

(A) $|z^2|$

(B) z^2

(C) $\bar{z}^2$

(D) $z\bar{z}$

(iii) Let A and B be two arbitrary square matrices. Then

যদি A আৰু B যিকোনো দুটা বৰ্গ মৌলকক্ষ হয়, তেতিয়াহ'লে

(A) $AB = BA$

(B) $AB \neq BA$

(C) $A + B = AB$

(D) $(AB)^{-1} = A^{-1}B^{-1}$

(iv) If $\sqrt{-1} = i$ and n is any positive integer, then $i^{4n+3} =$

যদি $\sqrt{-1} = i$ আৰু n এটা যিকোনো ধনাত্মক অখণ্ড সংখ্যা হয়, তেতিয়াহ'লে $i^{4n+3} =$

(A) i

(B) 1

(C) $-i$

(D) -1

(v) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation

$-2x^4 - 5x^3 - 15x^2 + 10x + 8 = 0$, then

$\alpha\delta(\beta + \gamma) + \beta\gamma(\alpha + \delta) =$

যদি $\alpha, \beta, \gamma, \delta$

$-2x^4 - 5x^3 - 15x^2 + 10x + 8 = 0$ সমীকৰণটোৰ মূল হয়, তেন্তে

$\alpha\delta(\beta + \gamma) + \beta\gamma(\alpha + \delta) =$

(A) -5

(B) 5

(C) $15/2$

(D) $-15/2$

(vi) If each root of the equation $x^3 + 3x^2 - 8x + 1 = 0$ is increased by 1, then the resultant equation is

$x^3 + 3x^2 - 8x + 1 = 0$ সমীকৰণটোৰ প্ৰত্যেকটো মূল 1কৈ বৃদ্ধি কৰিলে, নতুনকৈ গঠন কৰা সমীকৰণটো হ'ল

(A) $y^3 - 11y - 11 = 0$

(B) $y^3 + 11y + 11 = 0$

(C) $y^3 - 11y + 11 = 0$

(D) $y^3 + 11y - 11 = 0$

(vii) If α is a root of $x^n - 1 = 0$, then which of the following is not true?

যদি α এটা $x^n - 1 = 0$ সমীকৰণটোৰ মূল হয়, তেন্তে তলৰ কোনটো শুদ্ধ নহয়?

(A) α^m is a root of $x^n - 1 = 0$

α^m, $x^n - 1 = 0$ ৰ মূল

(B) α^{m+n} is a root of $x^n - 1 = 0$

α^{m+n}, $x^n - 1 = 0$ ৰ মূল

(C) α^{m-n} is a root of $x^n - 1 = 0$

α^{m-n}, $x^n - 1 = 0$ ৰ মূল

(D) $\alpha^{m/n}$ is a root of $x^n - 1 = 0$

$\alpha^{m/n}$, $x^n - 1 = 0$ ৰ মূল

(viii) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Then (here $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$)

যদি $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, তেন্তে (ইয়াত $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$)

(A) $A^{-1} = \dfrac{1}{2}(A - 5I)$

(B) $A^{-1} = -\dfrac{1}{2}(A + 5I)$

(b) Find the square root of $a^2 - 1 + 2ia$.

$a^2 - 1 + 2ia$ ৰ বৰ্গমূল নিৰ্ণয় কৰা।

(ii) Expand $\tan^{-1}\left(\dfrac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}\right)$ as a power series in $\tan\theta$.

$\tan^{-1}\left(\dfrac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}\right)$ ক $\tan\theta$ ৰ শ্ৰেণী হিচাপে প্ৰকাশ কৰা।

(iii)   2½+2½=5

If $\sin(\alpha + i\beta) = x + iy$, then prove that

যদি $\sin(\alpha + i\beta) = x + iy$ হয়, তেন্তে প্ৰমাণ কৰা যে

(a) $x^2\csc^2\alpha - y^2\sec^2\alpha = 1$

(b) $x^2\operatorname{sech}^2\beta + y^2\operatorname{cosech}^2\beta = 1$

(iv) Prove that

প্ৰমাণ কৰা যে

$$\sin^{-1}x = x + \frac{1}{2}\cdot\frac{x^2}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^3}{5} + \ldots$$

(vi) Find A^{-1} of $A = \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ by elementary row operations.

$A = \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ মৌলকক্ষটোৰ A^{-1} নিৰ্ণয় কৰা।

(Elementary row operation প্ৰয়োগ কৰি)

(vii) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then find the value of $\sum \alpha^2$.

যদি α, β, γ, $x^3 + qx + r = 0$ সমীকৰণটোৰ মূল হয়, তেন্তে $\sum \alpha^2$ ৰ মান নিৰ্ণয় কৰা।

3. Answer the following questions : *(any six)*

5×6=30

তলৰ প্ৰশ্নসমূহৰ উত্তৰ লিখা ঃ (যিকোনো ছয়টা)

3+2=5

(i)

(a) If $\sqrt{a + ib} = x + iy$, then show that

$\sqrt{a - ib} = x - iy$.

যদি $\sqrt{a + ib} = x + iy$, তেন্তে দেখুওৱা যে

$\sqrt{a - ib} = x - iy$.

2. Answer the following questions : *(any five)*

2×5=10

তলৰ প্ৰশ্নসমূহৰ উত্তৰ লিখা ঃ (যিকোনো পাঁচটা)

(i) Express $(5 - 3i)^3$ in the form of $A + iB$.

$(5 - 3i)^3$ ক $A + iB$ আকাৰত প্ৰকাশ কৰা।

(ii) Prove that (প্ৰমাণ কৰা যে)

$$\frac{\pi}{8} + \frac{1}{1 \cdot 3} + \frac{1}{5 \cdot 7} + \frac{1}{9 \cdot 11} + \dots$$

(iii) Find the modulus of (মাপাংক নিৰ্ণয় কৰা)

$(i + i)/(1 - i)$.

(iv) Find the eigenvalues of the following matrix :

তলৰ মৌলকক্ষটোৰ eigenvalue নিৰ্ণয় কৰা ঃ

$A = \begin{bmatrix} 7 & -1 \\ 6 & 2 \end{bmatrix}$

(v) Examine whether the following matrix A is singular or not.

তলৰ মৌলকক্ষটোৰ ক্ৰিয়ামান সম্বন্ধে পৰীক্ষা কৰা।

$A = \begin{bmatrix} -4 & 1 & -6 \\ 1 & 2 & -5 \\ 6 & 3 & -4 \end{bmatrix}$

(C) $A^{-1} = \frac{1}{2}(A + 5I)$

(D) $A^{-1} = \frac{1}{2}(-A + 5I)$

(ix) The system of equations $x + ay = 4$, $ax + 9y = b$ has a unique solution if and only if

$x + ay = 4$ আৰু $ax + 9y = b$ সমীকৰণ দুটাৰ এটা অদ্বিতীয় মূল থাকিব যদি আৰু যদিহে

(A) $a \neq \pm 3$

(B) $a = \pm 3$

(C) $a = 3$

(D) $a = -3$

(x) $e^{i(4n+1)\pi/2}$ is equal to

$e^{i(4n+1)\pi/2}$ ৰাশিটোৰ সমান হয়

(A) 1

(B) i

(C) $-i$

(D) -1

(v) Solve the following system of equations by matrix method :

তলৰ সমীকৰণৰ থুপাটোক মৌলকক্ষ পদ্ধতি প্ৰয়োগ কৰি সমাধান কৰা ঃ

$$x_1 + x_2 - 2x_3 + 4x_4 = 5$$
$$2x_1 + 2x_2 - 3x_3 + x_4 = 3$$
$$3x_1 + 3x_2 - 4x_3 - 2x_4 = 1$$

(vi) Reduce the following matrix A into echelon form and hence find its rank.

তলৰ A মৌলকক্ষটোক echelon আকাৰলৈ পৰিণত কৰা আৰু তাৰদ্বাৰাই ইয়াৰ কোটি নিৰ্ণয় কৰা।

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$$

(vii) Prove that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.

প্ৰমাণ কৰা যে প্ৰত্যেক বৰ্গ মৌলকক্ষক অদ্বিতীয়ভাৱে এটা সমমিত আৰু এটা বিষম সমমিত মৌলকক্ষৰ যোগফল হিচাবে প্ৰকাশ কৰিব পাৰি।

(viii) If α, β, γ are the roots of the equation
$x^3 + qx + r = 0 \, (r \neq 0)$, show that

$$\sum \frac{\alpha^2}{\beta} = \frac{2q^2}{r}.$$

যদি α, β, γ, $x^3 + qx + r = 0 \, (r \neq 0)$ সমীকৰণটোৰ মূল হয়, তেন্তে দেখুওৱা যে

$$\sum \frac{\alpha^2}{\beta} = \frac{2q^2}{r}.$$

(ix) If α, β, γ are the roots of the equation
$x^3 + px^2 + qx + r = 0$, find the equation
whose roots are $\alpha\beta + \beta\gamma, \beta\gamma + \gamma\alpha, \gamma\alpha + \alpha\beta$.

যদি α, β, γ, $x^3 + px^2 + qx + r = 0$ সমীকৰণটোৰ মূল হয়, তেন্তে সেই সমীকৰণটো নিৰ্ণয় কৰা যাৰ মূল $\alpha\beta + \beta\gamma, \beta\gamma + \gamma\alpha, \gamma\alpha + \alpha\beta$.

4. Answer the following questions : **(any two)**

অলৰ প্ৰশ্নসমূহৰ উত্তৰ লিখা ঃ **(যিকোনো দুটা)**

$$10 \times 2 = 20$$

(i) (a) If ω is a cube root of unity, then
find the value of :

যদি ω, 1ৰ এটা ঘনমূল হয়, তেন্তে মান নিৰ্ণয়
কৰা ঃ
$$4 + 6 = 10$$

$$(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^3$$

(b) Find the value of $(1 + i)^{1/3}$.

মান নিৰ্ণয় কৰা ঃ $(1 + i)^{1/3}$.

(ii) State and prove De Moivre's theorem.

De Moivre উপপাদ্যটো লিখি আৰু প্ৰমাণ কৰা।

(iii) State Cayley-Hamilton theorem. Verify it
for the following matrix A. $2 + 8 = 10$

Cayley-Hamilton ৰ উপপাদ্যটো লিখা আৰু তলৰ
মৌলকক্ষ A ৰ কাৰণে পৰীক্ষা কৰা ঃ

(iv)
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{bmatrix}$$

(a) Solve by Cardon's method :
$$6 + 4 = 10$$

Cardon ৰ পদ্ধতি প্ৰয়োগ কৰি সমাধান কৰা ঃ

$$x^3 + 8x - 35 = 0$$

(b) Solve the equation
$$2x^3 - x^2 - 18x + 9 = 0 \text{ if two roots}$$
are equal in magnitude but
opposite in sign.

$2x^3 - x^2 - 18x + 9 = 0$ সমীকৰণটো
সমাধান কৰা যদিহে ইয়াৰ দুটা মূল সমান কিন্তু
চিহ্ন বেলেগ হয়।

2024

MATHEMATICS

Paper : MATMIN2014

(Analytical Geometry)

Full Marks : 70

Pass Marks : 28

Time : Three hours

The figures in the margin indicate full marks for the questions.

1.   Choose the correct answer :   1×10=10

শুদ্ধ উত্তৰটো বাছি উলিওৱা :

   (i)   Let P be a point in a two-dimensional plane whose polar coordinates are $\left(\sqrt{2}, \pi/3\right)$. Then its Cartesian coordinates are

ধৰাহ'ল কোনো এখন দ্বিমাত্রীয় সমতলৰ এটা বিন্দু P ৰ polar স্থানাংক হ'ল $\left(\sqrt{2}, \pi/3\right)$। তেন্তে ইয়াৰ cartesian স্থানাংক বিন্দু হ'ব

(A) $\left(\dfrac{1}{2}, \sqrt{3/2}\right)$

(B) $\left(\dfrac{1}{\sqrt{2}}, \dfrac{\sqrt{3}}{2}\right)$

(C) $\left(\dfrac{1}{\sqrt{2}}, \sqrt{3/2}\right)$

(D) $\left(\dfrac{1}{\sqrt{2}}, \dfrac{3}{\sqrt{2}}\right)$

(ii) If the pair of straight lines represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are perpendicular, then

যদি $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ সমীকৰণটোৱে প্ৰতিনিধিত্ব কৰা যুৰীয়া সৰলৰেখাদ্বয় লম্ব হয়, তেন্তে

(A) $a - b = 0$

(B) $\dfrac{a}{h} = \dfrac{h}{b} = \dfrac{g}{f}$

(C) $a + b = 0$

(D) $h^2 = ab$ and $af^2 = bg^2$

(iii) If $ax^2 + 2hxy + by^2 = 0$ represents a conic, then which of the following is not the tangent at (x', y') ?

যদি $ax^2 + 2hxy + by^2 = 0$ সমীকৰণটোৱে এটা conic বুজায়, তেন্তে তলৰ কোনটো সমীকৰণ (x', y') বিন্দুত এডাল স্পৰ্শক নহয় ?

(A) $axx' + h(xy' + x'y) + byy' = 0$

(B) $(ax' + hy')x + (by' + hx')y = 0$

(C) $\dfrac{ax' + hy'}{by' + hx'} + \dfrac{y}{x} = 0$

(D) $(ax + hy)x' + (by + hx)y' = 0$

(iv) The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a hyperbola, if—

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ সমীকৰণটোৱে এটা অধিবৃত্ত বুজাৰ, যদিহে—

(A) $ab - h^2 < 0$

(B) $ab - h^2 > 0$

(C) $ab - h^2 = 0$

(D) $ab - h^2 \neq 0$

(v) The straight line $lx + my = n$ is a tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$, if—

$lx + my = n$ সৰলৰেখাডাল $x^2/a^2 + y^2/b^2 = 1$ উপবৃত্তটোৰ এডাল স্পৰ্শক হ'ব, যদিহে—

(A) $a^2m^2 + b^2l^2 = n^2$

(B) $a^2l^2 - b^2m^2 = n^2$

(C) $a^2m^2 - b^2l^2 = n^2$

(D) $a^2l^2 + b^2m^2 = n^2$

(vi) The parametric coordinates of the hyperbola $xy = c^2$ are—

$xy = c^2$ অধিবৃত্তটোৰ parametric স্থানাংক বিন্দু হ'ব—

(A) $(c/t, ct)$

(B) $(ct, c/t)$

(C) $(c^2t, 1/t)$

(D) $(1/t, c^2t)$

(vii) The centre of the sphere

$$x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$$

is

$$x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$$

গোলকটোৰ কেন্দ্ৰবিন্দু হ'ব

(A) $(3, -4, 5)$

(B) $(-3, -4, -5)$

(C) $(-3, -4, 5)$

(D) $(3, -4, -5)$

(viii) Every section of a right circular cone by a plane perpendicular to its axis is

যদি এটা right circular cone ক এখন ইয়াৰ অক্ষডালৰ লম্ব হৈ থকা সমতলে ছেদ কৰে, তেন্তে ছেদাংশটোৰ আকাৰ হ'ব

(A) a parabola

এটা পৰাবৃত্ত

(B) a circle

এটা বৃত্ত

(C) an ellipse

এটা উপবৃত্ত

(D) a hyperbola

এটা অধিবৃত্ত

(ix) The equation $yz + zx + xy = 0$ represents—

$yz + zx + xy = 0$ সমীকৰণটোৰে বুজায়—

(A) a pair of planes

এখন যুৰীয়া সমতল

(B) a sphere

এটা গোলক

(C) a cone

এটা শংকু

(D) a cylinder

এটা চিলিণ্ডাৰ

(x) The guiding curve of a right circular cylinder is—

এটা right circular চিলিণ্ডাৰৰ guiding curve—

(A) an ellipse

এটা উপবৃত্ত

(B) a circle

এটা বৃত্ত

(C) a pair of straight lines

যুৰীয়া সৰলৰেখা

(D) any closed curve

যিকোনো বদ্ধ বক্ৰৰেখা

2. Answer the following questions : *(any five)*

2×5=10

তলৰ প্ৰশ্নসমূহৰ উত্তৰ লিখা : (যিকোনো পাঁচটা)

(i) Transform the following equation to polar form:

তলৰ সমীকৰণটো polar form-ত লিখা :

$$(x^2 + y^2)^2 = a^2(x^2 - y^2)$$

(ii) Find the equation of the line $3x + 4y - 10 = 0$ where the origin is transformed to the point $(2,1)$.

যদিহে originটো $(2,1)$ বিন্দুলৈ স্থানান্তৰ কৰা হয়, তেন্তে $3x + 4y - 10 = 0$ ৰেখাডালৰ স্থানান্তৰীয় সমীকৰণটো লিখা।

(iii) Find the angle of the straight lines represented by the equation—

$$2x^2 - 5xy + 3y^2 - 2x + 3y = 0$$

$2x^2 - 5xy + 3y^2 - 2x + 3y = 0$ সমীকৰণটোৱে প্ৰতিনিধিত্ব কৰা যুৰীয়া ৰেখাদ্বয়ৰ মাজৰ কোণটো নিৰ্ণয় কৰা।

(iv) Does the equation

$$3x^2 - 8xy + 7y^2 - 4x + 2y - 7 = 0$$

represents a central conic? If so, find its centre.

যদিহে $3x^2 - 8xy + 7y^2 - 4x + 2y - 7 = 0$ সমীকৰণটোৱে এটা central conic বুজায়, তেন্তে ইয়াৰ কেন্দ্ৰটো নিৰ্ণয় কৰা।

(v) Prove that the line $y = x + \sqrt{5/6}$ touches the ellipse $2x^2 + 3y^2 = 1$.

প্ৰমাণ কৰা যে $y = x + \sqrt{5/6}$ ৰেখাডালে $2x^2 + 3y^2 = 1$ উপবৃত্তটোক স্পৰ্শ কৰে।

(vi) Find the radius of the following sphere :

তলৰ গোলকটোৰ ব্যাসাৰ্ধ নিৰ্ণয় কৰা:

$$2x^2 + 2y^2 + 2z^2 - 3x + 5y + 7z + 3 = 0$$

(vii) Find the equation of the cone whose vertex is the point $(1,1,0)$ and whose guiding curve is $y = 0, x^2 + y^2 = 4$.

সেই শংকুটোৰ সমীকৰণ নিৰ্ণয় কৰা যাৰ শীৰ্ষ বিন্দু $(1,1,0)$ আৰু guiding curve $y = 0, x^2 + y^2 = 4$।

3. Answer the following questions : **(any six)**

$5 \times 6 = 30$

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া : (যিকোনো ছয়টা)

(i) Find the value of k, so that the equation

$$kx^2 + 3xy - 5y^2 + 7x + 14y + 3 = 0$$

may represent a pair of straight lines.

যদিহে $kx^2 + 3xy - 5y^2 + 7x + 14y + 3 = 0$ সমীকৰণটোৱে এযোৰ সৰলৰেখাক বুজায়, তেন্তে k ৰ মান নিৰ্ণয় কৰা।

(ii) Define polar of a point to a conic. Find the pole of the line $lx + my + n = 0$ with respect to the parabola $y^2 = 4ax$.

$$1+4=5$$

এটা conic সাপেক্ষে কোনো এটা বিন্দুত polar ৰ সংজ্ঞা লিখা। $lx + my + n = 0$ ৰেখাডাল $y^2 = 4ax$ পৰাবৃত্ত সমীকৰণটোৰ সাপেক্ষে pole হোৱাৰ সমীকৰণটো লিখা।

(iii) Prove that the locus of the point whose polar with respect to the ellipse $x^2/a^2 + y^2/b^2 = 1$ subtends a right angle at the centre is the ellipse

$$x^2/a^4 + y^2/b^4 = 1/a^2 + 1/b^2 .$$

যদিহে এটা বিন্দুৰে $x^2/a^2 + y^2/b^2 = 1$ উপবৃত্তটোৰ সাপেক্ষে কৰা polar ৰেখাডালে কেন্দ্রবিন্দুত এটা সমকোণ সৃষ্টি কৰে, তেন্তে প্রমাণ কৰা যে সেই বিন্দুটোৰ গতিপথ হ'ব

$$x^2/a^4 + y^2/b^4 = 1/a^2 + 1/b^2$$

(iv) Reduce the following equation to the standard form:

তলৰ সমীকৰণটো standard আকাৰলৈ পৰিৱৰ্তন কৰা :

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

(v) Find the equation of the tangent to the conic $4x^2 + 3xy + 2y^2 - 3x + 5y + 7 = 0$ at the point $(1,-2)$. (Give detail procedure).

তলৰ conic টোৰ $(1,-2)$ বিন্দুত স্পৰ্শকডালৰ সমীকৰণটো নিৰ্ণয় কৰা:

$$4x^2 + 3xy + 2y^2 - 3x + 5y + 7 = 0$$

(সম্পূৰ্ণ পদ্ধতি বৰ্ণনা কৰিব লাগিব)

(vi) Find the equation to the right circular cone whose vertex is (2, –3, 5) and the axis makes equal angles with the axes and semi-vertical angle is 30°.

সেই right circular cone টোৰ সমীকৰণটো নিৰ্ণয় কৰা যাৰ শীৰ্ষবিন্দু (2, –3, 5) আৰু যাৰ অক্ষডালে x, y আৰু z অক্ষৰ লগত সমান কোণ কৰি থাকে আৰু semi-vertical যাৰ কোণৰ মান 30°।

(vii) Find the equation of the cylinder whose guiding lines have the direction cosines (l, m, n) and which passes through the circle $x^2 + z^2 = a^2$, $y = 0$.

সেই চিলিণ্ডাৰটোৰ সমীকৰণ নিৰ্ণয় কৰা যাৰ guiding line বিলাকৰ direction cosine বিলাক (l, m, n) আৰু $x^2 + z^2 = a^2$, $y = 0$ বৃত্তটোৰেদি পাৰ হৈ যায়।

(viii) Find the equation of the cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$.

সেই চিলিণ্ডাৰটোৰ সমীকৰণ নিৰ্ণয় কৰা যাৰ generator বিলাক $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ ৰেখাডাল সমান্তৰাল হয় আৰু যাৰ guiding curve এই $x^2 + 2y^2 = 1, z = 3$ উপবৃত্তটো।

(ix) Find the equation of the sphere which passes through the points $(0, 0, 0)$, $(0, 1, -1)$, $(-1, 2, 0)$, $(1, 2, 3)$.

সেই গোলকটোৰ সমীকৰণটো নিৰ্ণয় কৰা যি $0, 0, 0)$, $(0, 1, -1)$, $(-1, 2, 0)$ আৰু $(1, 2, 3)$ বিন্দুৰ মাজেদি পাৰ হৈ যায়।

4. Answer the following questions : **(any two)**
   $10 \times 2 = 20$

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া : *(যিকোনো দুটা)*

(a) (i) Find the condition under which the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of straight lines intersecting at some point (x', y').
   6

কি চৰ্ততত

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

সমীকৰণটোৰে এযুৰীয়া সৰলৰেখাক প্ৰতিনিধিত্ব কৰিব যিয়ে (x', y') বিন্দুত কটাকটি কৰে।

(ii) Show that the equation

$$4x^2 + 12xy + 9y^2 + 8x + 12y = 0$$

represents of pair of parallel straight lines and find the distance between them. $2+2=4$

দেখুওৱা যে তলৰ সমীকৰণটোৰে এযোৰ সমান্তৰাল সৰলৰেখাক প্ৰতিনিধিত্ব কৰে আৰু ৰেখা দুডালৰ মাজৰ দূৰত্ব নিৰ্ণয় কৰা :

$$4x^2 + 12xy + 9y^2 + 8x + 12y = 0$$

(b) Find the equation of the polar of a given point $P(x_1, y_1)$ with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$$

কোনো এটা বিন্দু $P(x_1, y_1)$, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ conicটোৰ সাপেক্ষে polar ৰ সমীকৰণ নিৰ্ণয় কৰা।

(c) (i) If the tangent plane to the sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts a, b, c on the coordinate axes, show that

$$1/a^2 + 1/b^2 + 1/c^2 = 1/r^2$$ 5

যদি $x^2 + y^2 + z^2 = r^2$ গোলকটোক স্পৰ্শক সমতলখন অক্ষৰ লগত কৰা ছেদাংশ বিলাক ক্ৰমে a, b, c হয়, তেন্তে দেখুওৱা যে

$$1/a^2 + 1/b^2 + 1/c^2 = 1/r^2$$

(ii) Prove that the equation

$$ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$$

represents a cone, if

$$u^2/a + v^2/b + w^2/c = 0.$$ 5+5=10

যদিহে $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ 5

সমীকৰণটোৱে এটা শংকু বুজায়, তেন্তে প্ৰমাণ কৰা যে

$$u^2/a + v^2/b + w^2/c = 0 ।$$

(d) (i) Prove that the cones
$$ax^2 + by^2 + cz^2 = 0 \text{ and}$$
$$x^2/a + y^2/b + z^2/c = 0 \text{ are}$$
reciprocal. 5

প্ৰমাণ কৰা যে $ax^2 + by^2 + cz^2 = 0$ আৰু $x^2/a + y^2/b + z^2/c = 0$ শংকু দুটা প্ৰতিক্ৰমী হয়।

(ii) Find the equation of the pair of tangents from a given point (x', y') to the ellipse

$$x^2/a^2 + y^2/b^2 = 1.$$

কোনো এটা বিন্দু (x', y') ৰ পৰা $x^2/a^2 + y^2/b^2 = 1$ উপবৃত্তটোলৈ টনা স্পৰ্শক দুডালৰ যুৰীয়া সমীকৰণটো নিৰ্ণয় কৰা।

2024

MATHEMATICS

Paper : MATSEC2013

(Scilab)

Full Marks : 40

Pass Marks : 16

Time : Two hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : 1×5=5

    (i) Which one of the following functions is true about Scilab ?

    (a) They are all user-defined.

    (b) Functions do not have arguments

    (c) User-defined functions are a subset of all functions

    (d) User-defined functions are a super-set of all functions

(ii) Which one of the following functions is not a mathematical function in Scilab ?

(a) Trigonometric functions

(b) Absolute value function

(c) Gamma function

(d) Logarithmic function

(iii) Key building depend on which of the following?

(a) Operating system

(b) Text bar

(c) Printer

(d) Keyboard

(iv) Which one of the following is not used to write algorithms for numerical computation ?

(a) FORTAT

(b) Python

(c) Java

(d) Calculator

(v) Which of the following is not a mathematical operation in Scilab ?

(a) Division

(b) Multiplication

(c) Addition

(d) Computation

2. Answer the following questions : *(any five)*
2×5=10

(i) Write any four physical constants in Scilab.

(ii) What is Scilab ?

(iii) What is Autocompletion in Scilab ?

(iv) What is Tab Completion ?

(v) Write four mathematical operations in Scilab in precedence order.

(vi) Write *any two* predefined mathematical functions in Scilab.

(vii) Write *any two* mathematical functions in Scilab.

3. Answer the following questions : **(any three)**

$5 \times 3 = 15$

(i) What are the five main data types ?

(ii) What are the five parts of Scilab workspace ?

(iii) How to write a text file in Scilab ?

(iv) What are the different types of plots in Scilab ?

(v) Write the steps to install Scilab in PC.

4. Answer the following question : **(any one)**

$10 \times 1 = 10$

i. How to plot 2D and 3D graphs in Scilab ?

ii. What are matrix and vector ? How to do matrix and vector in Scilab ?

Total number of printed pages = 8

2024

MATHEMATICS

Paper : MATMAJ2014

(Elements of Real Analysis)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : 1×6=6

    (a) The set $\mathbb{N}$ of Natural numbers is

    (i) bounded above

    (ii) not bounded above

    (iii) bounded below

    (iv) not bounded below

Contd.

(b) Between two distinct real numbers, there always lies

 (i) a rational number

 (ii) two rational numbers

 (iii) finitely many rational numbers

 (iv) infinitely many rational numbers

(c) A series Σu_n is called absolutely convergent if

 (i) the series $\Sigma |u_n|$ is devergent

 (ii) the series Σu_n is divergent

 (iii) the series $\Sigma |u_n|$ is convergent

 (iv) the series Σu_n is convergent

(d) If X and Y are countable sets then $X \cap Y$ is also

 (i) Countable Set.

 (ii) Uncountable Set.

 (iii) Both countable and uncountable Set.

 (iv) Absolutely uncountable Set.

(e) If a and b are any two positive real numbers such that $a<b$ then there exists a positive integer n such that

 (i) $na > nb$

 (ii) $na < nb$

 (iii) $na \le nb$

 (iv) $na \ge nb$

(f) A positive term series $\Sigma \dfrac{1}{n^p}$ is convergent if and only if

 (i) $P \ge 1$

 (ii) $P < 1$

 (iii) $P > 1$

 (iv) $P \le 1$

2. Answer the following questions : *(any five)*
$$2\times5=10$$

(a) Find the supremum and infimum for the set

$$X = \left\{ \frac{1}{n} \,/\, n \in N \right\}$$

(b) Prove that a non-empty finite set is not a neighbourhood of any point.

(c) Prove that the sequence $\{a_n\}$ is bounded where

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}, n \in N$$

(d) Show that one series whose n^{th} term is $\sin\dfrac{1}{n}$ is divergent.

(e) Prove that the sequence $\{x_n\}$ where $x_n = \dfrac{1}{n}$ is a Cauchy sequence.

(f) Define Cauchy's root test for the convergence of a series.

(g) Show tht the series $\sum \dfrac{\underline{|n}}{n^n}$ is convergent.

3. Answer the following questions : **(any six)**
5×6=30

(a) State and prove Bolzano-Weierstrass theorem for a sequence. 1+4=5

(b) Test the series for convergence

$$1 + \frac{x}{\underline{|1}} + \frac{x_2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \cdots \infty$$

(c) If a sequence of closed intervals $[a_n, b_n]$ is such that each member $[a_{n+1}, b_{n+1}]$ is contained in the preceding one $[a_n, b_n]$ and $\lim(b_n - a_n) = 0$. Prove that there is one and only one point common to all the intervals of the sequence.

(d) Show that sequence $\{s_n\}$ where

$$s_n = \left(1 + \frac{1}{n}\right)^n \quad \text{is convergent and that}$$

$$\lim\left(1 + \frac{1}{n}\right)^n \quad \text{lies between 2 and 3.}$$

(e) Is the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ is conditionally convergent? Justify your answer.

(f) Show that the set of all rational numbers Q is not complete ordered set.

(g) Test the convergence of the following series by Cauchy root test :

$$\sum \left(1+\frac{1}{n}\right)^{-n^2}$$

(h) State Squeeze theorem. Use it to show that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad 2+3=5$$

(i) Show that the sequence

$$x_1 = \sqrt{2},\ x_{n+1} = \sqrt{2x_n} \text{ converges to 2.}$$

4. Answer the following questions : *(any two)*

$$12 \times 2 = 24$$

(a) If $\sum u_n$ is a positive term series such that

$$\lim_{n \to \infty} \frac{u_{n+1}}{u_n} = l, \text{ then prove that the series}$$

(i) converges if $l < 1$

(ii) diverges if $l > 1$

(iii) the test fails if $l = 1$ $5+5+2=12$

(b) (i) Prove that a necessary and sufficient condition for a monotonic sequence to be convergent is that it is bounded.

8

(ii) Show that the series

$$\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \cdots + \frac{1}{(\log n)^p} + \cdots$$

diverges for $p > 0$

4

(c) (i) If $\sum u_n$ is a positive term series such that

$$\lim_{n \to \infty} (u_n)^{\frac{1}{n}} = l, \text{ then prove that the series}$$

(i) Converges if $l < 1$

(ii) diverges if $l > 1$ $4+4=8$

(ii) If $\lim a_n = a$ and $a_n \geq 0$ for all n, then prove that $a \geq 0$

4

(d) (i) If x_1, x_2 are positive and

$x_{n+1} = \dfrac{1}{2}(x_n + x_{n-1})$ then prove that

the two sequences with values

$x_1, x_3, x_5, \cdots$ and $x_2, x_4, x_6, \cdots$

One is decreasing and the other is increasing both converge to the

same limit $\dfrac{1}{3}(x_1 + 2x_2)$ 8

(ii) Show that the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{n^2+1}$ is

convergent.   4

2024

MATHEMATICS

Paper : MATMAJ2024

[Differential Equations (ODE)]

Full Marks : 50

Pass Marks : 20

Time : Two hours

**The figures in the margin indicate
full marks for the questions.**

1. Choose the correct answer :   1×5=5

   a.   The order of the differential equation

   $$\frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 3x = \sin t \ \text{ is}$$

   (i)   1

   (ii)   2

   (iii)   3

   (iv)   4

b. Which one of the following ordinary differential equation is linear ?

(i) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 6y^2 = 0$

(ii) $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = 0$

(iii) $\dfrac{d^2y}{dx^2} + 5\left(\dfrac{dy}{dx}\right)^3 + 6y = 0$

(iv) $\dfrac{d^2y}{dx^2} + 5y\dfrac{dy}{dx} + 6y = 0$

c. If the Wronskian of the functions $y_1(x)$ and $y_2(x)$ is zero, i.e. $W(y_1, y_2) = 0$, then the functions $y_1(x)$ and $y_2(x)$ are called

(i) linearly dependent

(ii) non-linearly dependent

(iii) linearly independent

(iv) non-linearly independent

d. The particular integral of $(D^2 - 3D + 2)y = e^{5x}$ is

(i) xe^{5x}

(ii) $\dfrac{e^{5x}}{5}$

(iii) $\dfrac{e^{5x}}{12}$

(iv) $\dfrac{e^{5x}}{25}$

e. The differential equation

$$\dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + x^3y = 0$$

is a second-order

(i) linear homogeneous

(ii) non-linear homogeneous

(iii) linear non-homogeneous

(iv) non-linear non-homogeneous

2. Answer the following questions : *(any five)*
$$2 \times 5 = 10$$

a. Show that the function f defined for all real x by $f(x) = 2\sin x + 3\cos x$ is an explicit solution of the differential equation

$$\frac{d^2y}{dx^2} + y = 0 .$$

b. Find the integrating factor of

$$\left(1 + y^2\right)dx = \left(\tan^{-1}y - x\right)dy$$

c. Examine the following differential equation is exact or not :

$$\left(3x + 2y\right)dx + \left(2x + y\right)dy = 0$$

d. Draw input-output compartmental diagram for CO_2.

e. Find the general solution of the differential equation

$$y'' - 4y = 0 .$$

f. Find the Wronskian of the functions x, x^2, x^3. Are they linearly independent ?

g. Write the formulation of the equation of limited growth with harvesting.

3. Answer the following questions : *(any five)*
$$5 \times 5 = 25$$

a. Show the relation $x^2 + y^2 - 25 = 0$ is an implicit solution of the differential equation $x + y\dfrac{dy}{dx} = 0$ on the interval I defined by $-5 < x < 5$.

b. Solve : $\left(3x^2 + 4xy\right)dx + \left(2x^2 + 2y\right)dy = 0$

c. Solve : $\left(x - 4\right)y^4 dx - x^3\left(y^2 - 3\right)dy = 0$

d. Solve the initial value problem (IVP) in equation $\dfrac{dN}{dt} = -kN ,$

with initial condition $N(t_0) = n_0$.

e. Find a differential equation for the amount of salt in the tank at any time t. (Note that concentration can be difined as the mass of salt per unit volume of mixture).

f. Using undetermined co-efficients method to solve

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = x^2 - 2x + 2.$$

g. Solve by the method of variation of parameter the equation

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$

h. Solve :

$$(D^2 - 2D + 5)y = e^{2x} \sin x.$$

(b) Write the differential equation of drug assimilation model of a single cold pill and solve it.

2+8=10

4. Answer the following questions : **(any one)**

10×1=10

(a) (i) Solve the following differential equation by finding an integrating factor

5

(ii) Write down Bernoulli's equation and solve it.

1+4=5

(5xy + 4y² + 1)dx + (x² + 2xy)dy = 0